**DYNAMIC ECONOMETRIC MODELS**
Vol. 7 – Nicolaus Copernicus University –Toruń – 2006

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# Bayesian Analysis of Main Bivariate GARCH and SV models for PLN/USD and PLN/DEM (1996-2001)

## 1. Introduction

In a previous Bayesian comparison of 16 GARCH and 4 SV (stochastic variance) bivariate models Osiewalski, Pajor and Pipień (2006) have shown that even simple SV specifications fit the data much better than very sophisticated GARCH structures. This phenomenon can be attributed to describing volatility by latent AR(1) processes, which is the main feature of the SV class and yields its distributional flexibility and ease in modelling outliers.

The aim of the paper is to present and compare posterior inferences (for the main quantities of interest) obtained using different models. Here we take into account our previous results on model comparison and focus only on three leading SV specifications and two representative GARCH structures: the best one, i.e. the *t*-BEKK(1,1) model, and the parsimonious *t*-DCC model, based on the one proposed by Engle (2002). As in our previous papers, we use Markov chain Monte Carlo (MCMC) techniques to conduct our Bayesian approach and, for the sake of comparison, the daily growth rates of two exchange rates: PLN/USD and PLN/DEM (6.02.1996-31.12.2001). We show that sequences of estimates of the conditional standard deviations and correlation coefficients can be moderately similar for good SV and reasonable GARCH models, despite huge differences in model fit and incomparability of the conditional covariance matrices (we condition on different variables in GARCH and SV models).

Due to space limitations, we do not present the Bayesian methodology, described in our other papers. In the next section we review the main models from previous studies. In section 3 we summarise the results of the Bayesian model comparison. Section 4 is devoted to the presentation and comparison of posterior results obtained within particular models.

## 2. Main Bayesian Models from the SV and GARCH Classes

We denote by $y_t = (y_{1,t} \ \ y_{2,t})'$ bivariate observations on growth (or return) rates, and we model them using the basic VAR(1) framework:

$$y_t - \delta = R(y_{t-1} - \delta) + \varepsilon_t \tag{1}$$

with $\varepsilon_t$ described by competing bivariate time-varying volatility processes. More specifically,

$$\begin{pmatrix} y_{1,t} \\ y_{2,t} \end{pmatrix} - \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \left\{ \begin{pmatrix} y_{1,t-1} \\ y_{2,t-1} \end{pmatrix} - \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \right\} + \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix}, \quad t=1,\dots,T.$$

The elements of $\delta$ and $R$ are common parameters, which we treat as a priori independent of all other (model-specific) parameters and assume for them the multivariate standardised Normal prior $N(0, I_6)$, truncated by the restriction that all eigenvalues of $R$ lie inside the unit circle.

### 2.1. Bivariate Stochastic Volatility Specifications

Assume that $\varepsilon_t$ in (1) is conditionally Normal (given parameters and latent variables in $\theta_{(i)}$) with mean vector 0 and covariance matrix $\Sigma_t$, depending on latent variables, i.e.

$$\varepsilon_t \mid M_i, \theta_{(i)} \sim N(0_{[2\times 1]}, \Sigma_t).$$

Thus, the corresponding conditional distribution of $y_t = (y_{1,t} \ \ y_{2,t})'$ (given its past **and** latent variables) is Normal with mean $\mu_t = \delta + R\cdot(y_{t-1} - \delta)$ and covariance matrix $\Sigma_t$. The competing bivariate SV models $M_i$ are defined through different specifications of the latent processes and different structures of $\Sigma_t$ (always, by construction, positive definite symmetric).

#### 2.1.1. The Stochastic Discount Factor Model (SDF)

This simplest MSV specification ($M_1$) uses just one latent process $g_t$ to describe the dynamics of the whole conditional covariance matrix (see Jacquier, Polson and Rossi (1995)):

$$\varepsilon_t = \xi_t \sqrt{g_t}, \quad \ln g_t = \gamma + \phi(\ln g_{t-1} - \gamma) + \sigma_g \eta_t,$$
$$\xi_t \sim iiN(0_{[2\times 1]}, H), \quad \eta_t \sim iiN(0,1), \quad \xi_t \perp \eta_s; \ t, s \in Z.$$

The conditional covariance matrix of $\varepsilon_t$ takes the very simple form

$$\Sigma_t = g_t H = \begin{bmatrix} g_t h_{11} & g_t h_{12} \\ g_t h_{12} & g_t h_{22} \end{bmatrix},$$

which leads to the invariable conditional correlation coefficient $\rho_{12,t} = \rho = h_{12} / \sqrt{h_{11} h_{22}}$.

