DYNAMIC ECONOMETRIC MODELS
Vol. 6 – Nicolaus Copernicus University – Toruń – 2004

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The TAR-GARCH Models with Application to Financial Time Series

1. Introduction

The most distinctive feature of a threshold model is its ability to analyse a complex stochastic system by decomposing it into simpler subsystems. The basic idea is the local approximation over the states, i.e. the introduction of regimes via thresholds (see: Tong (1990)). On the other hand, a threshold model consists of several linear models nested in a non-linear structure. Its idea comes form the switching regression concept which capture a variety of different types of switching rules. The „threshold principle" in time series modelling was introduced by Tong in 1977, and developed by Tong and Hansen (see: Tong (1990) and Hansen (1996)). Threshold models are widely used in the analysis of many economic processes such as business cycles (Proietti (1998)) and unemployment (Hansen (1997)).

Models of financial returns usually combine two parts: i.e. the conditional mean and the conditional variance. One of the simple univariate cases is the ARMA-GARCH representation. On the other hand, financial time series (mainly returns) are frequently asymmetrically distributed. This is because investors may react in one way in the case of high returns and in another when the returns are low. It seems that the threshold models may be a very useful alternative in the analysis of the asymmetric behaviour of the investors.

In the presented paper the class of TAR-GARCH models is used to describe both a conditional mean due to regimes given by threshold parameters and a conditional variance. The aim of the paper is to present some methods of inference within a threshold framework with application to Polish financial time series. The paper consists of five sections. In the next section the model is considered. The third section presents the statistical inference using the self-exciting threshold autoregressive model, namely testing for linearity in the presence of ARCH, parameter estimation and forecasting. The empirical results are presented in the section 4. The final remarks are summed up in the fifth section.

2. Threshold model representations

Let Y_t denote k-dimensional random vector. Let us consider the model

$$Y_t = B^{J_t} Y_t + A^{J_t} Y_{t-1} + H^{J_t} \varepsilon_t + C^{J_t}, \tag{2.1}$$

where J_t is a random variable taking values of a finite set of natural numbers $\{1,2,3,\dots,p\}$, B^{J_t}, A^{J_t}, H^{J_t} are $k\times k$ - dimensional matrices of the coefficients, ε_t is the k – dimensional white noise and C^{J_t} is a constant vector. The model (2.1) is called a canonical form of the threshold model. It defines a wide class of the models depending on the choice of J_t.

When J_t is the function of Y_t., we obtain SETAR models (self-exciting threshold autoregressive model). The SETAR(p; k_1, k_2,...,k_p) model is defined in the following way

$$Y_t = \alpha_0^j + \sum_{i=1}^{k_j} \alpha_i^j Y_{t-i} + h^j \varepsilon_t \tag{2.2}$$

conditionally on $Y_{t-d} \in R_j, j = 1,\dots,p$.

The more convenient form of (2.2) is the following

$$Y_t = \begin{cases} \alpha_0^1 + \alpha_1^1 Y_{t-1} + \dots + \alpha_{k_1}^1 Y_{t-k_1} + h^1 \varepsilon_t & for\ Y_{t-d} \le r_1 \\ \alpha_0^2 + \alpha_1^2 Y_{t-1} + \dots + \alpha_{k_2}^2 Y_{t-k_2} + h^2 \varepsilon_t & for\ r_1 < Y_{t-d} \le r_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & \\ \alpha_0^p + \alpha_1^p Y_{t-1} + \dots + \alpha_{k_p}^p Y_{t-k_p} + h^p \varepsilon_t & for\ Y_{t-d} > r_{p-1} \end{cases}. \tag{2.3}$$

The threshold variable is in (2.3) lagged Y_t, but it can be also an exogenous variable, say lagged Z_t.

