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Some Aspects of Seasonality in Co-integration Analysis*

1. Introduction

In the integration and co-integration analysis of non-stationary processes with reference to time series of high frequency of observations it is most often assumed that they do not contain deterministic components such as trend function or seasonal effects. However, many economic processes observed in monthly or quarterly cycles are characterised by clear seasonal features.

At the turn of the 1980s new terms connected with the definition of seasonal integration and co-integration published in the works of Engle et al. (1989), Hylleberg *et al*. (1990), Lee (1992) and Hylleberg (1992) contributed to this field worldwide. In this period some new applications of this new approach in empirical analyses could be found in the English language. They most frequently contained time series observed in quarterly periods[1]. The authors of these analyses often emphasised a certain amount of arbitrariness in the selection of the so-called seasonal differentiation (seasonal increments) in connection with the necessity to introduce non-seasonal differentiation leading the series examined to stationarity. It is also worth paying attention to the approximate nature of the critical values to the tests applied which are set by means of a simulation rather than analysis, which means they depend on the structure of the model applied for the simulation[2].

In Polish scientific literature it is not possible to indicate too many works concerning analyses based on the idea of seasonal integration and co-

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[1] See: e.g. Engle *et al*. (1989), Engle *et al*. (1993).

[2] See: e.g. Charemza, Deadman (1997) p. 115.

integration. In recent years, works of J. Kotłowski ((2002), (2004)) appeared, where certain macroeconomic processes (observed quarterly) of the Polish period of transformation were investigated with the application of the analysis of seasonal co–integration

An attempt to apply the relatively simple procedure of eliminating seasonal components is the object of consideration of this article. The procedure consists in *periodical smoothing* of non-stationary time series observed in monthly cycles. This denotes the application of quarterly, semi-annual and annual smoothing, which should lead to a partial or total elimination of seasonal components, while the trend structure of processes analysed was maintained or strengthened. A basic problem here is an answer to the question: "What influence on the inference of integration and co-integration of the analysed economic processes do the periodical smoothing and the alternative elimination of the trend-and-seasonal effects have?"

The above issues will be illustrated by an example of monthly observations of selected characteristics of the economic and financial activity of Meat Plant Grudziądz S.A.

2. Methodology

2.1. Elimination of deterministic trend-and-seasonal components

An elimination of deterministic trend-and-seasonal components for economic processes observed in monthly cycles carried out with the utilisation of:

a) the additive seasonal variable (dummies)

S_{it} = 1 for month i , 0 for the remaining months. (1)

b) the deterministic trend function

$f_k(t) = \alpha_k \cdot t^k$, for k = 1,2, (2)

c) multiplicative seasonal variable

$ST_t(i,k) = S_{it} \cdot f_k(t)$, for month i and trend of degree k . (3)

2.2. Periodical smoothing[3]

The alternative approach aimed at the clearing of time series from seasonal effects consists in the introduction of the periodical smoothing with the application of:

a) quarterly smoothing

[3] Such approach can be treated as a kind of filtration. Detail discussion of this topic will be a subject of author's dissertation.

$$W3X_t = (\sum_{i=0}^{2} X_{t-i})/3, \quad (4)$$

b) semi-annual smoothing

$$W6X_t = (\sum_{i=0}^{5} X_{t-i})/6, \quad (5)$$

c) annual smoothing

$$W12X_t = (\sum_{i=0}^{11} X_{t-i})/12. \quad (6)$$

2.3. Testing for integration and co–integration

The testing for integration of non-stationary processes cleared of the seasonal-trend effects and the periodically smoothed ones carried out with the application of Dickey-Fuller tests (DF and ADF)[4]

a) Dickey-Fuller (DF) testing of the unit root on the basis of the equation:

$$DX_t = const. + \delta X_{t-1} + \varepsilon_t, \quad (7)$$

b) the augmented Dickey-Fuller (ADF) test

$$DX_t = const. + \delta X_{t-1} + \sum_{i=1}^{l} \delta_i DX_{t-i} + \varepsilon_t. \quad (8)$$

The testing of be-factorial and of multi-factorial (vector) co-integration for non-stationary processes cleared of seasonal-trend effects and those smoothed periodically will be conducted with the application of a two-phase procedure proposed by Engle and Granger [1987]:

a) the co-integrating equation (linear regression):

$$Y_t = \beta_0 + \beta_1 X_t + \eta_t \quad (9)$$

b) the testing of the stationarity of the residual process η_t by means of the DF or ADF test.

