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Econometric Model of “Promotion Bubble”: Identification, Analysis and Use

1. Characteristics of “Promotion Bubble” Phenomena

The phenomena of short term disturbances of price system are often observable on many markets, especially on consumer markets. Sales promotions are introduced by producers (so then it has global range and is correlated with strong advertisement in media), as well as markets chains and it can concern new product as well as stocked ones. Figure 1 presents graphs of prices of chosen washing powders expressed as value per mass unit. In given period total sales of small and medium-sized packaging of washing powders was 19681 items. Sales of chosen brands was about 81% of total number of sold items.
Because market is often consumer market, sales promotions are temporary and cyclical, which is the effect of dynamic stability of the market. As a result, temporary and aggressive price reductions can be treated as conscious attempt of destabilizing market. In this way producers try to make use of high (in absolute value) price elasticity and achieve short term increase of sales. Such situation we will call “promotion bubble”. Moment of appearing price promotion can be treated as an market impulse. Figure 2 presents two examples of “promotion bubbles”, which were observed for Vizir washing powder.
Identification the effect of “promotion bubble” need little care. Many sales promotions consist on adding bonus item to the main product or selling products in “2 in 1” or even “3 in 1” packages. Because of that, in order to identify the period of “promotion bubble”, prices should be recalculated and expressed as value of unit of mass, capacity, and so on…

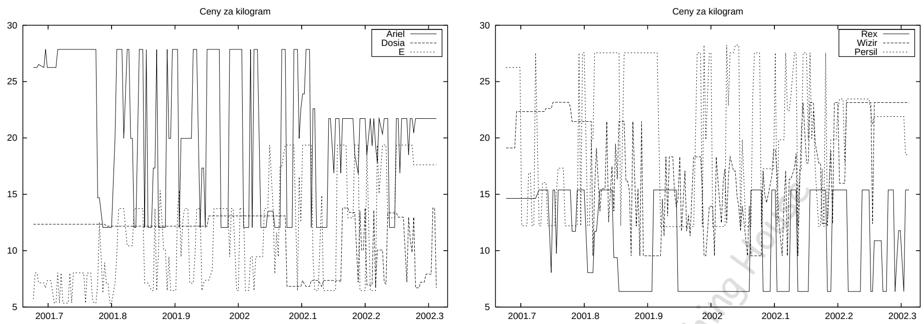


Fig. 1. Price per one kilogram for chosen washing powders in middle package group in 2001.09.05-2002.04.25 (n=200)
Source: Own study.

Periods with temporary price reductions and parallel increase of sale are just "promotion bubbles".

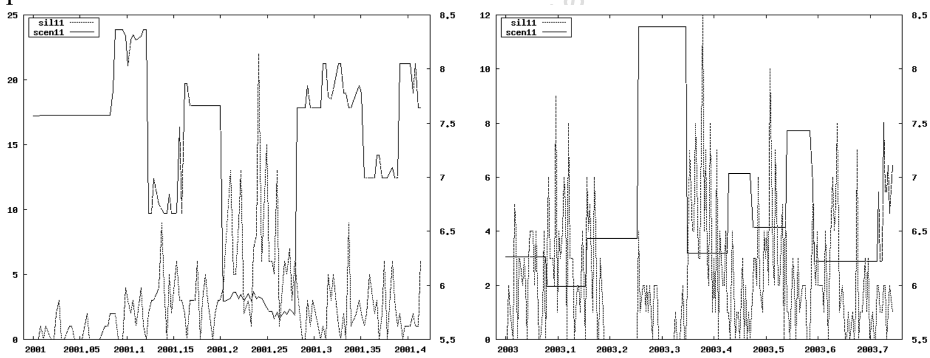


Fig. 2. Examples of promotion bubbles for Vizir washing powder
Source: Own study.

2. Modeling the Effects of "Promotion Bubbles"

2.1. Market Response Models and Impulse Response Functions

Treating "promotion bubble" as an impulse effect suggests use of econometric tools from wide range of market analyzing tools known as "market response models" (MRM). One of the MRM's ways of analyzing market is to treat the market as black box with many inputs and only one output. In such case market response model will be some function, which can be written as[1]:

$$Y_t = c + \sum_i \frac{A_i(L)}{B_i(L)} L^{b_i} X_{it} + \frac{\Theta(L)}{\Phi(L)} \varepsilon_t \qquad (1)$$

[1] See: Stawicki (1994), p. 22, Hanssens (1990), p. 140.

