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Identification of Non-linearity in Economic Time Series♦

1. Introduction

Employment of non-linear models describing data generating mechanisms becomes more and more frequent in modern econometrics. In this respect financial econometrics is one of the important domains of non-linear modeling (Doman, Doman (2004)). One of recently developed model specifications is related to stochastic unit roots processes STUR (Granger, Swanson (1997)). The mentioned processes constitute a class of random parameters models and should be understood as non-stationary neither in levels nor in differences of any order. It can be shown that after some re-arrangements STUR processes can be transformed to non-linear models like bi-linear or GARCH (Tsay(2002)).

In such a case, identification of non-linearity in empirical time series becomes the important problem. As there is a wide class of non-linear models generating slightly different processes, we often cannot distinguish among them. On the other hand, the tools of identification are not satisfactory yet (Bruzda (2002)).

The purpose of the paper is to compare identification capability of different methods applied to GARCH, stochastic volatility (SV), bilinear (BL) and STUR models *via* the Monte Carlo experiment. The white noise, SETAR and random walk models are also considered, as their typical properties are already known. We also applied the identification procedures to empirical time series: the Polish financial series and some macroeconomic ones. The results are presented in the final part of the article.

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2. The Methodology of Non-linearity Identification

In the literature one can find several tests for non-linear relationships in econometrics. Our attention is just turned to the methods related to identification of the non-linear models generating time series. We can divide the tests into two groups. We will include for the first group all those tests that do not formulate the exact form of non-linear function under the alternative. The most known representative is the BDS test (Brock, Hsieh, Scheinkman, LeBaron (1996)). The other tests in this group are based on the spectral representation of higher orders, like bi-covariance and bi-spectrum, for example the Hinich test. The second group of tests consists of those tools that verify linearity against exact alternative like smooth transition regression model or self-exciting threshold autoregressive model (Granger, Teräsvirta (1993)) as well as the stochastic unit roots tests. Hereby we use: the Hinich test (1982) and the STUR tests formulated by Leybourne, McCabe and Tremayne (1996), and Leybourne. McCabe and Mills (1996).

The Hinich test is based on the bi-spectrum characteristics and is used to detect the third order relationship in the time series. Let us assume that y_{t} is the realisation of the stochastic process with mean equal to zero and stationary up to the third order. The latter means that $\mathrm{E}\,|\,\mathrm{y}_{\mathrm{t}}\,|^3 = \mathrm{K} < \infty$ and moments $E(y_1, y_2, y_3)$ do not depend on any translation in time. Bi-spectrum $B_y(\omega_1\omega_2)$ of the time series y_t we define as the Fourier transform of the function $c_3(r,s) = E(y_t y_{t+r} y_{t+s})$ written as:

$$B_y(\omega_1\omega_2) = \sum_{r=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} c_3(r,s)\exp[-2\pi i(\omega_1 r + \omega_2 s)]. \tag{1}$$

The function (1) is a two-period function of the variables $(\omega_1\omega_2)$ with the basic domain $\{0 < \omega_2 < \frac{2}{3}\pi; \omega_2 < \omega_1 < -\frac{1}{2}\omega_2 + \pi\}$. The Hinich statistics is based on the bi-coherence coefficient, which estimator is given as:

$$\Psi^2(\omega_1\omega_2) = \frac{|\,\hat{B}_y(\omega_1\omega_2)|^2}{\hat{f}_y(\omega_1)\hat{f}_y(\omega_2)\hat{f}_y(\omega_{1+2})} \tag{2}$$

where $\hat{B}_y(\omega_1\omega_2)$ denotes estimator of the series bi-spectrum, and $\hat{f}_y(\omega_j)$ $j = \{1,2\}$ is a mean periodogram for frequency ω_{j}. The formula (2) is applied to hypotheses verification in two ways: first of all we test the distribution of the time series to be Gaussian, and in the next step those series which are not normally distributed, are checked for linearity.