In order to ensure identifiability, the restriction $\gamma$=0 is imposed, while $H$ is a symmetric positive definite matrix consisting of three free entries. We assume independence among parameters and use the following prior distributions: $H \sim IW(2I_2, 2)$, i.e. inverse (or inverted) Wishart with 2 degrees of freedom and parameter matrix $2I_2$; $\sigma_g^2 \sim IG(0.01, 2)$, an inverse Gamma distribution; $\phi \sim N(0, 100)\ I_{(-1,1)}(\phi)$, i.e. Normal with mean 0 and variance 100, truncated to (-1, 1); and $\ln(g_0) \sim N(0, 100)$. Here we use the same parameterisation of the inverse Wishart and Gamma class as O'Hagan (1994). That is, for the hyperparameter values chosen here, the prior expectations of $H$ and $\sigma_g^2$ do not exist. Equivalently, $H^{-1}$ has a Wishart prior with mean $I_2$, and $\sigma_g^{-2}$ has a Gamma prior with mean 200 and variance 40000.

#### 2.1.2. The JSV Model

The SDF specification is very restrictive since it assumes the same dynamics for all entries of the conditional covariance matrix. The JSV model ($M_2$), proposed by Pajor (2005b) and based on the spectral decomposition of $\Sigma_t = P\,\Lambda_t P^{-1}$, uses a separate latent process to describe each eigenvalue of $\Sigma_t$. More specifically, $\Lambda_t$ is the diagonal matrix of eigenvalues of $\Sigma_t$, $\Lambda_t = Diag(\lambda_{1,t}, \lambda_{2,t})$, and $P$ is the orthogonal matrix of eigenvectors of $\Sigma_t$, depending on $p_{11}$, a parameter from (0, 1]:

$$P = \begin{bmatrix} p_{11} & \sqrt{1-p_{11}^2} \\ \sqrt{1-p_{11}^2} & -p_{11} \end{bmatrix}.$$

Hence, the conditional covariance matrix of $\varepsilon_t$ has a non-trivial structure:

$$\Sigma_t = \begin{bmatrix} \lambda_{1,t} p_{11}^2 + \lambda_{2,t}(1-p_{11}^2) & (\lambda_{1,t}-\lambda_{2,t})p_{11}\sqrt{1-p_{11}^2} \\ (\lambda_{1,t}-\lambda_{2,t})p_{11}\sqrt{1-p_{11}^2} & \lambda_{2,t} p_{11}^2 + \lambda_{1,t}(1-p_{11}^2) \end{bmatrix},$$

which leads to the varying conditional correlation coefficient:

$$\rho_{12,t} = \frac{(\lambda_{1,t}-\lambda_{2,t})p_{11}\sqrt{1-p_{11}^2}}{\sqrt{(\lambda_{1,t}-\lambda_{2,t})^2 p_{11}^2(1-p_{11}^2) + \lambda_{1,t}\lambda_{2,t}}}.$$

Note that we have a pair of positive latent processes, $\Theta_t = (\lambda_{1,t}, \lambda_{2,t})'$, where

$$\ln\lambda_{1,t} - \gamma_{11} = \phi_{11}(\ln\lambda_{1,t-1} - \gamma_{11}) + \sigma_{11}\eta_{1,t},$$
$$\ln\lambda_{2,t} - \gamma_{22} = \phi_{22}(\ln\lambda_{2,t-1} - \gamma_{22}) + \sigma_{22}\eta_{2,t},$$
$$\eta_t = (\eta_{1,t}, \eta_{2,t})', \qquad \eta_t \sim iiN(0_{[2\times 1]}, I_2).$$

We impose prior independence and assume the following priors: $(\gamma_{ii}, \phi_{ii})' \sim N(0, 100I_2)\ I_{(-1,1)}(\phi_{ii})$; $\sigma_{ii}^2 \sim IG(0.01, 2)$; $\ln\lambda_{i,0} \sim N(0, 100)$, $i = 1, 2$; $p_{11} \sim U([0, 1])$, where $U(A)$ is the uniform distribution over $A$.

