DYNAMIC ECONOMETRIC MODELS
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Maria Szmuksta-Zawadzka, Jan Zawadzki
Technical University of Szczecin, Agricultural Academy of Szczecin

On Hierarchic Models for Decade Data with Seasonal Fluctuations

Hierarchic predictive modelling and forecasting economical variables with 12-month cycle seasonal fluctuations are discussed in Zawadzki (2003) and Szmuksta-Zawadza, Zawadzki (2002). As it has been proven, the essential advantage of these models is decreasing the number of estimated parameters. It allows for forecasting by using short time series limited in the extreme case to two cycles.

In the present paper hierarchic time-series models will be considered with decade fluctuations both for full series and series with unsystematic gaps.

A consequence of shortening the period from a month to a decade is an increase in the number of hierarchic regular models. The number of permutations and permutations with repeated divisors of cycle fluctuations, which satisfy the following conditions: they are not less than 2 and not greater than m/2 and their product equals the length of the fluctuation cycle, equals 25.

This number is composed of seven two-level models, twelve three-level models and six four-level models. The models for periodic fluctuations or relatively constant ones having 36-decade cycle together with the denotations of matrices corresponding to the hierarchy levels are given in Table 1.

The last column contains the numbers of the estimated parameters defining the seasonal fluctuations. The number ranges from 18 for two-factor models H218 and H182 to 6 for the four- factor models. We note for comparison that 35 parameters should be estimated in classical models. Thus in the case of a 36-decade cycle, advantages related to the increased number of degrees of freedom are entirely clear. Noncorrelation of the factors (hierarchy levels) for the considered model is an important property of the hierarchic models for full data. This property allows measuring the participation of variance of various factors in the explanation of the seasonal variance. In the case of the series with the

unsystematic gaps, e.g. when we have at least one observation for each sub-period this property does not appear.
For example a part of matrix X_{433} comprising the period of a year (36 decades for model H_{433}) is presented below

$X_{433} =$

KW1	KW2	KW3	MCKW1	MCKW2	DTD1	DTD2
1	0	0	1	0	1	0
1	0	0	1	0	0	1
1	0	0	1	0	-1	-1
1	0	0	0	1	1	0
1	0	0	0	1	0	1
1	0	0	0	1	-1	-1
1	0	0	-1	-1	1	0
1	0	0	-1	-1	0	1
1	0	0	-1	-1	-1	-1
0	1	0	1	0	1	0
0	1	0	1	0	0	1
0	1	0	1	0	-1	-1
0	1	0	0	1	1	0
0	1	0	0	1	0	1
0	1	0	0	1	-1	-1
0	1	0	-1	-1	1	0
0	1	0	-1	-1	0	1
0	1	0	-1	-1	-1	-1
0	0	1	1	0	1	0
0	0	1	1	0	0	1
0	0	1	1	0	-1	-1
0	0	1	0	1	1	0
0	0	1	0	1	0	1
0	0	1	0	1	-1	-1
0	0	1	-1	-1	1	0
0	0	1	-1	-1	0	1
0	0	1	-1	-1	-1	-1
-1	-1	-1	1	0	1	0
-1	-1	-1	1	0	0	1
-1	-1	-1	1	0	-1	-1
-1	-1	-1	0	1	1	0
-1	-1	-1	0	1	0	1
-1	-1	-1	0	1	-1	-1
-1	-1	-1	-1	-1	1	0
-1	-1	-1	-1	-1	0	1
-1	-1	-1	-1	-1	-1	-1
,	,	,	,	,	,	,
,	,	,	,	,	,	,
,	,	,	,	,	,	,

Source: Authors' elaboration.

Negative components of the matrix mean that the corresponding parameters are equal to zero.

The presented theoretical investigations will be illustrated by an empirical example concerning the forming of the decade deposits on the current accounts in Bank C. Four full 36-decade cycles constituted the estimating period. Equations were estimated using full series as well as series with gaps. It was assumed in the considered example that unsystematic gaps appear in the three consecutive decades of odd months. Due to the unsystematic type of gaps it was assumed that the data were accessible for February and April in the first year, June and August in the second year and October and December in the last year. Since the estimating period was a four-year time (144 decades), 54 gaps appear in the data.