where:
$A(L), B(L)$ are lag operators of order, respectively, s and r,
X_{it} is a matrix of explanatory (input) variables,
ε_t is a white noise process.
All processes are stationary and invertible ARMA processes. It's worth to notice, that model (1) has in fact properties of conformable model in Zieliński's sense. Transfer function $\frac{A_i(L)}{B_i(L)} L^{b_i}$ is said to be of order (r, s, b). Model (1) can be written in less complicated, so called impulse response, form[2]:

$$Y_t = c + V_1(L)x_{1t} + V_2(L)x_{2t} + \ldots + \eta_t \quad (2)$$

$V_i(L)$ operators are finite, when r = 0 and infinite when r > 0, $\eta_t = \frac{\Theta(L)}{\Phi(L)} \varepsilon_t$. Estimation procedure of function (2) is explained in Stawicki (1994) and Hanssens (1990).
Model (1) and its response form (2) are single-equation models and it seems to be too simplified way of econometric modeling "promotion bubbles" effects. Number of relations between competitive products and different customer's reactions to short term price promotions[3] won't be possible to explain by single-equation model. Because system (market) is expected to be in equilibrium and market is consumer market, between different brands of products there will be strong substitution[4]. Furthermore, one should notice, that producers often sale several brands of the same product type simultaneously. In such case, producers introducing new promotions on market have to take into account disadvantageous of possible substitution between their own brands. Thus price impulse in one brand will have an influence on sale of all brands. Therefore, to get correct estimates of all parameters of MRM, model should be multi equation system. This multi equation system should include typical features of sales processes, i.e. nonstationarity in mean, particular analysis of periodicity[5].
Multi equation market response model can be based on two methodologies:

- structural multi equation model, where coefficients of market response function can be estimated by dynamic simulation or multiplier analysis,
- vector autoregressive models.

Presented examples are based upon VAR methodology.

[2] See: Hanssens (1994), p. 143.
[3] Often promotions last 2-3 days.
[4] What yields in high values of price elasticity of demand.
[5] Problem of identification internal structure of daily time series is described i.e. in Kufel (1997), p. 1–6.

2.2. Impulse Response Functions in VAR Models

Price impulses, introduced by producers or wholesalers, appear in different days of week. Because week periodicity in analyzed time series is noticeable, one can expect shifts of sales reactions (τ) in time. So it seems, that in order to model such market, VAR can be written as [6]:

$$Bx_t = \Gamma_0 D_t + \Gamma_1 x_{t-1} + \Gamma_2 x_{t-2} + \ldots + \Gamma_k x_{t-p} + \xi_t \tag{3}$$

where:
x_t vector of all processes included in model,
D_t vector of deterministic explanatory,
Γ_0 coefficients matrix of deterministic explanatory,
B coefficients matrix of temporary x_t values,
Γ_i matrix of coefficients of lagged x_t values,
ξ_t vector of error processes.
Multiplying equation (3) by B^{-1} and making following substitutions: $\Psi_i = B^{-1}\Gamma_i, i = 1,2,\ldots,p$ and $e_t = B^{-1}\xi_t$, yield in standard form VAR model:

$$x_t = \Psi_0 D_t + \Psi_1 x_{t-1} + \Psi_2 x_{t-2} + \ldots + \Psi_k x_{t-p} + e_t \tag{4}$$

Model (4) is then direct extension of impulse response form (2) in multi equation case. Using relationship between variance – covariance matrixes of models (3) and (4):

$$\Sigma_\xi = B\Sigma_e B^T$$

where:
Σ_ξ diagonal variance – covariance matrix of error processes of model (3),
Σ_e variance – covariance matrix of error processes of model (4),
model (4) can be rewritten in structural form with "orthogonal innovations"[7]. As a result, VAR in form (4) is estimated. Construction of impulse response functions is made by inverting VAR model into its infinite vector moving average representation VMA[8]:

$$x_t = \sum_{i=0}^{\infty} A_i e_{t-1} + \sum_{i=0}^{\infty} G_i \omega_{i-i}, i = 1,2,\ldots,T \tag{5}$$

where:
$A_i = \Phi_1 A_{i-1} + \Phi_2 A_{i-2} + \ldots + \Phi_p A_{i-p}, i = 1,2,\ldots$

[6] See: Kusideł (2000), p. 35 and Osińska (2002), p. 139.
[7] See: Kusideł (2000), p. 36 and Charemza, Deaman (1997), p. 156.
[8] See: Pesaran, Shin (1998), p. 18.

Coefficients of impulse response function can be estimated with use of Cholesky decomposition. In presented example all models and impulse response functions were estimated in GNU Regression, Econometrics and Time-series Library, (http://gretl.sourceforge.net).