3. Empirical data

Twelve variables characterising the productive and economic activity of the Meat Plant Grudziądz S.A.[5] will be analysed empirically taking into account an examination of integration and co-integration of economic processes after the elimination of deterministic trend-and-seasonal components and periodical

[4] See: Dickey, Fuller (1978).

[5] I wish to thank my MA student, Kopińska, for letting me use her data (2003).

smoothing. Data expressed in PLN contain monthly observations for the period from 1994M01 to 2002M12.

Table 1. Specification of processes analysed empirically

No.	Name of the process	Symbol
1	Depreciation	AMO
2	Income costs	KUP
3	Receivables	NAL
4	Processing output	PRP
5	Receipts from sales	PZS
6	Purchasing	SKU
7	Total sales	SPR
8	Slaughtering	UBO
9	Compensation	WYN
10	Stocks	ZAP
11	Employment	ZAT
12	Liabilities	ZOB

4. Results of the trend–and–seasonal analysis

All of twelve original and periodically smoothed processes is examined from the point of view of the trend–and–seasonal analysis including additive and multiplicative seasonal variables and the trend function of the first, second and third degree. The best versions of the estimated models taking the criterion of the best fit (max R^2) will be subjected to the integration and co-integration analysis.

Table 2. The trend-and-seasonal structure of process X_t = ZAP

X_t		Trend	S01	S02	S03	S04	S05	S06	S07	S08	...	S12	R^2
ZAP	S	–	(+)	(+)	(+)	(+)	(+)	–	–	–	–	–	0.283
	ST	(–)	(+)	(+)	(+)	(+)	(+)	–	–	–	–	–	0.362
	ST2	(–)	(+)	(+)	(+)	(+)	(+)	–	–	–	–	–	0.359
	ST3	(–)	(+)	(+)	(+)	(+)	(+)	–	–	–	–	–	0.353
W3	S	–	–	(+)	(+)	(+)	(+)	(+)	–	–	–	–	0.256
	ST	(–)	(+)	(+)	(+)	(+)	(+)	(+)	–	–	–	–	0.341
	ST2	(–)	–	(+)	(+)	(+)	(+)	(+)	–	–	–	–	0.303
	ST3	(–)	–	(+)	(+)	(+)	(+)	(+)	–	–	–	–	0.300
W6	S	–	–	–	–	(+)	(+)	(+)	(+)	–	–	–	0.134
	ST	–	–	–	(+)	(+)	(+)	(+)	(+)	–	–	–	0.193
	ST2	–	–	–	–	–	–	–	–	–	–	–	–
	ST3	(–)	–	–	–	–	–	–	–	–	–	–	0.038
W12	S	–	–	–	–	–	–	–	–	–	–	–	–
	ST	–	–	–	–	–	(+)	(+)	(+)	–	–	–	0.105
	ST2	–	–	–	–	–	–	–	–	–	–	–	–
	ST3	–	–	–	–	–	–	–	–	–	–	–	–

Legend: Symbols (+) and (–) denote significant positive or negative seasonal or trend effects, respectively.

The majority of processes which were subject to examination of the trend-and-seasonal structure, generally shows seasonal effects in single months and clear trend effects. Periodical smoothing permits, in the prevailing number of cases, to clear particular processes of seasonal components with a simultaneous maintenance of a distinct trend structure. Process ZAP is exceptional (stocks – table 2) which is characterized by strong seasonal effects over 5 subsequent months and even if subject to annual smoothing, it shows certain vestigial seasonal effects, whereas both the linear and the higher-degree trend were nearly totally eliminated.