### 2.1.3.The TSV Model

The non-trivial structure of the JSV conditional covariance matrix is based on as many latent variables as there are time series under consideration. Hence, the covariance dynamics is not completely free, as it is related to volatilities. The third MSV model ($M_3$), proposed by Tsay (2002) – thus called TSV – and used by Pajor (2005a, 2006), is based on as many separate latent processes as there are distinct elements of the conditional covariance matrix. The TSV model relies on the Cholesky decomposition $\Sigma_t = L_t\, G_t L_t'$, where

$$L_t = \begin{bmatrix} 1 & 0 \\ q_{21,t} & 1 \end{bmatrix} \; G_t = \begin{bmatrix} q_{11,t} & 0 \\ 0 & q_{22,t} \end{bmatrix},$$

$$\ln q_{11,t} - \gamma_{11} = \phi_{11}(\ln q_{11,t-1} - \gamma_{11}) + \sigma_{11}\eta_{11,t}\,,$$

$$\ln q_{22,t} - \gamma_{22} = \phi_{22}(\ln q_{22,t-1} - \gamma_{22}) + \sigma_{22}\eta_{22,t}\,,$$

$$q_{21,t} - \gamma_{21} = \phi_{21}(q_{21,t-1} - \gamma_{21}) + \sigma_{21}\eta_{21,t}\,,$$

$$\eta_t = (\eta_{11,t}, \eta_{21,t}, \eta_{22,t})', \qquad \eta_t \sim iiN(0_{[3\times 1]}, I_3)\,.$$

Now $\Theta_t = (q_{11,t},\ q_{22,t},\ q_{21,t})'$ is a trivariate latent process with two positive components, and the conditional covariance matrix of $\varepsilon_t$ takes the form:

$$\Sigma_t = \begin{bmatrix} q_{11,t} & q_{11,t}q_{21,t} \\ q_{11,t}q_{21,t} & q_{11,t}q_{21,t}^2 + q_{22,t} \end{bmatrix},$$

which leads to the following variable conditional correlation coefficient:

$$\rho_{12,t} = \frac{q_{21,t}\sqrt{q_{11,t}}}{\sqrt{q_{22,t} + q_{11,t}q_{21,t}^2}}\,.$$

We make similar assumptions about the prior structure as previously:
$(\gamma_{ij},\ \phi_{ij})' \sim N(0, 100I_2)\, I_{(-1,1)}(\phi_{ij})$, $\quad \sigma_{ij}^2 \sim IG(0.01, 2) \quad (i,j \in \{1,2\},\ i \geq j)$;
$\ln q_{ii,0} \sim N(0, 100)$, $i = 1, 2$; $\quad q_{21,0} \sim N(0, 100)$.

Finally, note that the diagonal entries of $\Sigma_t$ are not modelled in a symmetric way and, thus, the order of appearance (numbering) of financial instruments matters in the TSV specification, contrary to other models.

## 2.2. Bivariate GARCH Specifications

We assume that the conditional distribution of $\varepsilon_t$ (given its past, $\psi_{t-1}$, and parameters) is Student $t$ with location vector 0, inverse precision matrix $H_t$ and degrees of freedom $\nu > 2$, i.e.

$$\varepsilon_t \mid M_i, \theta_{(i)}, \psi_{t-1} \sim St(0_{[2x1]}, H_t, \nu)\,, \quad H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{12,t} & h_{22,t} \end{bmatrix}. \tag{2}$$

Note that we now use the Student $t$ distribution instead of conditional Normality assumed for the SV class. However, these distributional assumptions are not directly comparable as the conditioning variables are different in the SV and GARCH classes. As regards initial conditions for $H_t$, we take $H_0 = h_0 I_2$ and treat $h_0$ as an additional parameter. We assume prior independence for $h_0$, $\nu$ and the remaining parameters; $h_0$ follows the Exponential prior with mean 1, $Exp(1)$, and $\nu$ has the $Exp(10)$ prior, truncated by the condition $\nu > 2$.