In the former case, assuming the null to be true, the test statistics

$$H = 2\Psi^2(\omega_1\omega_2) \tag{3}$$

has asymptotic chi-squared distribution $\chi^2(2P)$, where P means the number of frequency pairs $(\omega_1\omega_2)$ within the basic domain. The acceptance of the null brings us to the end of research. When, however, the null is rejected, we further check, whether the series is linear or not. This means that the time series may be non-Gaussian but linear or non-Gaussian and non-linear. When the null of time series linearity is true, the statistics (3) has non-centered chi-squared distribution $\chi^2(2,\lambda)$, where λ is the mean of the series. Then the estimator $2\Psi^2(\omega_1\omega_2)$ provides P independent values from the $\chi^2(2,\lambda)$ distribution. If then the examined series is actually linear, the sample dispersion is consistent with the distribution dispersion. On the other hand the statistics $2\Psi^2(\omega_1\omega_2)$ brings P independent values form non-centered χ^2 distribution which changing mean value and the dispersion of such a distribution is obviously different. The testing procedure consists in the comparison of dispersion of both distributions, using for example the deciles range or quartiles range. The latter was used in the presented research. The null hypothesis of linearity is rejected when the empirical and theoretical values of the quartiles range are significantly different. Hinich (1982) has shown that the test does not require pre-filtering of linear relationship.

In the paper concerning the STUR identification, Leybourne, McCabe and Mills (1996) suggest the following simple random coefficient autoregressive model describing a stochastic unit root:

$$y_t = \alpha_t y_{t-1} + \varepsilon_t \tag{4}$$

where:

$$\begin{aligned} &\alpha_t = \alpha_0 + \delta_t \\ &\delta_0 = 0 \\ &\delta_t = \rho\delta_{t-1} + \eta_t \\ &\alpha_0 = 1 \text{ and } |\rho| \le 1. \end{aligned} \tag{5}$$

Stochastic processes $\varepsilon_t \sim N(0,\sigma^2)$ and $\eta_t \sim N(0,\omega^2)$ are assumed to be independent. If $|\rho| < 1$, then α_t constitutes the AR(1) with mean equal to one, and for $\rho = 1$ it is a random walk. The latter is true also for $\alpha_0 = 1$ and $\omega^2 = 0$. If $\alpha_0 = 1$ and $\omega^2 > 0$ a process with a unit root in mean, called a stochastic unit root process is observed.

Leybourne, McCabe and Tremayne (1996) have proposed a testing procedure (LMT test). Under the null the exact unit root is tested, while under the alternative the stochastic unit root is assumed (see also Leybourne, McCabe and Mills (1996)). Hypotheses in the LMT test concern the variance ω^2 in the model (5). The null is $H_0 : \omega^2 = 0$, that means the random walk process or

ARIMA(p,1,0), while the alternative is $H_1 : \omega^2 > 0$. The interpretation of the alternative depends on the value of the ρ parameter in (5). When $|\rho| < 1$, δ_t is a stationary process with a zero mean, for $\rho = 1$ it follows a random walk.

To avoid the influence of deterministic trend and autocorrelation, the model may include the linear or quadratic time trend, and the autoregressive lags of the dependent variable as well, so it takes the following form:

$$y_t^* = \alpha_t y_{t-1}^* + \varepsilon_t \tag{6}$$

where:

$$y_t^* = y_t - P_t - \sum_{i=1}^{p} \varphi_i y_{t-i} \tag{7}$$

P_t means a deterministic component, usually trend in the following forms:

$P_{1t} = \beta + \gamma t$ or $P_{2t} = \beta + \gamma t + \theta t(t+1)/2$.

The autoregressive part in (7) is stationary and its role is similar to the augmentation in the ADF test.