For two regimes we have the following *I(y)* function

$$I(y) = \begin{cases} 0 & when\ y \le 0 \\ 1 & when\ y > 0 \end{cases}. \tag{2.4}$$

and the corresponding SETAR(2, k, k) model

$$\begin{aligned} Y_t = &(\alpha_0 + \alpha_1 Y_{t-1} + \dots + \alpha_k Y_{t-k}) + \\ &(\beta_0 + \beta_1 Y_{t-1} + \dots + \beta_k Y_{t-k}) \cdot I(Y_{t-d}) + \varepsilon_t \end{aligned}. \tag{2.5}$$

When all $\beta_0, \beta_1, \dots, \beta_k$ parameters are zeros then (2.5) becomes the linear autoregressive model.

Letting ε_t to be a martingale difference sequence, we can modify the classic SETAR model by allowing conditional heteroscedasticity. Let us consider the case when the conditional variance changes over time, but it does not change within the regimes. As the result we have the second equation defining a GARCH-type model

$$\varepsilon_t | \Psi_{t-1} \sim N(0, h_t) \tag{2.6}$$

where:

$$h_t = a_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} , \tag{2.7}$$

$p \geq 0$, $q > 0$ and $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i = 1,2,\dots,q$, $\beta_i \geq 0$ for $i = 1,2,\dots,p$. (see: Bollerslev (1986)).

3. Statistical inference within the TAR framework

3.1. Testing for the TAR model vs. the linear one in the presence of ARCH

In testing for threshold non-linearity vs. the linear alternative (e.g. $H_0 : \alpha = \beta$ in (2.5)) one has to remember that the threshold parameter r is unknown and unidentified, as a rule. Thus the asymptotic distribution of LM statistics is non-standard. Usually the LR type tests are used. The testing procedure while the residuals constitute the white noise process is described in Tong (1990) and Osińska, Witkowski (1997).

Hansen (1996), (1997) indicates, that the presence of ARCH affects the testing for non-linearity in the TAR models. In the case of the changing the conditional variance the following procedure is recommended. An appropriate test is the Wald statistics, which is consistent in the case of heteroscedasticity. It is constructed for each value of the threshold parameter r. The test has the following form:

$$W_n(r) = (R\theta(r))' \left[R\left(M_n(r)\right)^{-1} V_n(r) \left(M_n(r)\right)^{-1} R' \right]^{-1} R\theta(r) \tag{3.1}$$

where

$\theta = [\alpha \quad \beta]$,

$R = [I \quad -I]$,

$M_n(r) = \sum y_t(r) y_t(r)'$,

$V_n(r) = \sum y_t(r) y_t(r)' \varepsilon_t^2$,

$y_t(r)$ - is a set of lagged values of Y_t in each regime.

An appropriate statistics for H_0 is

$$W_n = \sup_{r \in R} W_n(r). \tag{3.2}$$

The critical values are generated using the bootstrap technique in the following way: let u_t^* be a sequence of random numbers such as $u_t^* \sim n.i.d.$, t=1,2, ..., n and let $x_t^* = \varepsilon_t u_t^*$. Using empirical observations y_t, regress x_t^* conditionally on y_t and $y_t(r)$. Taking the first regression we obtain the residual variance σ_t^{*2}, and the second regression gives $\sigma_t^{*2}(r)$. Assuming that W_n statistics converges to the F distribution, which is the limit distribution when the threshold parameter r is known, we may compute $F_n^*(r) = n\left(\sigma_n^{*2} - \sigma_n^{*2}(r)\right)/\sigma_n^{*2}$ and $F_n^* = \sup_{r \in R} F_n^*(r)$. Hansen (1996) showed that the distribution of F_n^* converges to W_n distribution, then repeating the bootstrap procedure, and computing F_n^* we obtain the asymptotic distribution of W_n. The asymptotic p-values are given by adding the ratio of bootstrap samples for which F_n^* exceeds the computed value of W_n.