Using the estimated equations, the interpolative (for 54 missing data) and extrapolative forecasts (for 9 decades) were developed. Both types of the forecasts were empirically verified which were obtained using the models with gaps while the extrapolative forecasts were verified only in the case of models without gaps.

Synthetic estimations of the stochastic structure parameters which are simultaneously the estimations of the predictive properties both for the case of models with and without gaps are given in Table 2. To distinguish between these types of the models, the models for the full data are denoted by P.

We note from the last column that the estimations of determination coefficients of equations without gaps are very similar. Maximum estimation of the parameter is for model P123 (0.9938), while the minimum for model P3322 (0.9870). The difference is thus 0.68%.

Estimations of the standard deviations of the random components are more diversified. They are in the range from 462,400 PLN for model P123 to 562,400 PLN for model P334. Three concentrations of the estimations can be seen. The first concentration is formed by models of estimations less than 420,120 PLN (eleven models). The second concentration is formed by models of the estimations in the range 450,000-470,000 PLN (four models). Ten models having estimations greater than 497,000 PLN fall in the third concentration.

Composition of the group, especially extreme groups, is not casual. A specific feature of the models of the first group is that the last number in the denotation of the model, being the divisor of the cycle length, is three or a multiplicity of three whereas the models having the divisors of two or a multiplicity of two belong to the third group.

The above-defined principle is related to the occurrence of the three-decade cycle in the month resulting from the payments in or out of the accounts.

Estimations of the determination coefficients of equations estimated using the data with unsystematic gaps are given in the seventh column of Table 2.

The estimations are little diversified, similarly to the case of the full data. This is why it is important to measure the estimation of the predictive capabilities using the estimations of the standard deviations of the random components. They are generally a few or sometimes even by more then ten percent more than

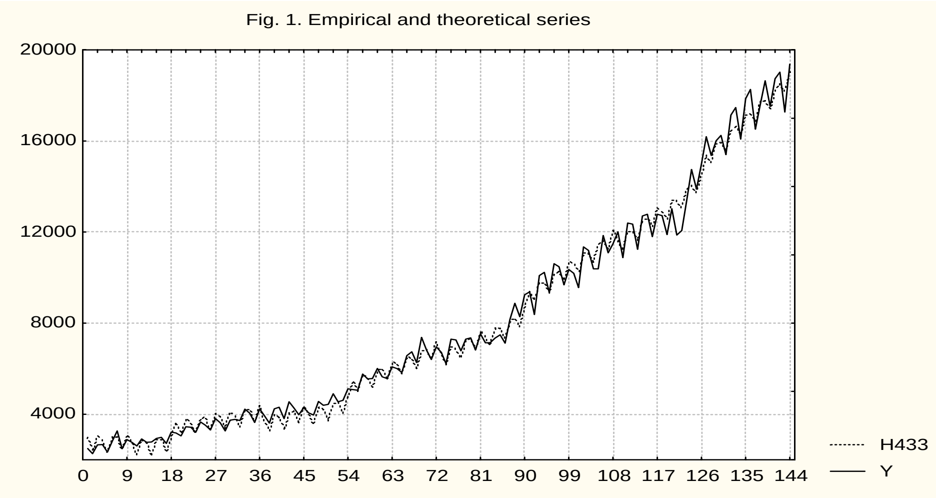
Table 2. Estimations of selected parameters of stochastic structure of hierarchic equations for decade data