If in H_1 $|\rho| < 1$ then the *Z* statistic is computed in the following way:

1. estimate the equation ordinary last squares (OLS)

$$\Delta y_t = \Delta P_t + \sum_{i=1}^{p} \varphi_i \Delta y_{t-i} + \varepsilon_t \tag{8}$$

2. compute the statistics

$$Z = T^{-\frac{3}{2}} \sigma^{-2} \kappa^{-1} \sum_{t=2}^{T} \left(\sum_{j=1}^{t-1} \varepsilon_j \right)^2 \left(\varepsilon_t^2 - \sigma^2 \right) \tag{9}$$

where: $\sigma^2 = T^{-1} \sum_{t=1}^{T} \varepsilon_t^2$ and $\kappa^2 = T^{-1} \sum_{t=1}^{T} \left(\varepsilon_t^2 - \sigma^2 \right)^2$.

If in H_1 $\rho = 1$ then the following *E* statistic is preferred (see Leybourne, McCabe and Mills (1996)):

$$E = T^{-3} \sigma^{-4} \sum_{i=2}^{T} \left\{ \left[\sum_{t=i}^{T} \varepsilon_t \left(\sum_{j=1}^{t-1} \varepsilon_j \right) \right]^2 - \sigma^2 \sum_{t=i}^{T} \left(\sum_{j=1}^{t-1} \varepsilon_j \right)^2 \right\} \tag{10}$$

Depending on the trend model choice P_{1t} or P_{2t} the statistics are denoted Z_1, Z_2 or E_1, E_2 respectively. The distribution of the tests does not converge to any standard distribution, so critical values computed by the authors of the tests were used (see also Osińska (2004)).

Generated and empirical time series are additionally examined using standard descriptive statistics. For STUR processes their values are much more higher than for the remained models. This fact confirms the explosive characteristics of the process. Moreover the autocorrelation in levels using the Box-Ljung test and in squares using the Engle and McLeod-Li tests was checked. These results are not presented here to save space, but are available form the authors on request.

3. Monte Carlo Experiment

The aim of simulation is defined as sensitivity analysis of the tests for stochastic unit root. Different forms of the alternatives are used, while the null always means the exact unit root. Very similar research for chosen alternatives was made by Taylor and van Dijk (1999). Additionally the standard Dickey-Fuller test was computed. The next part of analysis is based on testing for Gaussianity and linearity of the time series under study, using Hinich test. The properties of the tests based on bi-spectrum were analyzed in Bruzda (2002) and Doman, Doman (2004), but stochastic unit root processes were not considered so far. One possible reason is that spectral methods require stationarity of the processes up to the order of the moment examined. The STUR processes by definition do not possess such a property. Despite of this we made an attempt at the Hinich test application for these models, just to determine the limits to linearity identification of the empirical time series generated by STUR.

The following data generating models were considered:

WN	ε_t	$\varepsilon_t \sim N(0,1)$
RW	$y_t = y_{t-1} + \varepsilon_t$	$\varepsilon_t \sim N(0,1)$
BI[1]	$y_t = \sigma_t \cdot u_t$	$\varepsilon_t \sim N(0,1)$
	$\sigma_t = \sigma_{t-1} + \varepsilon_t$	$u_t \sim N(0,1)$
BL diag	$y_t = -1 - 0.5 y_{t-1}\varepsilon_{t-1} + \varepsilon_t$	$\varepsilon_t \sim N(0,1)$
BL nad	$y_t = -1 - 0.5 y_{t-2}\varepsilon_{t-1} + \varepsilon_t$	$\varepsilon_t \sim N(0,1)$
BL pod	$y_t = -1 - 0.5 y_{t-1}\varepsilon_{t-2} + \varepsilon_t$	$\varepsilon_t \sim N(0,1)$
ARCH(2)	$y_t = \varepsilon_t \sqrt{h_t}$	$\varepsilon_t \sim N(0,1)$
	$h_t = 0{,}1 + 0.5\varepsilon_{t-1}^2 + 0.4\varepsilon_{t-2}^2$	
GARCH(1,1)	$y_t = \varepsilon_t \sqrt{h_t}$	$\varepsilon_t \sim N(0,1)$
	$h_t = 0{,}1 + 0.89 h_{t-1} + 0.1\varepsilon_{t-1}^2$	
SV[2]	$y_t = \varepsilon_t \sqrt{h_t}$	$\varepsilon_t \sim N(0,1)$
	$\ln h_t = 0{,}1 + 0.95 \ln h_{t-1} + \sqrt{0.009}\eta_t$	$\eta_t \sim N(0,1)$
SETAR(2,1,2)	$y_t = \begin{cases} 0.5 - 0.22 y_{t-1} + 0.1\varepsilon_t & y_{t-2} < 0.07 \\ 0.82 y_{t-1} + 0.4\varepsilon_t & y_{t-2} \geq 0.07 \end{cases}$	$\varepsilon_t \sim N(0,1)$