3.2. The parameter estimation of the TAR model

The parameters of the TAR models are estimated using the OLS method, conditionally on whether the parameters d, r and k are known or not. The parameters are usually not known and have to be estimated (see: Witkowski (1999)).
Let us consider the following modification of (2.3) model:

$$Y_t = \begin{cases} \alpha_0^1 + \alpha_1^1 Y_{t-1} + \ldots + \alpha_{k_1}^1 Y_{t-k_1} & \text{for } Y_{t-d} < r \\ \alpha_0^2 + \alpha_1^2 Y_{t-1} + \ldots + \alpha_{k_2}^2 Y_{t-k_2} & \text{for } Y_{t-d} \geq r \end{cases} \tag{3.3}$$

The estimation proceeds in two steps (see Tong (1983), (1990)):

The estimation of parameters standing with lagged variables with fixed *d, r*, *k*$_1$, *k*$_2$

Let

$$\overline{\alpha}_i = \left[\alpha_0^i, \alpha_1^i, \ldots, \alpha_{k_i}^i\right] \quad i = 1, 2,$$

$$k = \max\left(k_1, k_2, d\right).$$

The data $\left[y_{k+1}, \ldots, y_N\right]$ may be divided into two groups $\overline{y}_1, \overline{y}_2$ satisfying:

$$\begin{aligned} y_j \in \overline{y}_1 &\Leftrightarrow y_{j-d} < r, \\ y_j \in \overline{y}_2 &\Leftrightarrow y_{j-d} \geq r. \end{aligned} \tag{3.4}$$

Let

$$\bar{y}_1 = \left[y^1_{j_1}, y^1_{j_2}, \ldots, y^1_{j_{n_1}} \right], \quad y_2 = \left[y^2_{j_1}, y^2_{j_2}, \ldots, y^2_{j_{n_2}} \right],$$

$$n_1 + n_2 = N - k,$$

and

$$A_i = \begin{bmatrix} 1 & y^i_{j_1-1} & y^i_{j_1-2} & \cdots & y^i_{j_1-k_i} \\ 1 & y^i_{j_2-1} & y^i_{j_2-2} & \cdots & y^i_{j_2-k_i} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & y^i_{j_{n_i}-1} & y^i_{j_{n_i}-2} & \cdots & y^i_{j_{n_i}-k_i} \end{bmatrix} \quad i = 1,2.$$

The estimate of $\bar{a}_i$ may be expressed in the following way:

$$\bar{a}_i = \left(A_i^T A_i \right)^{-1} A_i^T \bar{y}_i, \quad i = 1,2.$$

The estimation of all parameters set

Let d, r be fixed at d_0 r_0. Let L denote the maximum order for each linear autoregressive model within the regimes. Denote:

$$AIC(d_0, r_0) = AIC(\hat{k}_1) + AIC(\hat{k}_2), \tag{3.5}$$

where

$$AIC(\hat{k}_1) = \min_{0 \le k_1 \le L} \left[n_1 \ln\left\{ \|\varepsilon_1\|^2 / n_1 \right\} + 2(k_1 + 1) \right],$$

$$AIC(\hat{k}_2) = \min_{0 \le k_2 \le L} \left[n_2 \ln\left\{ \|\varepsilon_2\|^2 / n_2 \right\} + 2(k_2 + 1) \right], \tag{3.6}$$

$$\bar{\varepsilon}_i = \bar{y}_i - A_i \bar{a}_i, \ i = 1,2. \tag{3.7}$$

Hence, minimising (3.6) we obtain $\hat{k}_1$ and $\hat{k}_2$ with fixed d, r. Under (3.5), $AIC(d_0, r_0)$ is determined.

Finally, we estimate the delay parameter d and threshold parameter r:

$$AIC(\hat{d}, \hat{r}) = \min_{d \in \{1,2,\ldots,T\}} \left\{ \min_{r \in \{\tau_1, \tau_2, \ldots, \tau_m\}} AIC(d, r) \right\} \tag{3.8}$$

where T is the maximum value of d and $\{\tau_1, \tau_2, \ldots, \tau_m\}$ is a set of potential candidates for the estimation of r.

3.3. Forecasting procedures using threshold models

Forecasting based on the non-linear models is mostly often based on the Monte Carlo method (see Brown, Mariano (1984), Clements, Smith (1997)). The MC method gives an asymptotically unbiased predictor, while the standard deterministic predictor is usually biased. Taking a great number of replications the MC predictor is usually more efficient – taking the mean squared error – than the deterministic one. There are however some disadvantages. The strong requirement of the MC method is a prior assumption of the innovations distributions. While it is improperly specified, the predictor becomes asymptotically biased. The alternative method is based on the bootstrap technique, which uses the estimated residuals of the model instead of the generated innovations.