No.	Models for full data				Models for nonsystematic gaps			
	Model	R^2	Se	DW	Model	R^2	Se	DW
1	P218	0.9934	418.95	1.41	H218	0.9936	472.22	1.58
2	P312	0.9916	462.40	1.47	H312	0.9913	511.3	1.44
3	P49	0.9932	414.00	1.41	H49	0.9935	452.23	1.64
4	P66	0.9929	420.12	1.40	H66	0.9933	457.86	1.62
5	P94	0.9892	522.36	2.04	H94	0.9914	522.15	1.96
6	P123	0.9938	397.15	1.51	H123	0.9936	455.85	1.67
7	P182	0.9907	497.53	1.87	H182	0.9924	524.49	1.87
8	P229	0.9931	414.21	1.40	H229	0.9933	547.17	1.62
9	P236	0.9929	418.12	1.39	H236	0.9931	459.07	1.62
10	P263	0.9933	406.84	1.43	H263	0.9931	456.67	1.62
11	P292	0.9900	500.30	1.80	H292	0.9911	525.05	1.9
12	P326	0.9913	461.84	1.15	H326	0.9911	517.06	1.51
13	P334	0.9870	562.40	1.74	H334	0.9893	565.72	1.79
14	P343	0.9916	454.85	1.17	H343	0.9917	497.42	1.44
15	P362	0.9894	535.35	1.55	H362	0.9898	555.35	1.74
16	P433	0.9931	410.23	1.42	H433	0.9933	447.24	1.63
17	P623	0.9929	417.24	1.40	H623	0.9932	452.93	1.61
18	P632	0.9897	503.32	1.74	H632	0.9913	515.21	1.91
19	P922	0.9892	520.48	2.04	H922	0.9914	518.93	1.97
20	P2233	0.9931	410.45	1.41	H2233	0.9931	452.07	1.61
21	P2323	0.9929	415.30	1.39	H2323	0.9930	454.24	1.61
22	P2332	0.9897	500.53	1.72	H2332	0.9911	514.39	1.9
23	P3223	0.9913	458.31	1.51	H3223	0.9917	495.58	1.43
24	P3232	0.9891	536.76	1.15	H3232	0.9897	551.77	1.72
25	P3322	0.9870	560.24	1.74	H3322	0.9893	562.39	1.78

Source: Authors' elaboration.

for the models without gaps. The least value of the standard deviation was obtained for model H433 (447,240 PLN) and the highest for model H334 (565,720 PLN). Division of the models into three concentrations, according to the standard deviations, is valid also for this model class.

The courses of the investigated variable (Y) and fitted values on the base of hierarchical model H433 are presented in Fig.1.



Source: Authors' elaboration.

As it has been earlier remarked using the estimated equations, inter- and extrapolative forecasts were calculated and the ex post analysis of their accuracy was carried out (for the models without gaps only the extrapolative forecasts were analysed). Mean relative errors of the inter- and extrapolative forecasts are given in Table 3. It follows from the third column that the estimations of mean relative errors of the interpolative forecasts were in the range from 4.61 (H123) to 7.50% (H922). Estimations less than five percent were obtained jointly for ten models, from five to six percent for four models, and more than six percent for ten models. The models of the third group belonged to the last case. Model H312 was the only model of this group with error estimation less than five percent.

Very interesting results were obtained for the extrapolation forecasts. Error estimations of the forecasts with gaps are generally less than the forecast errors obtained using models without gaps. The average difference was 0.63%. The least estimation among the models with gaps was for model H433 (2.57%) and among models without gaps P326 (2.73%). The greatest estimations were for H94 (3.35%) and P182 (4.08%), respectively.

For all the models with unsystematic gaps belonging to the first and second group the error did not exceed 3%. The only model of the third group with such an estimation was model H312. The limit for the models without gaps was a little higher, equal to 3.32%. The only model of the third group above the limit was model P312. Little higher effectiveness of the forecasts obtained using models with gaps can be explained by disturbance of the examined process in the periods where gaps appear.