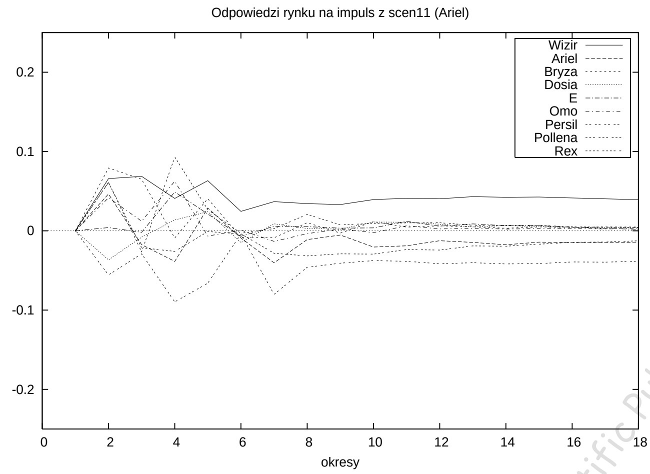
3. Empirical Example

Main assumption of this analysis is, that examined market is in equilibrium and after period of price promotion this market is able to back to this equilibrium. As a result, impulse response functions should diminish to zero after finite time. Such property of impulse response functions means, that modeled system is stable. Data used in an example come from one of Toruń's supermarket. Processes of daily sales data are divided into two groups: sales of small packages $(0;720g]$ and sales of middle packages $(720g;3kg]$. Two periods, 2001.09.05 - 2002.04.25 and 2003.04.10 - 2003.11.28 (n = 200), were analyzed. Two VAR models (VAR(3) and VAR(6)) with deterministic components were estimated, where order of polynomial of time and 0-1 matrixes for modeling periodicity were specified by analyzing internal structure of sales processes.
Results of estimation are satisfactory, error processes were not auto-correlated and had normal distribution[9]. Figure 3 presents reactions of sales in group of middle packages for chosen brands of washing powders as a response for 20% mean price impulses. Impulse response functions are presented in following configuration:

$$\textit{price impulse in one brand} \rightarrow \textit{response of all brands}$$

All estimated impulse response functions are diminishing to zero, which means that influence of price impulses are finite, so the whole system is stable.

[9] In several equations residuals were not normally distributed, but additional volatile could be modeled with use of ARCH.