For processes with significant seasonal-and-trend components, these models explain a considerable part of the variation of these processes, which in some cases exceeds 90%.

5. Results of integration analysis

The selected best versions of particular original and periodically smoothed processes were subject to integration testing with the use of either DF or ADF tests where critical values for the 5% significance level come from the work of Charemza and Deadman [1997] (pp.256–258).

Table 3. Specification of processes I(1) (the best versions after elimination of trend-and-seasonal components)

No.	X_t	$X_t \sim I(1)$	$SX_t \sim I(1)$	$SW3X_t \sim I(1)$	$SW6X_t \sim I(1)$	$SW12X_t \sim I(1)$
1	AMO	–	–	–	–	–
2	KUP	KUP	SKUP	ST3W3SKUP	STW6SKUP	STW12SKP
3	NAL	NAL	STNAL	STW3NAL	STW6NAL	STW12NAL
4	PRP	PRP	–	–	–	–
5	PZS	PZS	–	ST3W3PZS	W6PZS	W12PZS
6	SKU	SKU	–	–	–	–
7	SPR	SPR	–	–	–	–
8	UBO	UBO	–	–	–	–
9	WYN	WYN	–	ST3W3WYN	W6WYN	STW12WYN
10	ZAP	–	–	STW3ZAP	STW6ZAP	STW12ZAP
11	ZAT	–	ST3ZAT	–	–	–
12	ZOB	ZOB	–	–	–	–

The lack of elimination of trend-and-seasonal components in the case of eight out of twelve processes of integration analysis leads to the false conclusions about stationarity or integration I(1) of a given process, where in seven cases an uncleared process I(1) appears, to be a stationary process after the elimination of deterministic trend-and-seasonal components, and in one case the false inference of this type has an inverse character (ZAT).

In all these cases periodical smoothing leads to equal conclusions from the point of view of integration testing. With reference to three processes, the stationary, cleared, original processes (PZS.WYN.ZAP) become first order integrated processes after periodical smoothing, which makes detection of long–term dependencies between these processes possible.

6. Results of the co-integration analysis

According to the idea of Engle and Granger [1987], three cleared original processes I(1) i.e. SKUP.STNAL and ST3ZAT as well as five periodically smoothed processes in three variants (W3, W6 and W12) were subject to a be-factorial co-integration analysis (in pairs) and then vector integration respectively for three original processes I(1) and five periodically smoothed processes in three variants.

The results of co-integration analysis are shown in Tables 4.1. to 4.4., whereat critical values of tests DF and ADF for the 5% level of significance were derived from the work of Charemza and Deadman [1997] (pp. 256–258).

Table 4.1. The testing for be-factorial co-integration of processes SX_t ~I(1)

SX_t ~I(1)	ST3ZAT	STNAL
SKUP	(**) none	(*) CI (1,1)
STNAL	(**) none	

Appendix to Table 4.1. Co-integration testing for the vector of processes [$SKUP_t$, $STNAL_t$, $ST3ZAT_t$]
DW = 2.057, aug. = 0, DF = –4.892 (**) => CI(1,1)
Intervals of non-conclusiveness of test DF/ADF:
(*) without an intercept, n = 100, m = 1 => (–2.87; –2.64)
(**) without an intercept, n = 100, m = 2 => (–3.36; –3.20)