[1] The model was defined in Hansen (1992).
[2] The SV model was generated according to Pajor (2003).

RCA(1,1)	$y_t = b_t \cdot \varepsilon_t$	$\varepsilon_t \sim N(0,1)$
	$b_t = 0.3 b_{t-1} + 0.6\eta_t$	$\eta_t \sim N(0,1)$
STUR1	$\alpha_0 = 1$, $\rho = 0.98$, $\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$
STUR2	$\alpha_0 = 1$, $\rho = 0.95$, $\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$
STUR3	$\alpha_0 = 0.98$, $\rho = 0.98$, $\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$
STUR4	$\alpha_0 = 0.98$, $\rho = 0.95$, $\omega^2 = 0.01$	$\varepsilon_t \sim N(0,1)$
STUR5	$\alpha_0 = 1$, $\rho = 0.98$, $\omega^2 = 0.05$	$\varepsilon_t \sim N(0,1)$
STUR6	$\alpha_0 = 1$, $\rho = 0.95$, $\omega^2 = 0.05$	$\varepsilon_t \sim N(0,1)$
STUR7	$\alpha_0 = 0.98$, $\rho = 0.98$, $\omega^2 = 0.05$	$\varepsilon_t \sim N(0,1)$
STUR8	$\alpha_0 = 0.98$, $\rho = 0.95$, $\omega^2 = 0.05$	$\varepsilon_t \sim N(0,1)$
STUR9	$\alpha_0 = 1$, $\rho = 0.98$, $\omega^2 = 0.1$	$\varepsilon_t \sim N(0,1)$
STUR10	$\alpha_0 = 1$, $\rho = 0.95$, $\omega^2 = 0.1$	$\varepsilon_t \sim N(0,1)$
STUR11	$\alpha_0 = 0.98$, $\rho = 0.98$, $\omega^2 = 0.1$	$\varepsilon_t \sim N(0,1)$
STUR12	$\alpha_0 = 0.98$, $\rho = 0.95$, $\omega^2 = 0.1$	$\varepsilon_t \sim N(0,1)$

For each of the above models 500 observations were generated in 1000 replications. For STUR models we assumed linear or quadratic time trend and 5 autoregressive lags, respectively. The results of Z and E statistics and the Hinich test are presented in the table 1.

In the cases of stationary time series – the white noise, bi-integrated model, bilinear model, ARCH(2), GARCH(1,1), SV model, RCA(1,1) and SETAR model - the STUR tests accept the null hypothesis. This is rather obvious, because there is no stochastic unit root there. For non-stationary time series represented by random walk the Leybourne et all tests definitely indicate the exact unit root (the null). In the cases of time series generated as STUR the tests Z and E conclusions are rather promising. Depending on the values of the parameters, the ratios on rejecting the null are between 73% and 96%.

