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Fractional Integration Parameters Estimation for the PLN and for the Irish Pound Exchange Rates

1. Introduction

Since the beginning of transformation process the Polish zloty underwent several changes of exchange rate determination mechanisms. At the beginning of 1990s the exchange rate was fixed with respect to the US dollar, later – to a basket of currencies; then a pegged exchange rate was introduced and currency bands increased, until a full flexibility mechanism was reached in 2000. When searching for a similar gradual changes for a European currency, we notice that the Irish pound underwent a similar change – albeit over longer period. This suggests that a comparison of the behavior of two currencies would be useful.

Time series properties can be characterized with use of several measures. One of them is the so-called fractional integration parameter *d*. It can take integral values 0 – corresponding to a stationary series, and 1 – corresponding to a non-stationarity due to the presence of a unit root in the series. Perhaps more interesting for classification of the time-series behavior are non-integer values of *d*: we expect this parameter to be close to 1 for exchange rates and close to 0 for their logarithmic returns.

The results presented here are a continuation of the research on exchange rate modeling, in particular – comparisons of behavior of several currencies, published in the papers of Syczewska (2002a-d). Earlier we compared nominal exchange rate behavior, in particular checking whether permanent changes of USD/PLN fluctuations, detected by Załuska-Kotur, Krukowski and Orłowski (2002), took place also for other exchange rates. Among the methods applied we used non-stationarity (unit-root) tests and also a fractional integration parameter estimation method. The fractional integration was treated as an indicator of long-memory in the series. We noticed a marked change of the fluctuations pattern in 1997. This is due to the increased exchange rate volatility – as E. Pietrzak (2000) noted, since the beginning of 1997 the Polish zloty attained a degree of convertibility higher than one required by the IMF.

The comparison of the Polish zloty exchange rates behavior with the Irish pound exchange rates behavior would be eased by a different choice of sub-samples. Hence we decided to apply the fractional integration parameter estimations to sub-series corresponding to the following three periods:

1. the period from 4th January 1993 until 15th May 1995, which was a period of the rather stable behavior, the pre-announced peg with a relatively slow rate of change;
2. the period from 16th May 1995, when the degree of convertibility increased, until 12th April 2000, when the floating exchange rate was introduced;
3. the period from 12th April 2000 until the end of October 2004.

The fractional integration parameter estimation was also applied for the whole sample, from 4th January 1993 until 29th October 2004. We also compute the recursive estimates of d, starting with the sub-sample of 500 observations. We studied the following nominal exchange rates:

- USD/PLN – the US dollar versus the Polish zloty exchange rate, expressed as a number of zlotys per one US dollar,
- GBP/PLN – the British pound exchange rate,
- CHF/PLN – the Swiss franc exchange rate,
- AUD/PLN – the Australian dollar exchange rate,
- JPY/PLN – the Japanese yen exchange rate, expressed as a number of zlotys per a hundred of yens,
- CAD/PLN – the Canadian dollar exchange rate.

As a source of the data we use tables of average PLN exchange rates, published by the National Bank of Poland.

We analyze the behavior of the Irish pound exchange rates versus the following currencies:

USD/IEP – the US dollar versus the Irish pound;
GBP/IEP – the British pound versus the Irish pound;
CAD/IEP – the Canadian dollar versus the Irish pound;
JPY/IEP – the Japanese yen exchange rate, also expressed for a hundred yen.

2. Estimation of the fractional integration parameter for the Polish exchange rates

For a series with long memory, observations from the past are significantly correlated with current and future values. In the frequency domain such a series has nonzero power for lower frequencies. The fractional integration parameter can serve as an indicator of long memory. The Sowell's method of fractional integration parameter estimation requires normality of distribution. However, as is well known (see e.g. Bailie and Bollerslev (1989) or Campbell, Lo and MacKinlay (1997)), exchange rates have non-normal distributions, and are better characterized by *t*-Student, stable or a mixture of distributions.