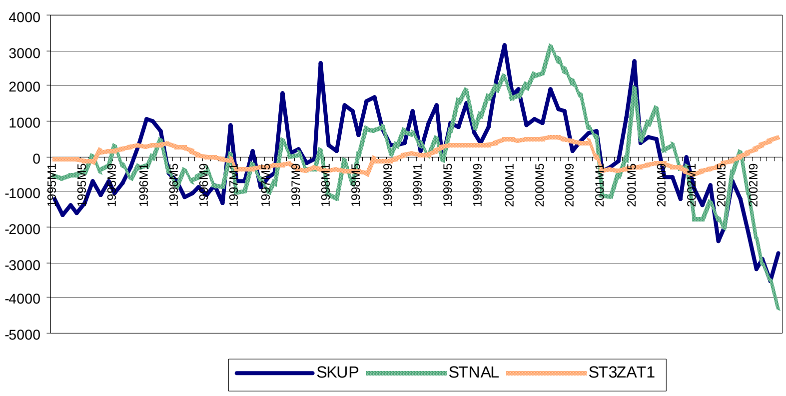

Fig. 1. Original processes SX_t ~I(1) (the best versions after elimination of trend-and-seasonal components)

Table 4.2. The testing for be-factorial co-integration of processes $SW3X_t$~I(1) (the best versions after elimination of trend-and-seasonal components)

$SW3X_t$ ~I(1)	STW3ZAP	ST3W3WYN	ST3W3PZS	STW3NAL
ST3W3KUP	(*) CI (1,1)	(*) CI (1,1)	(*) CI (1,1)	(*) CI (1,1)
STW3NAL	(*) none	(*) none	(*) none	
ST3W3PZS	(*) CI (1,1)	(*) CI (1,1)		
ST3W3WYN	(*) CI (1,1)			

Appendix to Table 4.2. Co-integration testing for the vector of processes $[ST3W3KUP_t\,,\, STW3NAL_t\,,\, ST3W3PZS_t\,,\, ST3W3WYN_t\,,\, STW3ZAP_t]$

DW = 1.867, aug. = 1, ADF = –6.072 (***) => CI(1,1)

Intervals of non-conclusiveness of test DF/ADF :
(*) without an intercept, n = 100, m = 1 => (–2.87; –2.64)
(**) without an intercept, n = 100, m = 2 => (–3.36; –3.20)
(***) without an intercept, n = 100, m = 4 => (–4.47; –3.81)

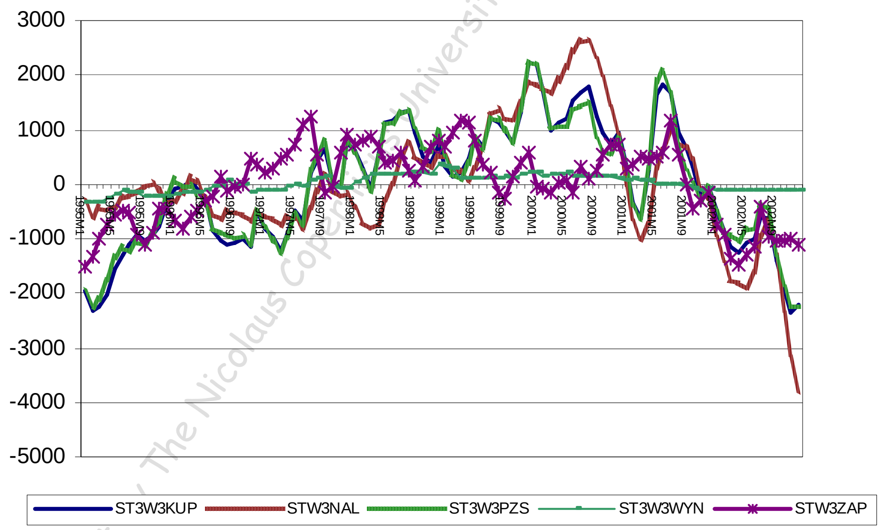

Fig. 2. Periodically smoothed processes $SW3X_t$ ~I(1) (the best versions after elimination of trend-and-seasonal components)

Table 4.3. The testing for be-factorial co-integration of processes $SW6X_t$ ~I(1) (the best versions after elimination of trend–and–seasonal components)