The Hinich test performs in a different way. Some of the results are not very reliable. It concerns mostly bi-linear processes but also ARCH, GARCH and SV models of conditional variance. For the STUR processes we observe non-Gaussianity and non-linearity almost in 100% of cases, however, this optimistic result may be biased by non-stationarity of the STUR processes.

The next experiment was designed to compare stochastic unit roots models. Several different specifications were generated, with different values of the variance ω^2, which is responsible for the parameter variability model and in fact for the presence of the STUR. We analysed data in the following panels: n=50, 100, 250, 500 and 1000 observations in 1000 replications. The results, excluding n=50, are shown in the table 2. All simulations were made for $\rho = 0{,}98$ (the results for other values of ρ are available on request).

Table 1. The results of the Z, E and Hinich tests. The ratios of rejection of the null are reported at 0.05 significance level

Model	Z		E		Hinich	
	Z1	Z2	E1	E2	Gaussianity	Linearity
WN	0.000	0.000	0.000	0.000	0.000	--
RW	0.170	0.060	0.000	0.000	--	--
BI	0.026	0.008	0.001	0.001	--	--
BL diag	0.109	0.085	0.003	0.002	0.647	0.445
BL nad	0.006	0.002	0.000	0.000	0.189	0.328
BL pod	0.108	0.105	0.004	0.002	0.590	0.412
ARCH(2)	0.163	0.165	0.025	0.021	0.703	0.703
GARCH(1,1)	0.012	0.008	0.000	0.000	0.087	0.322
SV	0.000	0.000	0.000	0.000	0.037	0.405
SETAR	0.004	0.000	0.000	0.000	0.391	0.205
RCA(1,1)	0.042	0.036	0.002	0.002	0.145	0.538
STUR1	0.851	0.919	0.793	0.855	0.996	0.996
STUR2	0.892	0.942	0.823	0.852	1.000	0.995
STUR3	0.845	0.923	0.794	0.851	0.997	0.997
STUR4	0.895	0.938	0.839	0.863	1.000	0.993
STUR5	0.885	0.940	0.806	0.835	0.999	1.000
STUR6	0.913	0.953	0.855	0.868	1.000	0.999
STUR7	0.895	0.929	0.759	0.786	1.000	0.998
STUR8	0.916	0.953	0.842	0.848	1.000	1.000
STUR9	0.866	0.926	0.741	0.755	1.000	0.998
STUR10	0.927	0.960	0.827	0.853	1.000	1.000
STUR11	0.878	0.930	0.732	0.745	1.000	0.996
STUR12	0.922	0.957	0.839	0.841	1.000	1.000

The results of the tests comparison show that the power and size of the analysed tests are better for large observation number, that is n=1000. The E test rejects the null less often than the Z test. The Hinich test does not consequently accept the Gaussianity hypothesis in STUR models, independently of the observations number. The hypothesis of linearity is also fairly rejected. It is also important, that the McLeod and Li test shows the presence of GARCH.

Table 2. The results of the Z, E and Hinich tests for STUR models. The ratios of rejection of the null are reported at 0.05 significance level