Three methods of forecasting the threshold models are discussed below: the mean squared error method, the Monte Carlo and the bootstrap.

The mean squared forecast error method

The mean squared forecast error method allows computing forecasts using any type of the TAR model. For the model (2.5) the practical way of taking the forecast is to compute a weighted average of the forecasts given separately from the first and second regimes. The weights are usually the probabilities that the forecasted series is in the first or in the second regime within the forecast horizon. Thus we have:

$$\hat{Y}_{n+k} = p_{k-1}\hat{Y}_{1,n+k} + \left(1 - p_{k-1}\right)\cdot \hat{Y}_{2,n+k} + \left(a_{2,1} - a_{1,1}\right)\hat{\sigma}_{n+k-1}\phi\left(\frac{r - \hat{Y}_{n+k-1}}{\hat{\sigma}_{n+k-1}}\right), \tag{3.9}$$

where

$k = 2,3....,$

$\hat{Y}_{1,n+k} = a_{1,0} + a_{1,1}\hat{Y}_{n+k-1}, \quad \hat{Y}_{2,n+k} = a_{2,0} + a_{2,1}\hat{Y}_{n+k-1},$

$p_{k-1} = \Phi\left(\frac{r - \hat{Y}_{n+k-1}}{\hat{\sigma}_{n+k-1}}\right).$

Φ, ϕ denote correspondingly the standard normal cumulative distribution and density $N(0,1)$. The formula (3.9) is recursive. The first step of the procedure is as follows:

$\hat{Y}_{n+1} = a_0 + a_1 Y_n + \left(b_0 + b_1 Y_n\right)\cdot I_n(r).$

The formula (3.9) requires the standard error of prediction $\hat{\sigma}_{n+k-1}$ to be estimated. It can be computed in the following way:

$$
\begin{aligned}
\hat{\sigma}_{n+k}^{2} &= \left\{\left(a_{1,0}+a_{1,1}\hat{Y}_{n+k-1}\right)^{2}+a_{1,1}^{2}\hat{\sigma}_{n+k-1}^{2}\right\}p_{k-1}+\\
&\left\{\left(a_{2,0}+a_{2,1}\hat{Y}_{n+k-1}\right)^{2}+a_{2,1}^{2}\hat{\sigma}_{n+k-1}^{2}\right\}\left(1-p_{k-1}\right)+\\
&+\left\{\begin{array}{l} a_{2,1}^{2}\left(r-\hat{Y}_{n+k-1}\right)+2a_{2,1}\left(a_{2,0}+a_{2,1}\hat{Y}_{n+k-1}\right)-\\ \left(a_{1,1}^{2}\left(r-\hat{Y}_{n+k-1}\right)+2a_{1,1}\left(a_{1,0}+a_{1,1}\hat{Y}_{n+k-1}\right)\right)\end{array}\right\}\cdot\\
&\cdot\hat{\sigma}_{n+k-1}p_{k-1}+\sigma_{\varepsilon}^{2}-\hat{Y}_{n+k}^{2}.
\end{aligned}
$$

The above formula is proper only in the case when the residual variances in each regime are mutually equal to σ_{ε}^{2}..

The Monte Carlo method

The Monte Carlo method is a simple simulation based method of forecasting used for a broad class of the non-linear models. The forecast for one period ahead is identical, i.e.