Geweke and Porter-Hudak (1993) introduced a simple method of the fractional integration parameter estimation, based on the dependence between spectral density and the value of the fractional integration parameter. Let N denote the number of observations. Let I(•) denote a periodogram value for a series. Let $I(\theta_j)$ denote the values of periodogram for a particular set of frequencies, $\theta_j = 2\pi j/N$, where $0<k<j<K<<N$. Bounds k and K are determined e.g. in Mills (1993); K is equal to a square root of the number of observations N. We run a regression of $I(\theta_j)$ with respect to the frequencies, parameter of this regression gives the estimate of the fractional integration parameter. The periodogram for a series can be determined with the use of the Fourier transform (see Talaga and Zieliński (1986)). The transform can be either a fast Fourier transform, FFT (which requires a sub-sample with the number of observations equal to the power of 2), or a discrete Fourier transform, DFT (which can be applied to the whole sample). The computations presented here were programmed and performed with the use of the GAUSS programming language.

In earlier paper by Syczewska (2002d) the fractional integration parameters were computed for the whole sample and for two sub-samples, with the breaking point at the assumed moment of change in volatility. Periodogram regression was run on the base of 48 values for the whole sample, 34 values for the first part of the sample, and 33 values – for the second. The normality hypothesis for regression errors was verified with the use of the χ^2 test and the Shapiro-Wilk test (values of parameters and critical values given by Domański and Pruska (2000), p. 35 and in tables 8 and 9, pp. 314–319). For example, in the case of periodogram regression for the USD/PLN exchange rates, the value of χ^2 test for regression errors is equal to 18.53[0.0001] for the whole sample, 0.82 [0.66] for the first part of the sample, 5.16 [0.076] for the second. Hence the null hypothesis of normality is rejected for the whole sample. However, it can not be rejected neither for the first nor for the second part. All values of the Shapiro-Wilk statistic are given in table 3 in Syczewska (2002d). For all currencies the null hypothesis of normality of errors is rejected, and the *t*-Statistic values, used to test whether a fractional parameter is equal to 1 (in the case of exchange rates) or 0 (for differences), have to be treated as approximation.

Table 1a. The GPH-Discrete Fourier Transform estimates of the *d* parameter for the PLN exchange rates

Period	USD	GBP	CHF	AUD	JPY
1	1.013 (0.039)	1.100 (0.048)	1.016 (0.037)	1.031 (0.054)	0.957 (0.029)
2	1.043 (0.044)	1.078 (0.050)	1.141 (0.104)	1.049 (0.073)	1.094 (0.108)
3	0.911 (0.090)	0.996 (0.097)	1.181 (0.129)	1.0005 (0.11)	1.008 (0.089)
	↓ (↑)	↓ (↑)	↑ (↑)	↓ (↓)	↑↓ (↑↓)

Source: Author's computations.

Table 1b. The GPH-Discrete Fourier Transform estimates of the *d* parameter for the logarithmic returns of the PLN exchange rates.

Period	USD	GBP	CHF	AUD	JPY
1	0.198 (0.148)	-0.055 (0.155)	0.074 (0.037)	0.070 (0.232)	0.355 (0.134)
2	0.021 (0.127)	-0.055 (0.113)	-0.023 (0.128)	0.074 (0.126)	0.105 (0.112)
3	-0.268 (0.125)	0.058 (0.104)	0.068 (0.106)	0.073 (0.115)	0.042 (0.104)
	↓ (↓)	→↑ (↓)	↓↑ (↑↓)	↑→ (↓)	↓ (↓)

Source: Author's computations.

The results of the Geweke and Porter-Hudak estimation of the fractional integration parameter are shown in Table 1a and 1b. The estimates for the logarithms of daily rates are close to 1, as we expected, but in the most of cases they are greater than 1. The standard errors of estimates are increasing in the most of cases, which seems to be a logical result in light of increased volatility of the corresponding rates due to the increasing degree of convertibility.

For logarithmic returns in case of all variables except the Swiss franc variances in the first, second and third period are decreasing, but their values are higher than in the respective periods for logarithms of the rates.