The conditional covariance matrix of $\varepsilon_t$ given $\theta_{(i)}$ and $\psi_{t-1}$ is $(\nu-2)^{-2}\nu\cdot H_t$. The competing bivariate GARCH models are defined by imposing different structures on $H_t$. Osiewalski, Pajor and Pipień (2006) consider two different groups of multivariate GARCH specifications: the VECH(1,1) structure together with its special cases, including a simple BEKK(1,1) model, and Bollerslev's CCC model and its generalisations proposed by Engle (2002) and Tse and Tsui (2002). Here we focus on Engle's DCC structure, which can easily be used in higher dimensional problems, as well as on the BEKK(1,1) case, which was the winner among bivariate GARCH models in our previous Bayesian comparisons.

### 2.2.1. The t-BEKK(1,1) Model

This model ($M_4$) is defined by the following structure of $H_t$ in (2):

$$H_t = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}' \left(\varepsilon_{t-1}\varepsilon_{t-1}'\right) \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}' H_{t-1} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \tag{3}$$

i.e., $H_t = A + B'\varepsilon_{t-1}\varepsilon_{t-1}'B + C'H_{t-1}C$, with $\rho_{12,t} = h_{12,t} \big/ \sqrt{h_{11,t}h_{22,t}}$ as the conditional correlation coefficient.

The parameters of the covariance structure (3) have the following prior distributions: $a_{11}$~$Exp(1)$, $a_{22}$~$Exp(1)$, $a_{12}$~$N(0,1)$, $b_{11}$~$N(0.5,1)$, $b_{12}$~$N(0,1)$, $b_{21}$~$N(0,1)$, $b_{22}$~$N(0.5,1)$, $c_{11}$~$N(0.5,1)$, $c_{12}$~$N(0,1)$, $c_{21}$~$N(0,1)$, $c_{22}$~$N(0.5,1)$, which are truncated by the restrictions of positive semi-definiteness of the symmetric (2x2) matrix $A$ and stability of the general (2x2) matrix $C$ (all eigenvalues of $C$ lie inside the unit circle). We also impose $b_{11}>0$ and $c_{11}>0$ in order to guarantee identifiability.

### 2.2.2. The t-DCC model

In Engle's dynamic conditional correlation (DCC) model the diagonal elements of $H_t$ are described as in the CCC specification of Bollerslev (1990):

$$h_{ii,t} = \alpha_{i0} + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{ii,t-1} \quad (i=1,2), \tag{4}$$

and for the off-diagonal element it is assumed

$$h_{12,t} = \rho_{12,t}\sqrt{h_{11,t}h_{22,t}}\ , \tag{5}$$

where $\rho_{12,t}$ is the time-varying conditional correlation coefficient, modelled as

$$\rho_{12,t} = q_{12,t} \big/ \sqrt{q_{11,t}q_{22,t}} \tag{6}$$

with $q_{ij,t}$'s being entries of a symmetric positive definite matrix $Q_t$ of the same order as the dimension of $\varepsilon_t$. A simple specification for $Q_t$, considered by Engle (2002), assumes that

$$Q_t = (1 - b - c)S + b\xi_{t-1}\xi_{t-1}^{'} + cQ_{t-1}, \tag{7}$$

where $b$ and $c$ are nonnegative scalar parameters ($b$+$c$<1), $\xi_t$ is the vector of standardised errors and $S$ is their unconditional correlation matrix. In the case of our bivariate conditionally Student $t$ specification, we keep Engle's basic structure and define $S$ as a square matrix with ones on the diagonal and $s_{12}=s_{21}=\rho_{12}$, an unknown parameter from the interval (-1, 1); this assures positive definiteness of $S$ and $Q_t$. Also, in our case

$$\xi_{i,t} = \varepsilon_{i,t}\sqrt{(\nu - 2)/(\nu\, h_{ii,t})} \quad (i=1,2). \tag{8}$$