Model		DF	McLeod Li	Z		E		Hinich	
ω^2	α_0		24	Z1	Z2	E1	E2	Gaussianity	Linearity
n=100									
0.01	1	0.243	0.757	0.507	0.610	0.476	0.510	0.975	0.870
0.01	0.98	0.232	0.767	0.501	0.604	0.461	0.516	0.970	0.879
0.05	1	0.311	0.658	0.580	0.704	0.553	0.605	0.991	0.893
0.05	0.98	0.280	0.695	0.611	0.725	0.576	0.633	0.993	0.905
0.1	1	0.514	0.689	0.680	0.781	0.622	0.644	0.998	0.948
0.1	0.98	0.465	0.684	0.651	0.755	0.611	0.645	0.999	0.936
0.01	1.02	0.185	0.757	0.511	0.630	0.477	0.542	0.981	0.893
0.01	1.05	0.177	0.755	0.533	0.655	0.517	0.579	0.973	0.909
n=250									
0.01	1	0.096	0.915	0.732	0.841	0.711	0.784	0.956	0.931
0.01	0.98	0.137	0.932	0.737	0.833	0.674	0.733	0.938	0.942
0.05	1	0.629	0.826	0.798	0.879	0.712	0.750	0.981	0.941
0.05	0.98	0.635	0.853	0.791	0.875	0.720	0.763	0.980	0.948
0.1	1	0.790	0.844	0.797	0.873	0.674	0.709	0.996	0.960
0.1	0.98	0.796	0.828	0.807	0.868	0.689	0.714	0.999	0.963
0.01	1.02	0.097	0.931	0.722	0.844	0.693	0.781	0.957	0.922
0.01	1.05	0.100	0.911	0.725	0.845	0.669	0.771	0.953	0.945
n=500									
0.01	1	0.337	0.976	0.851	0.919	0.793	0.855	0.996	0.996
0.01	0.98	0.358	0.980	0.845	0.923	0.794	0.851	0.997	0.997
0.05	1	0.840	0.925	0.885	0.940	0.806	0.835	0.999	1.000
0.05	0.98	0.864	0.932	0.895	0.929	0.759	0.786	1.000	0.998
0.1	1	0.874	0.912	0.866	0.926	0.741	0.755	1.000	0.998
0.1	0.98	0.884	0.919	0.878	0.930	0.732	0.745	1.000	0.996
0.01	1.02	0.346	0.982	0.845	0.918	0.783	0.837	0.995	0.995
0.01	1.05	0.338	0.980	0.835	0.913	0.773	0.830	0.998	0.995
n=1000									
0.01	1	0.800	0.998	0.907	0.951	0.839	0.874	1.000	0.997
0.01	0.98	0.820	0.997	0.921	0.954	0.853	0.874	1.000	1.000
0.05	1	0.907	0.971	0.930	0.966	0.824	0.846	1.000	1.000
0.05	0.98	0.925	0.977	0.947	0.976	0.844	0.847	1.000	1.000
0.1	1	0.899	0.940	0.897	0.937	0.769	0.768	1.000	1.000
0.1	0.98	0.913	0.938	0.936	0.961	0.790	0.777	1.000	1.000
0.01	1.02	0.816	0.997	0.904	0.955	0.863	0.886	1.000	0.998
0.01	1.05	0.795	0.994	0.906	0.952	0.854	0.885	1.000	0.997

4. The Empirical Results

We applied the above procedures of identification to economic time series. First of all we tried to identify the financial indices and their returns (in logs) observed daily at The Stock Exchange in Warsaw in 2.01.01 – 13.02.04. The results are presented in the table 3.

Table 3. The result of testing financial indices

Market index	Z		E		Hinich	
	Z1	Z2	E1	E2	Gaussianity	Linearity
WIG	-0.901	0.088	-0.107	0.023	---	---
Wieg-Return	0.015	0.015	0.002	0.002	5.516	---
WIG20	0.785*	0.667*	0.808*	0.627*	---	---
WIG20-Return	0.069	0.069	0.051	0.051	7.914	---
WIG-BANK	-0.295	-0.022	-0.010	-0.001	---	---
WIG-BANK-Return	0.024	0.024	0.001	0.001	7.229	---
WIG-BUDOW	-0.558	-0.372	-0.011	-0.010	---	---
WIG-BUDOW-Return	0.011	0.016	0.001	0.001	5.717	---
WIG-INFO	-0.174	0.222*	-0.048	0.000	---	---
WIG-INFO-Return	0.064	0.057	0.001	0.001	14.187	---
WIG-SPOZ	-1.792	0.198*	-0.047	0.018	---	---
WIG-SPOZ-Return	0.041	0.036	0.001	0.001	14.785	---
WIG-TELKOM	0.493*	0.374*	-0.133	-0.006	---	---
WIG-TELKOM-Return	0.035	0.036	0.001	0.001	12.929	---

* denotes stochastic unit root

The results of testing the economic time series bring us to the conclusion that the following indices: WIG20, WIG-info, WIG-spoz and WIG-telkom possess a stochastic unit root. The Hinich test for indices was not computed, because they are non-stationary. For logarithmic rates of return in any case we cannot reject the null, which seems to be rather strange, since other tests for normality shows that the return distributions are not Gaussian.