Table 2a shows the estimates of *d* computed according to the GPH method, with the use of the fast Fourier transform (based on the first 2048 observations – "Beginning", or on the last 2048 observations – "End"), the wavelet transform for the same samples, and the discrete Fourier transform for the whole sample. Only for the British pound and the Swiss franc the *t*-Student ratio computed is greater than the critical value, hence H_0 that a parameter *d* is equal to 1, has to be rejected. In other cases the *t*-Statistic values for the whole sample or for the second part suggest, that the null hypothesis $d=1$ cannot be rejected. This would mean that the exchange rate is integrated of order 1, in perfect agreement with results of the Dickey-Fuller test or the Phillips-Perron test.

Table 2a. The GPH estimates of the *d* parameter for logarithms of the PLN exchange rates.

Currency	Method of *d* estimation		*d*	Standard error	*t*-Statistic for H_0: *d*=1.
USD/PLN	End	FFT	1.114	0.090	1.270
		Wavelet	0.949		
	Whole	DFT	1.023	0.040	0.567
	Beginning	FFT	1.023	0.040	0.567
		Wavelet	0.981		
GBP/PLN	End	FFT	0.974	0.096	–0.272
		Wavelet	0.949		
	Whole	DFT	1.054	0.032	1.675
GBP/PLN	Beginning	FFT	1.085	0.036	2.321
		Wavelet	0.982		
CHF/PLN	End	FFT	1.188	0.093	2.021
		Wavelet	0.906		
	Whole	DFT	1.025	0.038	0.672
	Beginning	FFT	1.042	0.041	1.022
		Wavelet	0.985		
AUD/PLN	End	FFT	1.078	0.117	0.663
		Wavelet	0.878		
	Whole	DFT	0.955	0.063	–0.724
	Beginning	FFT	1.025	0.047	0.541
		Wavelet	0.979		
100JPY/PLN	End	FFT	1.203	0.103	1.970
		Wavelet	0.914		
	Whole	DFT	0.935	0.051	–1.272
	Beginning	FFT	1.009	0.047	0.184
		Wavelet	0.983		
CAD/PLN	End	FFT	1.024	0.092	0.262
		Wavelet	0.934		
	Whole	DFT	0.955	0.042	–1.078
	Beginning	FFT	1.027	0.047	0.575
		Wavelet	0.98		

Source: Author's computations.

Moreover, errors of estimates of the fractional parameter are lower in the first part of the sample than either for the whole sample or for the second part. This difference of results is caused by a higher volatility of exchange rates in the second part of the sample, which seems to dominate the results for the whole sample.

Table 2b. The GPH estimates of the *d* parameter for the logarithmic returns of the PLN exchange rates.

		d	Std.error	*t* for: H_0=0	*d*	Std.error	*t* for: H_0=0
Sample	Method	**USD**			**GBP**		
End	FFT	0.135	0.145	0.936	0.135	0.133	1.014
	Wavelet	–0.015			–0.033		
Whole	DFT	0.051	0.076	0.667	0.100	0.095	1.052
Beginning	FFT	0.040	0.121	0.326	0.008	0.081	0.093
		CHF			**AUD**		
End	FFT	–0.121	0.106	–1.143	0.040	0.139	0.286
	Wavelet	–0.061			–0.054		
Whole	DFT	0.036	0.105	0.345	0.052	0.097	0.532
Beginning	FFT	0.040	0.121	0.326	0.108	0.097	1.117
		100JPY			**CAD**		
End	FFT	0.169	0.089	1.885	0.119	0.103	1.159
	Wavelet	–0.020			–0.036		
Whole	DFT	0.227	0.087	2.604	0.017	0.101	0.169
Beginning	FFT	0.275	0.093	2.952	–0.135	0.105	–1.293

Source: Author's computations.

Table 2b shows the results of the *d* estimation for the logarithmic returns. The null hypothesis tested here is: *d*=0. This hypothesis cannot be rejected for all returns except the logarithmic returns for the Japanese yen.

The following graphs show the results of recursive estimation of the fractional integration parameters.