Thus, our DCC model (called $t$-DCC, $M_5$) generalises the conditionally Normal structure proposed by Engle (2002) to the Student $t$ conditional distribution; see Osiewalski and Pipień (2005). The original $N$-DCC model, corresponding to $\nu\to\infty$, is based on $\xi_{i,t} = \varepsilon_{i,t}\big/\sqrt{h_{ii,t}}$ . The initial condition for $Q_t$ is $Q_0 = q_0\, I_2$, with free $q_0$>0. The $t$-CCC model ($M_6$) is nested in $t$-DCC assuming $b$=$c$=0. We follow the exact Bayesian approach, which is fully feasible in the bivariate case. So, we do not use the approximate two-step estimation procedure suggested by Engle (2002). We assume that *a priori* $(b\ \ c)'$ is uniform over the unit simplex, $\alpha_{10}\sim Exp(1)$, $\alpha_{20}\sim Exp(1)$, $(\alpha_1,\alpha_2,\beta_1,\beta_2)\sim U([0,1]^4)$, $\rho_{12}\sim U([-1,1])$ and $q_0\sim Exp(1)$.

## 3. The Data and Results of Model Comparison

In order to compare the main bivariate GARCH and SV structures we use the same data set as Osiewalski and Pipień (2004,2005), Pajor (2005b), and Osiewalski, Pajor and Pipień (2006), who also present our Bayesian approach and MCMC techniques used. The data set consists of 1485 observations on the zloty (PLN) values of the US dollar ($x_{1,t}$) and German mark ($x_{2,t}$). They are the official daily exchange rates of the National Bank of Poland (NBP fixing rates), which cover the period from February 1, 1996 till December 31, 2001. The first three observations (February 1, 2 and 5, 1996) are used to construct initial conditions, $y_{(0)}$. Thus, $T$, the length of the modelled vector time series of daily growth rates of $x_{1,t}$ and $x_{2,t}$, is equal to 1482. We use the bivariate VAR(1) framework (1), where $y_{i,t}$ = 100 ln($x_{i,t}$ / $x_{i,t-1}$) for $i$=1,2. Both series ($y_{1,t}$ and $y_{2,t}$) are centred about zero, with several outliers and changing volatility. Their sample correlation coefficient (0.567) indicates positive correlation.

The overall ranking of the compared models $M_i$ as well as $log_{10}(B_{3,i})$, the decimal logarithms of the Bayes factors in favour of the TSV specification ($M_3$), calculated using the Newton and Raftery (1994) method, are shown in Table 1. Since Bayes factors differ by many orders of magnitude, the model ranking is numerically stable and robust with respect to reasonable changes in the prior distributions of the parameters. It is clear that even the SDF model, a very simple SV specification, beats the GARCH structures in terms of the marginal data density value (a natural Bayesian measure of fit). The SDF specification is better by 12 orders of magnitude than the best GARCH model from previous studies, i.e. the *t*-BEKK(1,1) model; see Osiewalski, Pajor and Pipień (2006). Using only one latent process (at the expense of common dynamics of the conditional variances and covariance) already helps a lot in modelling outliers. Of course, the use of more latent processes improves fit enormously.

Table 1. Logs of Bayes factors in favour of VAR(1) – TSV ($M_3$)

| Model ($M_i$) | Number of parameters (and latent variables) | Rank | $log_{10}(B_{3,i})$ |
|---|---|---|---|
| VAR(1) – TSV ($M_3$) | 18 (+3$T$) | 1 | **0** |
| VAR(1) – JSV ($M_2$) | 15 (+2$T$) | 2 | **15** |
| VAR(1) – SDF ($M_1$) | 12 (+$T$) | 4 | **92** |
| VAR(1) – *t*-BEKK(1,1) ($M_4$) | 19 | 5 | **104.5** |
| VAR(1) – *t*-DCC ($M_5$) | 18 | 6 | **122** |
| VAR(1) – *t*-CCC ($M_6$) | 15 | 7 | **168.5** |