$SW6X_t$ ~I(1)	STW6ZAP	W6WYN	W6PZS	STW6NAL
STW6KUP	(*) none	(**) none	(**) CI (1,1)	(*) none
STW6NAL	(*) none	(**) none	(**) none	
W6PZS	(**) none	(**) none		
W6WYN	(**) none			

Appendix to Table 4.3. Co-integration testing for the vector of processes
[$STW6KUP_t$, $STW6NAL_t$, $W6PZS_t$, $W6WYN_t$, $STW6ZAP_t$]
DW = 1.908, Aug. = 2, ADF = –3.582 (***) => none CI(1,1)
Intervals of non-conclusiveness of test DF / ADF :
(*) without an intercept, n = 100, m = 1 => (–2.87;–2.64)
(**) without an intercept, n = 100, m = 2 => (–3.36;–3.20)
(***) without an intercept, n = 100, m = 4 => (–4.47;–3.81)

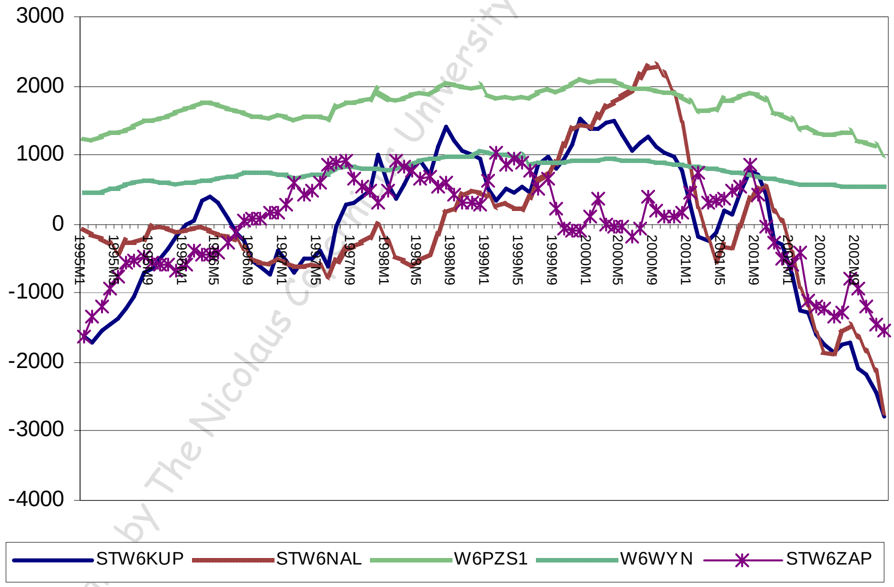

Fig. 3. Periodically smoothed processes $SW6X_t$ ~I(1) (the best versions after elimination of trend-and-seasonal components)

Table 4.4. The testing for be–factorial co-integration of processes $SW12X_t$ ~I(1) (the best versions after elimination of trend-and-seasonal components)

$SW12X_t$ ~I(1)	STW12ZAP	STW12WYN	W12PZS	STW12NAL
STW12KUP	(**) none	(**) CI (1,1)	(*) none	(**) CI (1,1)
STW12NAL	(**) none	(**) none	(*) none	
W12PZS	(*) CI (1,1)	(*) CI (1,1)		
STW12WYN	(**) none			

Appendix to Table 4.4. Co-integration testing for the vector of processes
[$STW12KUP_t$, $STW12NAL_t$, $W12PZS_t$, $STW12WYN_t$, $STW12ZAP_t$]
DW = 2.059, Aug = 3, ADF = –2.111 (***) => none CI(1,1)
Intervals of non-conclusiveness of test DF / ADF :
(*) without an intercept, n = 100, m = 1 => (–2.87; –2.64)
(**) without an intercept, n = 100, m = 2 => (–3.36; –3.20)
(***) without an intercept, n = 100, m = 4 => (–4.47; –3.81)