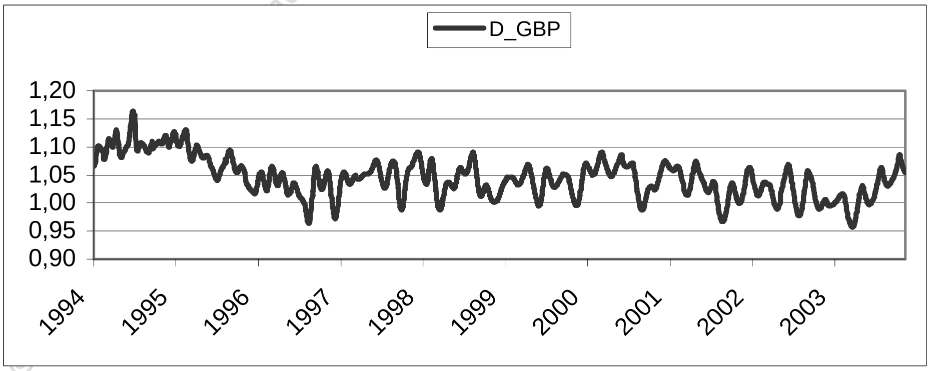

Fig. 1a. Recursive values of the fractional integration parameter estimates for the Polish zloty exchange rates
Source: Author's computations.

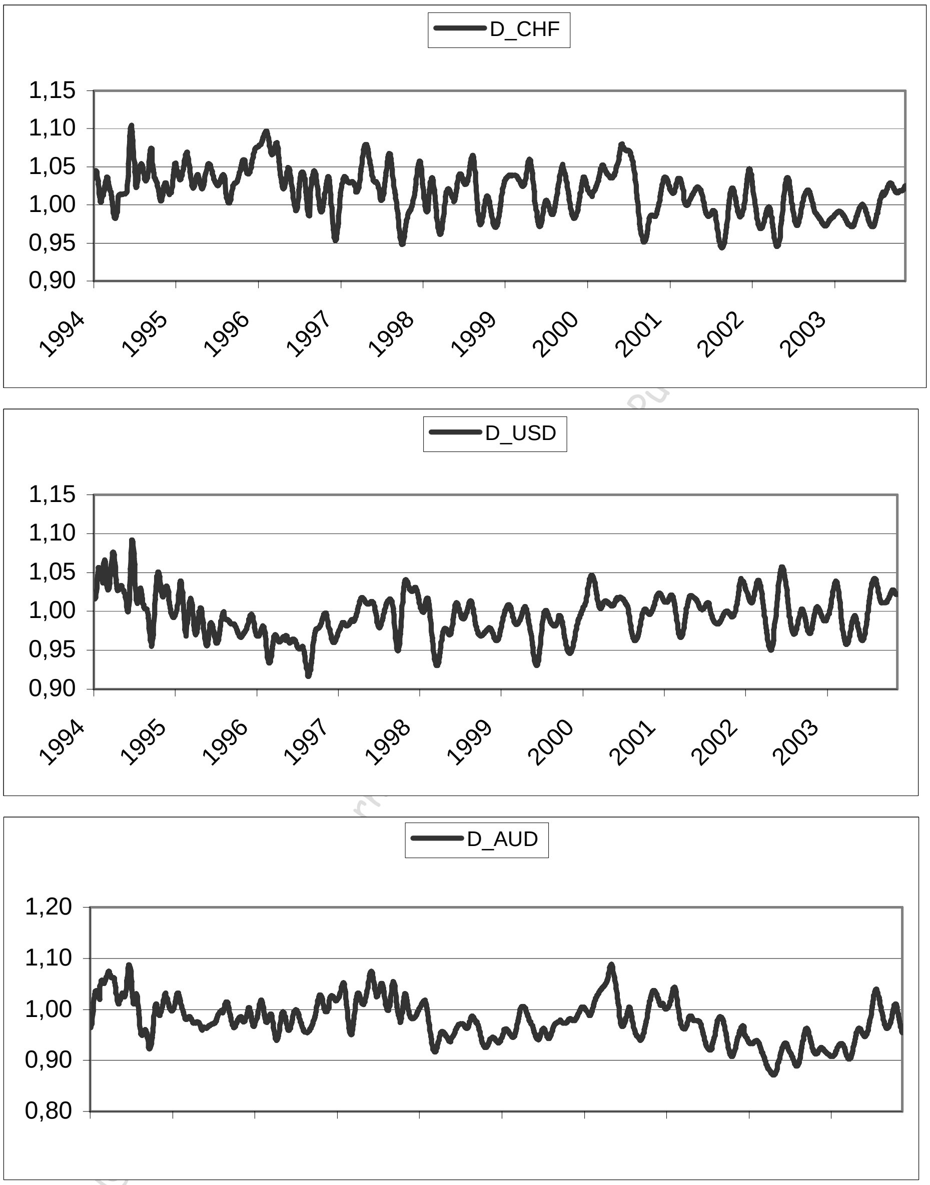

Fig. 1b. Recursive values of the fractional integration parameter estimates for the Polish zloty exchange rates.
Source: Author's computations.