$$\hat{Y}_{n+1}=a_0+a_1Y_n+\left(b_0+b_1Y_n\right)\cdot I_n(r). \tag{3.10}$$

For a longer forecast horizon ($k>1$) a following sequence of the forecasts is computed $\hat{Y}_{n+2}^{j},\hat{Y}_{n+3}^{j},...,\hat{Y}_{n+k}^{j}$, such as

$$\hat{Y}_{n+2}^{j}=a_0+a_1\hat{Y}_{n+1}+\left(b_0+b_1\hat{Y}_{n+1}\right)\cdot I_{n+1}(r)+\hat{\xi}_{2,j}^{h}, \tag{3.11}$$

$$\hat{Y}_{n+3}^{j}=a_0+a_1\hat{Y}_{n+2}^{j}+\left(b_0+b_1\hat{Y}_{n+2}^{j}\right)\cdot I_{n+2}(r)+\hat{\xi}_{3,j}^{h} \tag{3.12}$$

and

$$\hat{Y}_{n+k}^{j}=a_0+a_1\hat{Y}_{n+k-1}^{j}+\left(b_0+b_1\hat{Y}_{n+k-1}^{j}\right)\cdot I_{n+k-1}(r)+\hat{\xi}_{k,j}^{h}, \tag{3.13}$$

$$j=1,2,3,...N,$$

where $\xi_{2,j}^{h},\xi_{3,j}^{h},\xi_{k,j}^{h}$ constitute a set of independent random variables, normally distributed, independent from ε. The superscript h means, that the variance of the random variable depends on the regime of the process, i.e. $\xi_{i,j}^{h}\sim N\left(0,\sigma_h^2\right)$. Repeating the procedure given by the relations (3.11) - (3.13) for $j=1,2,3,...,N$ we are able to compute the final result as

$$\hat{Y}_{n+k}=\frac{1}{N}\sum_{j=1}^{N}\hat{Y}_{n+k}^{j}. \tag{3.14}$$

The bootstrap method

The idea is very similar to the Monte Carlo method, the difference is that the set $\hat{\xi}^h_{2,j}, \hat{\xi}^h_{3,j}, \hat{\xi}^h_{k,j}$ is the result of the independent sampling from the estimated error vectors $\hat{\varepsilon}_1, \hat{\varepsilon}_2$.

4. Estimated TAR-GARCH models for weekly returns from shares of banks

The objective of the research is the analysis of weekly returns of stocks listed at the Warsaw Stock Exchange using threshold models. The parameter estimates were obtained using EViews 4.0 software. The following assumptions were made:

a) there is one or two threshold parameters (i.e. two or three regimes),
b) the minimum and maximum value of parameter d is equal to one and five respectively,
c) the maximum order for each linear autoregressive model is equal to 5.

The results of the estimation are included in the following two tables. In the first table the SETAR-GARCH models are presented whereas in the second one the TAR-GARCH models are shown, where the Warsaw Stock Exchange Index is the threshold variable.

Table 1. Estimates of SETAR-GARCH model (see equation (2.3))

Independent	Parameters of SETAR-GARCH model							
WIG	α_0^1	h_1	r_1	α_0^2	α_1^2	α_2^2	α_3^2	h_2
	0.00469 (0.0026)	0.0403	0.00188	-0.0061 (0.0029)	-0.0712 (0.0765)	0.2437 (0.0719)	-0.1571 (0.0826)	0.0268
	r_2	α_0^3	α_1^3	h_3	d	SD		
	0.0187	0.00518 (0.0034)	0.1901 (0.096)	0.0395	4	0.0376		
	$h_t = \underset{(0.0001)}{0.000123} + \underset{(0.0246)}{0.0493}\,\varepsilon_{t-1}^2 + \underset{(0.0912)}{0.8604}\,h_{t-1}$							
BIG	α_0^1	α_1^1	h_1	r_1	α_0^2	α_1^2	α_2^2	α_3^2
	-0.0114 (0.0065)	-0.1840 (0.0914)	0.0809	0.00449	0.0128 (0.0074)	-0.0557 (0.1394)	0.0258 (0.0639)	0.1441 (0.0563)
	h_2	r_2	α_0^3	h_3	d	SD		
	0.0544	0.1194	-0.0086 (0.0351)	0.1646	1	0.0778		
	$h_t = \underset{(0.000148)}{0.0039} + \underset{(0.0525)}{0.152}\,\varepsilon_{t-1}^2 + \underset{(0.0436)}{0.158}\,\varepsilon_{t-2}^2$							