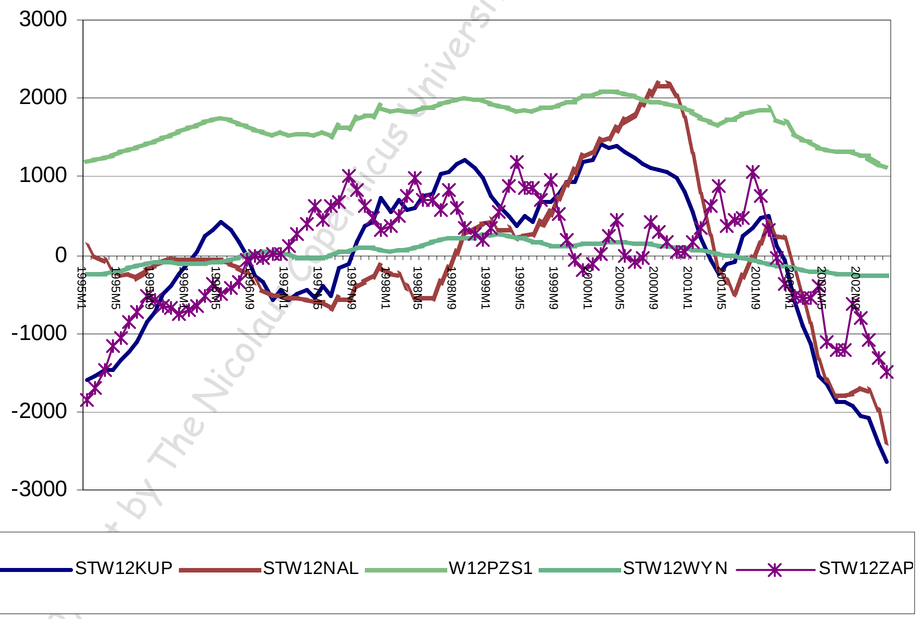

Fig. 4. Periodically smoothed processes $SW12X_t$ ~I(1) (the best versions after elimination of trend-and-seasonal components)

The testing of be-factorial co-integration (in pairs) for the three original processes which, after the elimination of trend-and-seasonal components, appeared to be first-order integrated processes SX_t ~I(1), makes it possible to infer that only one out of three potential pairs of processes of this type are co-integrated. However, the vector which contains all three processes together constitutes a set of co-integrated processes, which leads to the conclusion that there is a long–term dependence among these processes.

Among the five processes $SW3X_t$ ~ I(1) smoothed periodically in quarterly cycles and cleared of trend-and-seasonal components, out of the potential nine co-integrative pairs only three are not co-integrated processes. The analysis of vector co-integration for the five processes of this type permits us to infer that there is a strong long-term dependence of these processes in the understanding of co-integration CI(1,1).

Other conclusions result from the analysis of co-integration of five periodically smoothed processes in cycles longer than 3 months. After the elimination of trend-and-seasonal components only four out of ten pairs of smoothed processes in one-year cycles $SW12X_t$ ~ I(1) and only just one out of ten smoothed pairs in semi-annual cycles $SW6X_t$ ~ I(1) show long-term CI(1,1) type dependencies. In both cases, the vector co-integration analysis for sets of five periodically smoothed processes leads to the conclusion that there is no co-integration between these processes.

7. Conclusions

a) Empirically analysed processes are characterized by a distinct trend structure and in most cases also with seasonal effects.
b) Periodical smoothing generally permits the elimination of seasonal components with a clear trend structure retained. The ZAP process is an exception as it is characterized by a strong seasonality for which even the annual smoothing does not permit a full elimination of seasonal components.
c) The lack of elimination of trend-and-seasonal components leads most often to a false inference that the process examined is either stationary or integrated [6].
d) Periodical smoothing of monthly observations makes it possible to detect long-term dependencies between co-integrated processes. This refers especially to the quarterly smoothing.

[6] Issues related to the elimination of a trend from non-stationary processes are discussed, among others, in the works of Kufel (2002) and Piłatowska (2003).

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