Odpowiedzi rynku na impuls z scen11 (Ariel)
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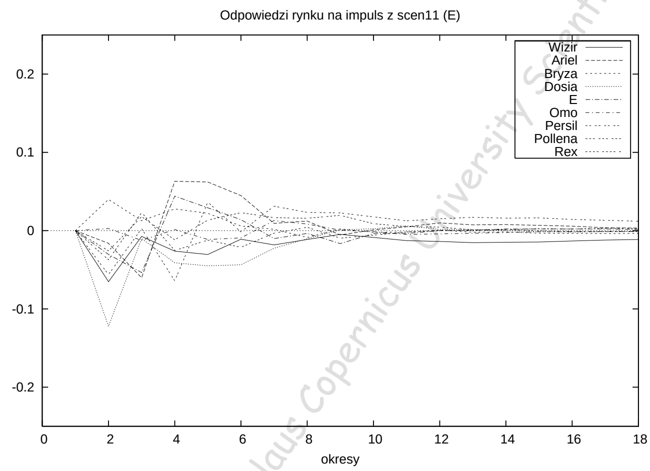
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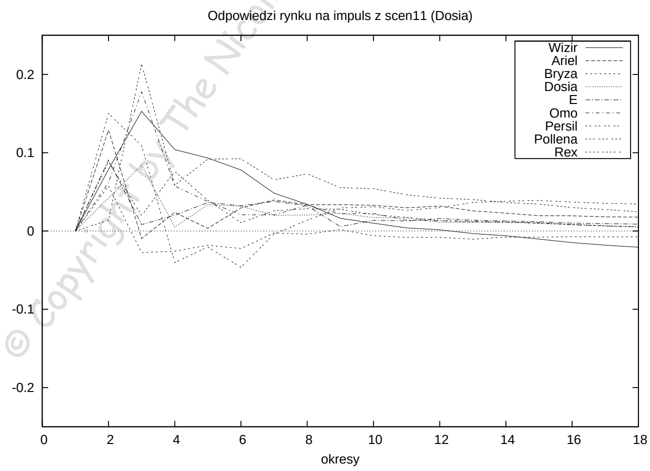
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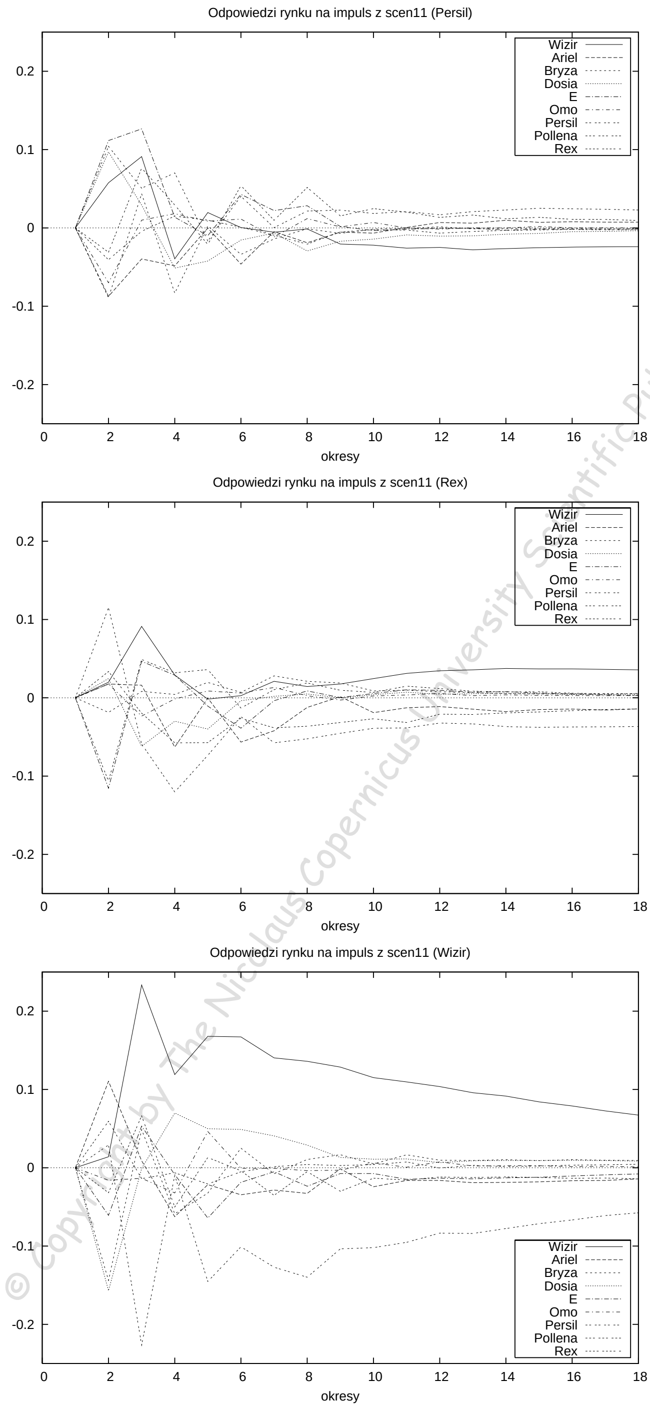

Fig. 3. Changes in quantity of sold middle packages of chosen 6 brands of washing powders in reaction to 20% price impulses in 2001.09.05-2002.04.25

4. Summary

In presented examples impulse introduced in price of only one brand had influence on sales of other brands for about 1,5 week (in 6 days week). Shifts of sales reactions in response to price impulse were up to 4 days. All impulse responses had tendency to diminish to zero, which means, that taken assumption, that analyzed market is in equilibrium, was correct. For the most endogenous variable (sales of Vizir), there were no simultaneous (with price impulse) reaction, but it was due to VAR specification. Week periodicity were confirmed with +0,249356 Friday effect. Unfortunately, results achieved with use of Cholesky decomposition depend on order of equations in the VAR model[10], which yield in “the most endogenous variable” and “the most exogenous variable”. To reduce disadvantages of it, one can try the following methods:

- make causality tests and reorder equations in model,
- make use of generalized impulse response functions defined by Pesaran and Shin,
- specifying structural model with simultaneous equations.

In most cases, causality test in Granger sense is used for VAR models. But this test “should be understand in the context of correlation between analyzed economic processes, so the decision whether examined relationship is causal is made by researcher”[11]. Maybe fallowing statement is true: “Immediate causality doesn’t exist, because there is always some time between independent actions”[12], so then multi equation model of analyzed market will never be the simultaneous equations system. But this not seems to be truth in case of consumer market, where analysis are based upon the daily sales data.

Presented example of analysis of “promotion bubble” effects is not exhaustive. Natural direction of next research is specification and estimation of structural model with simultaneous equations and comparing results, specially values of dynamic multipliers, with impulse response functions from VAR models.

Literature

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