## 4. Inference on Volatility and Conditional Correlation

The aim of this section is to compare posterior results for the dynamic correlation coefficient and individual volatilities. In this comparison we keep $M_1$ – $M_5$, omitting the worst model $M_6$ (*t*-CCC). It is important to know whether models that have so different fit lead to similar posterior inference on quantities of interest. Since the conditional distributions in GARCH and SV models condition on different variables, we interpret the conditional covariance matrices as based on the largest possible set of conditioning variables. That is, we formally condition on past observations and all latent variables used in our three SV models. The plots of the main posterior characteristics of $\rho_{12,t}$ (for each $t$=1,...,$T$; $T$=1482) are presented in Fig. 1 with two lines, showing $E(\rho_{12,t} \mid y, y_{(0)}) \pm D(\rho_{12,t} \mid y, y_{(0)})$. We focus on typical patterns, so only three models are represented. It is clear that constancy of conditional correlations (assumed in SDF and CCC) is not supported by the data. Also, the posterior standard deviations of $\rho_{12,t}$, $D(\rho_{12,t}|y,y_{(0)})$, are much higher in the case of TSV. This result can be explained by additional posterior uncertainty caused by the latent processes in case when the conditional correlation coefficient depends on these processes. In view of our model comparison, the high posterior precision in the GARCH cases is overly optimistic. See also Table 3, which shows the averages of the posterior means and standard deviations of $\rho_{12,t}$.

Similarities in the dynamics of the conditional correlation coefficients are summarised in Table 2, which shows empirical correlation between different sequences of $T$ estimates of $\rho_{12,t}$; it also shows empirical correlation between sequences of the conditional covariance estimates. The posterior means of the conditional correlation obtained using either GARCH (*t*-BEKK, *t*-DCC) or SV (TSV, JSV) models are quite highly correlated. However, the SDF specification (which fits the data much better than the GARCH models) assumes constancy of the conditional correlation coefficient, contrary to the evidence from the best SV models. Thus, the better fit of SDF (as compared to *t*-BEKK or *t*-DCC) need not mean more reasonable inference on the conditional correlation.

As regards the volatility estimates, plotted in Fig. 2 with averages in Table 3, the average volatilities of the SV specifications are much lower than in the case of the BEKK and DCC structures. Similarity of volatility dynamics of competing structures can be seen in Table 4, where the empirical correlation coefficients between sequences of volatility estimates from different models are presented. The three SV models show almost the same dynamics of volatilities; the results in the two GARCH models are also very highly correlated. What is most interesting, the volatility estimates obtained in models of different type show similar dynamics as well. The coefficients for *t*-BEKK and different SV models always exceed 0.7.

In our example, the dynamics of volatility is estimated quite similarly, despite huge differences in model fit. However, this is not the case for other important aspects of posterior inference. This means that we should rely on rich enough bivariate SV structures (TSV, JSV) in modelling pairs of financial time series. Unfortunately, there are serious problems with generalisations to $k$-variate time series (with $k$>3) as the JSV and especially TSV specifications are hard to estimate in highly multivariate cases. Since the GARCH models do not have enough flexibility to describe outliers, the question of good specifications for multivariate financial modelling is still open.

Table 2. Correlation coefficients between the posterior means of the conditional correlations (upper part) and covariances (lower part)

| Model | TSV | JSV | SDF | *t*-DCC | *t*-BEKK11 |
|---|---|---|---|---|---|
| TSV | 1 | 0.9428 | X | 0.6938 | 0.6946 |
| JSV | *0.9855* | 1 | X | 0.7497 | 0.7746 |
| SDF | *0.9475* | *0.9766* | 1 | X | X |
| *t*-DCC | *0.5453* | *0.5469* | *0.5212* | 1 | 0.9094 |
| *t*-BEKK11 | *0.5866* | *0.6023* | *0.5684* | *0.9172* | 1 |