Table 1 continued

BOS	α_0^1	h_1	r_1	α_0^2	α_1^2	α_2^2	h_2	r_2
	0.00158 (0.0042)	0.0472	-0.0135	-0.0053 (0.0036)	-0.0294 (0.0848)	-0.2287 (0.0705)	0.0308	0.00609
	α_0^3	h_3	d	SD				
	0.00386 (0.0041)	0.0478	4	0.0442				
	not significant ARCH efect							
KREDYT	α_0^1	α_1^1	h_1	r_1	α_0^2	α_1^2	α_2^2	α_3^2
	-0.0023 (0.0027)	-0.1367 (0.0544)	0.0472	0.0134	0.00949 (0.0262)	-0.1700 (0.110)	-0.2978 (1.195)	0.2945 (0.113)
	α_4^2	h_2	r_2	α_0^3	h_3	d	SD	
	-0.3381 (0.108)	0.0416	0.0307	0.0105 (0.0069)	0.0695	2	0.0521	
	$h_t = \underset{(0.0000108)}{0.0000217} + \underset{(0.0107)}{0.0521}\,\varepsilon_{t-1}^2 + \underset{(0.0109)}{0.939}\,h_{t-1}$							
KREDYT*	α_0^1	α_1^1	α_2^1	α_3^1	h_1	r_1	α_0^2	h_2
	-0.0026 (0.0028)	-0.1347 (0.0493)	-0.0261 (0.0657)	0.1013 (0.0489)	0.0469	0.0307	0.0105 (0.0069)	0.0695
	d	SD						
	2	0.0525						
	$h_t = \underset{(0.0000108)}{0.0000212} + \underset{(0.0107)}{0.0521}\,\varepsilon_{t-1}^2 + \underset{(0.0109)}{0.939}\,h_{t-1}$							

Table 2. Estimates of TAR-GARCH model (see equation (2.3))

Independent	Parameters of TAR-GARCH model							
BPH	α_0^1	h_1	r_1	α_0^2	α_1^2	α_2^2	α_3^2	α_4^2
	-0.0005 (0.0035)	0.0455	-0.0094	0.00627 (0.0030)	-0.101 (0.081)	0.0422 (0.067)	-0.0492 (0.065)	-0.0648 (0.066)
	α_5^2	h_2	r_2	α_0^3	h_3	d	SD	
	0.1928 (0.069)	0.0577	0.0700	0.0331 (0.017)	0.0776	1	0.0541	
	not significant ARCH effect							
BRE	α_0^1	α_1^1	h_1	r_1	α_0^2	α_1^2	α_2^2	α_3^2
	-0.0006 (0.0091)	0.2834 (0.116)	0.0643	-0.0431	-0.0284 (0.023)	0.6890 (0.271)	-0.0305 (0.306)	-0.6351 (0.280)
	α_4^2	h_2	r_2	α_0^3	h_3	d	SD	
	-0.9120 (0.367)	0.0870	-0.0319	0.00308 (0.0030)	0.0581	4	0.0601	
	$h_t = \underset{(0.00024)}{0.00305} + \underset{(0.0669)}{0.1605}\,\varepsilon_{t-1}^2$							

Table 2 continued

BZWBK	α_0^1	h_1	r_1	α_0^2	α_1^2	α_2^2	h_2	r_2
	-0.0028 (0.0074)	0.0649	-0.0318	-0.0008 (0.0057)	-0.4713 (0.124)	-0.266 (0.076)	0.0458	-0.0094
	α_0^3	α_1^3	α_2^3	h_3	d	SD		
	0.0072 (0.0046)	-0.0871 (0.069)	0.1544 (0.064)	0.0673	1	0.0627		
	not significant ARCH effect							

* denotes SETAR with 2 regimes, SD denotes standard deviation of residuals.

Taking into account the above results the conditionally heteroscedastic effect was rather strong. There is only one threshold model with two regimes due to models with three regimes fitted to data for almost all series better (according to AIC criterion) than the others. It should be said, that linear models explained dynamic of returns worse than threshold ones so they were out of our interest.
The estimated models show that rates of return behave in quite a different manner when compared between regimes. It can be specifically seen when the threshold variable changes its sign. For example, for BPH, we observe constant value with no autocorrelation when WIG_{t-1} is less than –0,0094 and bigger than 0,0700 (regime I and III) while in the middle regime we have significant autocorrelation. Such a case is very often in the presented examples. This means that returns demonstrate stronger tendency to be serially correlated in the middle regime than in the extreme one.