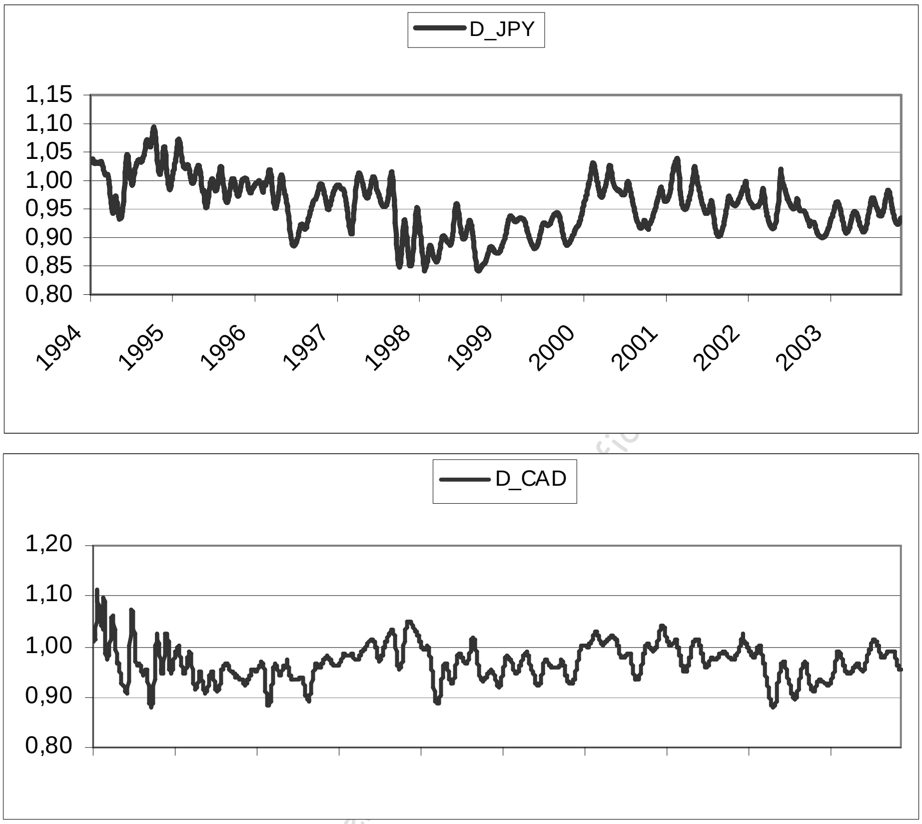

Fig. 1c. Recursive values of the fractional integration parameter estimates for the Polish zloty exchange rates.
Source: Author's computations.

Fig. 1 shows the results of recursive estimation of fractional integration parameters for sub-samples of growing number of observations. The first value was determined with the use of the GPH method based on the first 500 exchange rate values, next – for periodogram based on 501 exchange rate values, etc. until the whole sample was covered. The horizontal axis shows the years covered. This is an extension of similar graphs for the periodograms analyzed in Syczewska (2002d), for the whole period until the end of October 2004.

The recursive estimates of *d* for the GBP/PLN exchange rate until 1996/97 have values greater than 1. Later they take slightly lower values. For the CHF/PLN rate, estimates are higher than 1, later they seem to show higher volatility, also with slightly lower values (around 1). The same pattern is shown in the USD/PLN *d* recursive estimates – it also seems to concentrate around 1 with higher changes after 1997. The Yen and Australian dollar *d* recursive estimates give impression of a greater volatility than the estimates for the rest of currencies.