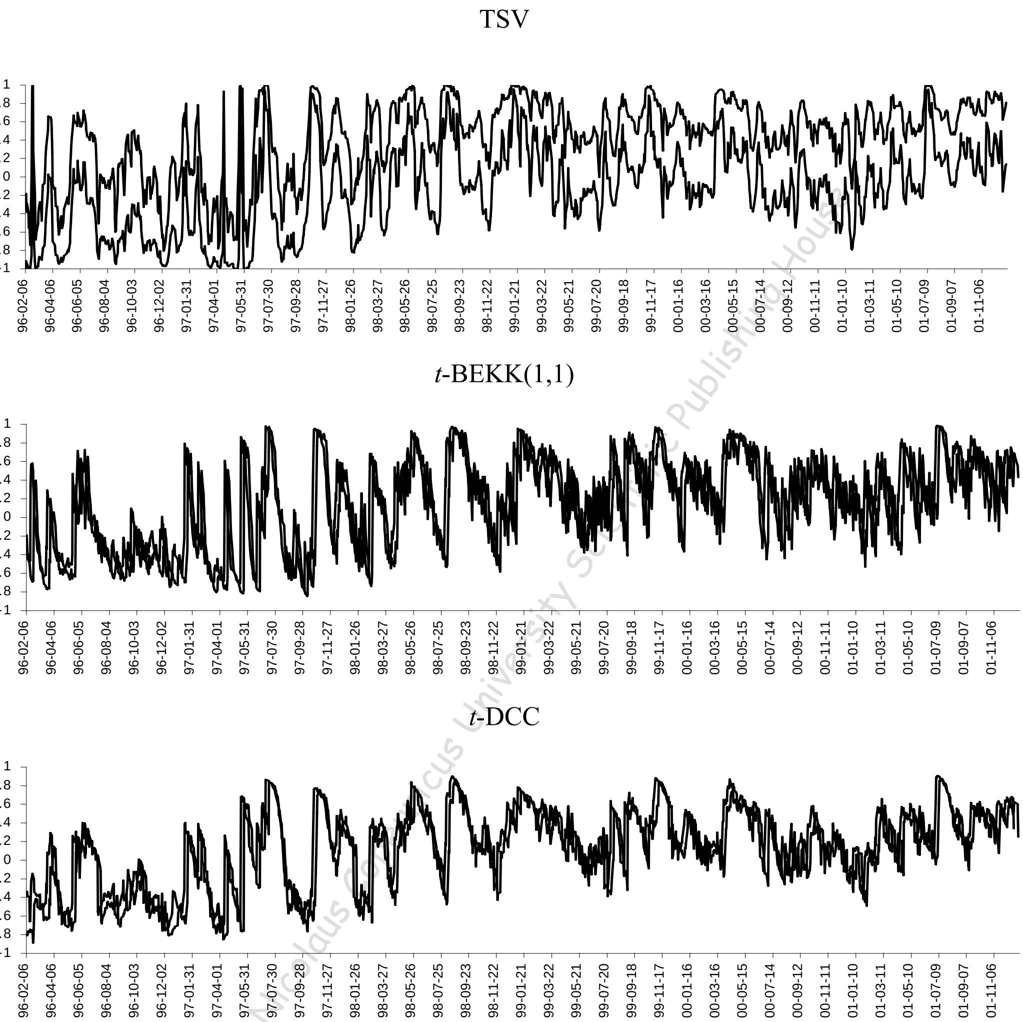


Fig. 1. Conditional correlation (posterior mean ± 1 standard deviation)

Table 3. Average posterior means of $\rho_{12,t}$ and average volatility estimates

| Model | Average $E(\rho_{12,t} \mid y, y_{(0)})$ (and $D(\rho_{12,t} \mid y, y_{(0)})$) | Average volatility of PLN/USD | Average volatility of PLN/DEM |
|---|---|---|---|
| TSV | 0.1509 (0.3042) | 0.5363 | 0.6005 |
| JSV | 0.0942 (0.2520) | 0.5733 | 0.5904 |
| SDF | 0.2162 (0.0298) | 0.5263 | 0.5864 |
| *t*-DCC | 0.1319 (0.0442) | 0.8116 | 0.9066 |
| *t*-BEKK(1,1) | 0.1617 (0.0469) | 0.7912 | 0.8816 |