The next table and graph shows what role the regimes play. Let us notice how often time series changes state over a whole considered time period.

Table 3. The "switching" property of SETAR model for BIG (see second row in table 1)

Regime	Number of observations	Number of switching between regimes	The longest period of staying in one regime
1	253	1→2: 93	8
		no switching: 147	
		1→3: 13	
2	176	2→1: 92	8
		no switching: 79	
		2→3: 5	
3	22	3→1: 14	4
		no switching: 4	
		3→2: 4	

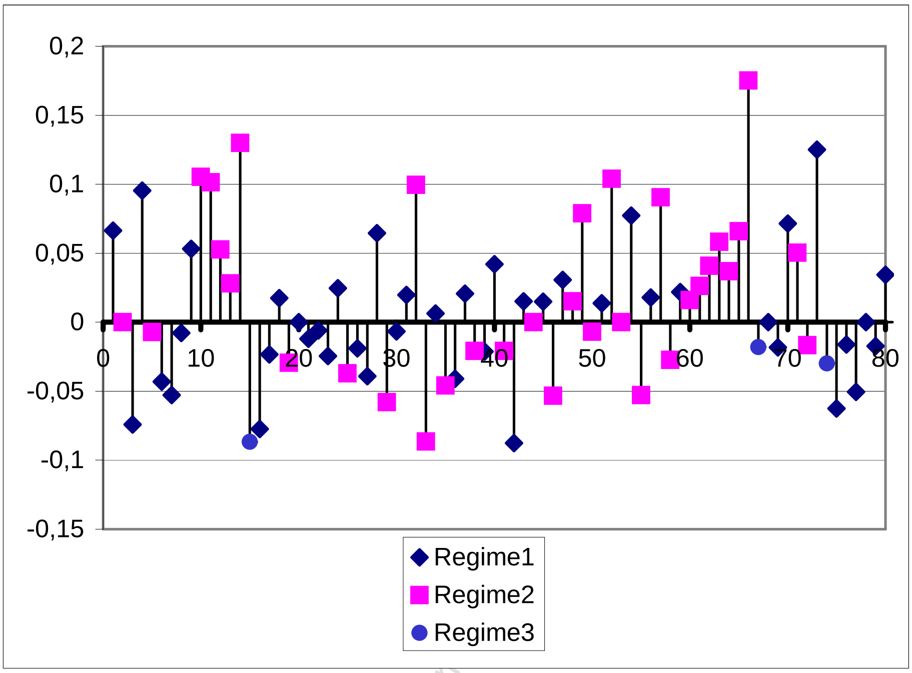

Fig. 1. First 80 observations for BIG return – regimes

The lagged variable determines the regime in the following way

$$BIG_t \in \text{Regime1} \Leftrightarrow BIG_{t-1} \in (-\infty, 0.00449)$$

$$BIG_t \in \text{Regime1} \Leftrightarrow BIG_{t-1} \in (0.00449, 0.1194)$$

$$BIG_t \in \text{Regime1} \Leftrightarrow BIG_{t-1} \in (0.1194, +\infty)$$

The prediction of returns in the case of the WSE stocks considered in section 4 within the threshold framework was presented in details in Jeziorska-Pąpka, Osińska, Witkowski (2004). The consistency of the direction of the forecasts was satisfactory in general. It was independent of the chosen method of forecasting. In many cases the forecast direction was the same as the realisation in 80% and even in 100%. The forecasting using threshold stationary models is recommended for shorter horizons (up to 5 periods ahead).

6. Final remarks

Non-linear models belonging to the switching regression class do not describe stock returns with the highest precision. We can still observe some other non-linearities, for example the GARCH effect. However taking the TAR-GARCH combination gives better results then the AR-GARCH, which was the most popular in the literature. The TAR-GARCH model is better from the loss of information point of view as well as for its forecasting behaviour.

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