3. Irish pound exchange rate evolution

One of the currencies undergoing a similar path of changes, namely starting with fixed system through more and more flexible mechanisms, was the Irish pound. Lothian and McCarthy (2002) presented a detailed analysis of real exchange rate of this currency with respect to the British pound, the Deutschmark and the US dollar. They used graphical and statistical methods, the unit root (Dickey-Fuller and Phillips-Perron) tests to confirm non-stationarity of the series, and the Chow test to check whether there were significant structural changes in 1973 and 1979. The number of lags on the ADF test was chosen with the Schwartz information criterion, and for the Phillips-Perron test – fixed at 3. Lothian and McCarthy (2002) used annual data, covering the period of 1922–1998. They describe the evolution of exchange rate determination in this period. At the beginning, with a strict monetary union with Great Britain, the exchange rate was fixed at level 1:1. In 1922–1943 the Currency Board performed the exchange rate control, in fact maintaining the fixed exchange rate. In 1943 the Irish Central Bank was established. Its policy until 1979 was aimed at maintaining parity with the British pound. In 1973 Ireland joined the European Union, in 1979 – the European Rate Mechanism, since then the Irish pound was pegged to the mark and other European currencies, and had a flexible exchange rate with respect to other currencies. The British pound joined the European Exchange Rate mechanism only for two years (October 1990 – 1992). Since the joining of the ERM the Central Bank of Ireland maintained a peg rate. Its interest rate policy was determined by a real exchange rate weighted by importance of trade partners. As Great Britain was the most important partner, the two currencies were highly correlated.

4. Fractional integration analysis for the Irish pound

We perform the analysis of the Irish pound behavior, and in particular of the fractional integration parameter for this currency, using daily data covering the period from 1979 until 1998, i.e. the moment of introducing the Euro. The daily historic statistics concerning the Irish pound exchange rate can be found at the Central Bank web site, http://www.centralbank.ie/eurorates/index.asp. This period covers moments of important changes in the Irish pound exchange rate determination, described by Lothian and McCarthy: 1979, 1986 and 1992.

As it was done for the PLN exchange rates, we perform computations for the three sub-periods. Firstly, we estimate the fractional integration parameter with the use of the FFT and the DFT. Secondly, we also apply the recursive integration parameter estimation for the Irish pound exchange rate series.

The aim is to check whether similar periods of the fixed, pegged and floating exchange rate determination would show a similar behavior to the one noted in case of the PLN exchange rates.

The introductory analysis was performed with the use of the Phillips-Perron test (as more robust to heteroskedasticity than the Dickey-Fuller test). The critical values, according to MacKinnon (1991), are equal to –2.57 for 1% level, –1.94 for 5% and 1.62 for 10%. Table 2 gives computed values of the Phillips-Perron test statistic.

Table 2. The Phillips-Perron Test Statistic for the Irish pound exchange rates

Exchange rate	The Phillips-Perron test statistic		Subperiod		
	For variable	For differences	I	II	III
LnUSD/IEP	–1.394	–70.09**	–1.767	0.208	–0.692
LnGBP/IEP	–1.127	–67.37**	–0.678	–0.658	–0.702
LnCAD/IEP	–0.594	–70.12**	–1.385	0.129	0.240
LnYen/IEP	–2.254*	–66.79**	–2.211*	–0.509	–0.960

** – significantly different from zero at 5%, * – at 1%.
Source: Author's computations.

Computations of the fractional integration parameter were performed both with the use of: the fast Fourier transform, which required a choice of a sub-sample containing 2^n observations, the discrete Fourier transform, which was applied to the whole sample, and the wavelet transform.

The whole series contained 4692 observations, hence the biggest sub-sample consists of $2^{12} = 4096$ observations. Hence the fractional parameter, in case of the FFT computations, was estimated for the first 4096 observations and for the last 4096 observations. To the same sub-samples the wavelet transform method was applied. Additionally the fractional integration parameter was computed for the three sub-periods, namely:
I – the period from1979 to the end of 1986, i.e. observations 1 – 1956;
II – the period from 1987 to the end of 1992, i.e. observations 1957 – 3462;
III – the period 1993 to the end of 1998, i.e. observations 3463 – 4962.