Table 4. Correlation coefficients between volatility estimates for PLN/USD (upper part) and for PLN/DEM (lower part)

| Model | TSV | JSV | SDF | *t*-BEKK11 | *t*-DCC |
|---|---|---|---|---|---|
| TSV | 1 | 0.9464 | 0.9511 | 0.7208 | 0.6945 |
| JSV | *0.9564* | 1 | 0.9823 | 0.7470 | 0.6937 |
| SDF | *0.9404* | *0.9799* | 1 | 0.7130 | 0.6676 |
| *t*-BEKK11 | *0.7286* | *0.7437* | *0.7086* | 1 | 0.9219 |
| *t*-DCC | *0.7163* | *0.6614* | *0.6328* | *0.8826* | 1 |

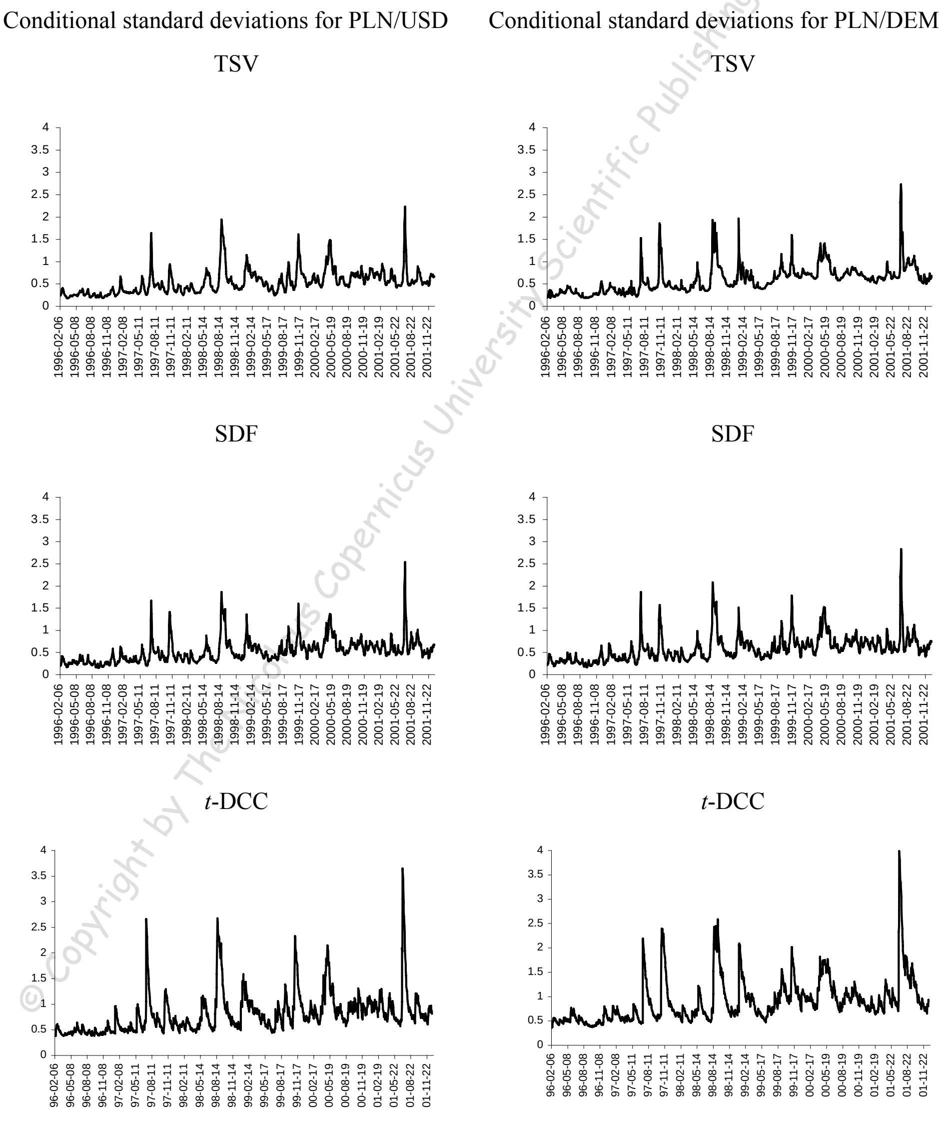


Fig. 2. Volatility estimates in TSV, SDF and t-DCC

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