Another example was related to the following macroeconomic time series: nominal wages, cpi, inflation, money supply M2, production in industry and unemployment rate, observed monthly, within ten years period (1994-2003). We considered the data seasonally adjusted and non-adjusted. As the conclusions remained almost the same, the results for original series are presented in the table 4. Having in mind that macroeconomic time series are non-stationary we do not present the Hinich tests results. However we can state that stochastic unit roots are present in some of the examined series. This fact cannot be ignored in the econometric modelling and forecasting, because exact unit root methodology is no longer valid in such a case. Random coefficient models or non-linear types of models should be than used for modelling a STUR type data.

Table 4. The results of STUR testing in macroeconomic time series

Series	Z		E	
	Z1	Z2	E1	E2
Wages	0.536*	0.381*	0.075*	0.056
CPI	0.016	0.020	-0.003	-0.001
Inflation	0.097	0.088	0.001	0.002
M2	2.276*	0.625*	0.407*	0.154*
Production in industry	0.144	0.138	0.023	0.021
Unemployment Rate	-0.092	0.0586	-0.005	0.009

* denotes stochastic unit root

References

Brock, W. A., Hsieh, D. A., Scheinkman, J. A., LeBaron, B. (1996) A test for independence Based on the Correlation Dimension, *Econometric Review*, v. 15.

Bruzda, J. (2002), Bispektra procesów ekonomicznych – kierunki zastosowań i analiza symulacyjna (Bi-spectra of economic processes – directions of applications and simulation analysis), *Acta Universitatis Nicolai Copernici*, Toruń.

Doman, M., Doman, R. (2004), *Ekonometryczne modelowanie dynamiki polskiego rynku finansowego (Econometric modelling of dynamics of Polish financial market),* Wyd. AE w Poznaniu, Poznań.

Granger, C. W. J., Swanson, N. R. (1997), An introduction to stochastic unit-root process, *Journal of Econometrics*, v. 80.

Granger, C. W. J., Teräsvirta, T. (1993) *Modelling Nonlinear Economic Relationships*. Oxford University Press.

Hansen, B. E. (1992), Heteroskedastic cointegration, *Journal of Econometrics*, vol. 54.

Hinich, M. J. (1982), Testing for Gaussianity and Linearity of a Stationary Time Series, *Journal of Time Series Analysis*, v. 3.

Leybourne, S. J., McCabe, B. P. M., Mills, T. C. (1996), Randomized unit root processes for modeling and forecasting financial time series: theory and applications, *Journal of Forecasting*, v. 15.

Leybourne, S. J., McCabe, B. P. M., Tremayne, A. R. (1996), Can economic time series be differenced to stationarity?, *Journal of Business and Economic Statistics*, v. 14

Osińska, M. (2004), Stochastic unit roots processes - properties and application, in *MACROMODELS'2003*, red. A. Welfe, W. Welfe, Wyd. Uniwersytetu Łódzkiego, Łódź.

Pajor, A. (2003), *Procesy zmienności stochastycznej SV w bayesowskiej analizie szeregów czasowych (Stochastic volatility processes in Bayesian analysis of time series),* Wydawnictwo AE , Kraków.

Taylor, A. M. R, van Dijk, D. (1999), Testing for Stochastic Unit Roots. Some Monte Carlo Evidence, *Econometric Institute Research Report EI-9922/A*.

Tsay R. S. (2002), *Analysis of Financial Time Series.* Wiley.