Tables 3 – 4 contain the results of the fractional parameter estimation for the sub-samples and for the whole series. "End" denotes the results based on the last 4096 observations, "Beginning" – the results based on the first 4096 observations. In the case of exchange rates we also compute the estimates of d for the three sub-samples, I, II and III. The number of values used in periodogram regression is equal to 70 for the whole sample, 64 in the case of the sub-sample containing 4096 observations, 44 for the sub-sample denoted I, 38 for the II and III sub-sample. Critical values for the t-Student statistics are equal to 2.000 for 70 and 68 observations, 2.021 for 38 or 44 observations (see Kuszewski and Podgórski (1998), table 1, p. 53).

The results are the following. On the base of computed values of the t-Statistic we do not reject the null hypothesis that the value of the fractional integration parameter was equal to 1 in the case of exchange rates, or 0 in the case of their differences. Estimates of fractional integration obtained with the use of the wavelet transform are in the most of cases lower than 1.

Table 3a. Fractional integration parameter estimates

Sub-sample	Method	USD/IEP		GBP/IEP	
		Estimate (Error)	*t*-Student H0: d=1	Estimate (Error)	*t*-Student H0: d=1
End	FFT	1.068 (0.076)	0.884		
	Wavelet transform	0.961		0.954	
Whole	DFT	1.048 (0.075)	0.645	1.124 (0.099)	1.25
Beginning	FFT	0.985 (0.107)	–0.136	0.985 (0.107)	–0.14
	Wavelet transform	0.942		0.943	
I		1.115 (0.105)	1.104	1.115 (0.105)	1.10
II		1.110 (0.098)	1.103	1.110 (0.098)	1.12
III	DFT	0.865 (0.061)	1.104	0.977 (0.119)	–0.19

Source: Author's computations.

Table 3b. Fractional integration parameter for log differences of the exchange rate

Sub-sample	Method	USD/IEP		GBP/IEP	
		Estimate of d (Error)	*t*-Student H0: d=0	Estimate of d (Error)	*t*-Student H0: d=0
End	FFT	–0.0064 (0.076)	0.084	–0.0808 (0.075)	–1.08
	Wavelet	0.0167		0.0186	
Whole	DFT	0.112 (0.082)	1.366	0.0748 (0.080)	0.94
Beginning	FFT	-0.00403 (0.103)	–0.039	–0.00398 (0.103)	–0.04

Source: Author's computations.

Table 4a. Fractional integration parameter estimates for the exchange rates

Sub-sample	Method	CAD/IEP		100JPY/IEP	
		Estimate (Error)	*t*-Student H0: d=1	Estimate (Error)	*t*-Student H0: d=1
End	FFT	0.964 (0.054)	0.884		
	Wavelet	0.988		2.400	
Whole	DFT	0.988 (0.054)	–0.22	1.114 (0.087)	1.31
Beginning	FFT	1.018 (0.035)	0.51	1.055 (0.073)	0.71
	Wavelet	0.978		0.936	
I		1.092 (0.051)	1.80	1.045 (0.136)	0.33
II		1.110 (0.095)	1.16	1.102 (0.142)	0.75
III	DFT	0.888 (0.118)	–0.95	1.140 (0.102)	1.37

Source: Author's computations.

Table 4b. Fractional integration parameter for log differences of the exchange rate

Sub-sample	Method	CAD/IEP		100JPY/IEP	
		Estimate of d (Error)	*t*-Student H0: d=0	Estimate of d (Error)	*t*-Student H0: d=0
End	FFT	0.120 (0.112)	1.07	–0.042 (0.098)	–0.42
	Wavelet transform	0.0289		0.0114	
Whole	DFT	0.200 (0.096)	2.08	0.109 (0.085)	1.28
Beginning	FFT	0.102 (0.076)	1.34	0.0498 (0.069)	0.72

Source: Author's computations.

The fast Fourier transform and the discrete Fourier transform method with the GPH estimation give higher estimates of the fractional parameter for the end 4096 observations and for the whole series than for the first 4096 observations. This feature is similar to the one for the PLN exchange rates.

A comparison of *d* estimates computed with the use of the GPH method for the fast and the discrete Fourier transform for the three sub-periods selected by Lothian and McCarthy brings the following comments. Fractional integration parameters estimates are greater than 1 in the I and II part, lower than 1 in part III for the USD, GBP and DEM /IEP exchange rates. The estimates for the yen exchange rates are similar for all three parts, greater than 1. Estimates errors are diminishing for the USD exchange rate, increasing for the CAD/IEP exchange rate, for the GBP – lower in part II than in I or III, and in case of yen exchange rate – rather constant.

Figure 2 shows graphs of recursive estimate of the fractional integration parameter, computed for windows (sub-samples) of constant length. For the GBP/IEP exchange rate is rather smooth, there is a change around the 700th window and the 2300th–2400th windows. Around the 3000th window there is an increase of value. For the Canadian dollar exchange rate changes are in similar places, but around the 300th window the change is negative.

For the USD/IEP exchange rate the results of the recursive fractional parameter estimation are more volatile than for other currencies, even more so around the 1600 – 1800th windows. After that there is a marked decrease in fractional parameter values.

For the Yen exchange rates fractional integration parameter estimates are more smooth than for other exchange rates. Especially for the 2100 – 3000 sub-sample recursive fractional integration estimates are quite similar and smooth.
Notice that windows (sub-samples) consist of 500 observations, hence the 1600th–1800th windows cover the point conjectured by Lothian and McCarthy change between I and II sub-sample. The 3000th window ends in the point of their II–III sub-sample.

We notice that the behavior of recursive estimates for the Irish pound exchange rates in the I period, i.e., fixed regime, is different than in the 1st period for the Polish zloty exchange rates – here the *d* estimates are close to 1 for all currencies.

Also in the II period the Irish estimates seem to be more volatile, increasing. This pattern is opposite to the Polish estimates behavior in the 2nd period, when the recursive estimates tended to stabilize and to decrease in value.

Only in the III period for the Irish rates and the 3rd period for the Polish rates, covering floating rate for both currencies, the recursive estimates of the fractional integration parameter behave in a similar way, showing relatively high volatility.

The overall result of the analysis and comparison of the PLN and the IEP exchange rate shows that the behavior of the fractional integration parameters is different in the case of the Irish pound. However, in the case of recursive estimation, changes of fractional integration parameter estimates correspond in a similar way to changes in the behavior of the exchange rates themselves; it can be used to detect structural and volatility changes for the rates.

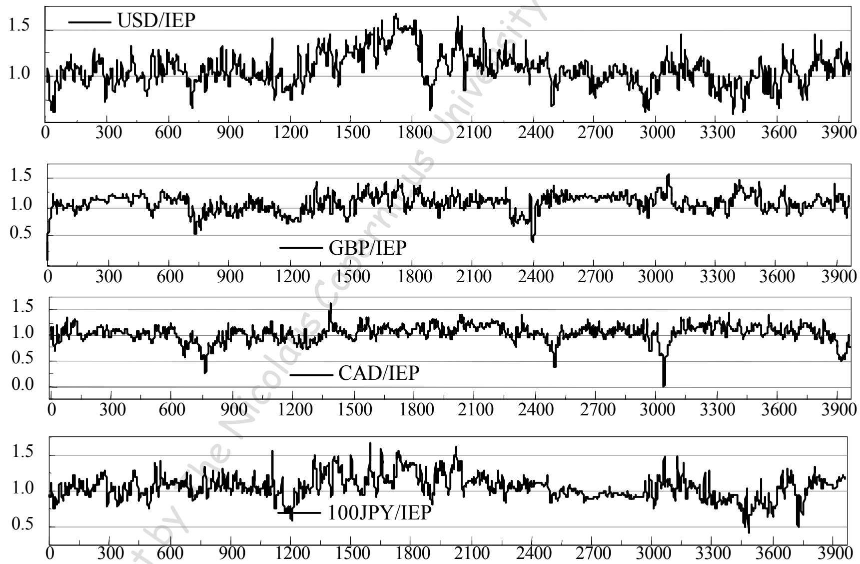

Fig. 2. Recursive estimates of the fractional integration parameters for the Irish pound exchange rates.
Source: Author's computations.

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