Table 3. Estimations of mean relative errors of inter- and extrapolative forecasts (%)

Model	Models with gaps		Models without gaps	
	Interpolative forecast	Extrapolative forecast	Model	Extrapolative forecast
H218	5.95	2.98	P218	3.19
H312	4.88	2.94	P312	*3.03*
H49	5.54	2.69	P49	3.16
H66	4.99	2.93	P66	3.16
H94	7.48	3.35	P94	3.92
H123	4.61	2.77	P123	3.13
H182	6.59	2.83	P182	4.08
H229	4.91	2.72	P229	3.06
H236	4.73	2.83	P236	2.73
H263	4.77	2.63	P263	3.05
H292	6.27	3.32	P292	3.93
H326	*5.99*	2.80	P326	*2.73*
H334	7.42	3.25	P334	3.63
H343	*4.63*	2.74	P343	*2.82*
H362	6.40	3.18	P362	3.55
H433	5.51	2.57	P433	3.08
H623	4.94	2.89	P623	3.12
H632	6.57	3.33	P632	3.95
H922	7.50	3.32	P922	3.91
H2233	4.81	2.59	P2233	2.98
H2323	4.63	2.89	P2323	3.01
H2332	6.40	3.28	P2332	3.34
H3223	*4.88*	2.66	P3223	3.32
H3232	6.57	3.10	P3232	3.4
H3322	7.42	3.22	P3322	3.61

Source: Authors' elaboration.

It follows from the presented example that the hierarchic models with decade fluctuations can be successfully used for predictive modelling and forecasting of economical processes with decade fluctuations. They require estimation of the less number of parameters than the classical models. This conclusion is related to both full series and series with unsystematic gaps.

References

Zawadzki, J. (ed.) (2003), Application of hierarchic models of time series for forcasting of economical variables with seasonal fluctuations (in Polish), Agriculture University of Szczecin.

Szmuksta-Zawadzka, M., Zawadzki, J. (2002), Hierarchic models of time series with seasonal fluctuations. Construction. Estimation. Forecasting, in: Zeliaś, A., *Spatial and time modelling and forecasting of economical processes* (in Polish), Economic Academy of Cracow.

Table 1. Specification of regular hierarchic model for decade data

Model	Type of variation (first factor)	Matrix	Type of variation (second factor)	Matrix	Type of variation (third factor)	Matrix	Type of variation (fourth factor)	Matrix	Number of estimated parameters
1	2	3	4	5	6	7	8	9	10
H218	half-year in a year	PR	decade in a half-year	DPR					18
H312	four-month period in a year	CZM	decade in a four-month period	DCZM					13
H49	quarter in a year	KW	decade in a quarter	DKW					11
H66	two-month period in a year	DWM	decade in a two-month period	DDWM					10
H182	two-decade period in a year	BIR	decade in a two-decade period	DD					18
H123	month in a year	MCR	decade in a month	DTD					13
H94	four-decade period in a year	CZD	decade in a four decade period	DCZD					11
H263	half-year in a year	PR	month in a half-year	MCPR	decade in a month	DTD			8
H236	half-year in a year	PR	two month period in a half-year	DWMPR	decade in a two-month period	DDWM			8
H362	four-month period in a year	CZM	two-decade period in a four-month period	BICZM	decade in a two-decade period	DD			8
H326	four-month period in a year	CZM	two-month period in a four-month period	DMCZM	decade in a two-month period	DDWM			8
H433	quarter in a year	KW	month in a quarter	MCKW	decade in a month	DTD			7
H343	four-month period in a year	CZM	month in a four-month period	MCCZM	decade in a month	DTD			7
H334	four-month period in a year	CZM	four-decade period in a four-month period	CZDCZM	decade in a four-decade period	DCZD			7
H623	two-month period in a year	DWM	month in a two-month period	MCDM	decade in a month	DTD			8

Table 1 continued

1	2	3	4	5	6	7	8	9	10
H632	two-month period in a year	DWM	two-decade period in two-month period	BISZ	decade in a two-decade period	DD			8
H292	half-year in a year	PR	two-decade period in a half-year	BIPR	decade in a two-decade period	DD			
H229	half-year in a year	PR	quarter in a half-year	KPR	decade in a quarter	DKW			10
H922		CZD	two-decade period in a four decade period	BICZD	decade in a two-decade period	DD			10
H2233	half-year in a year	PR	quarter in a half-year	KPR	month in a quarter	MCKW	decade in a month	DTD	6
H2332	half-year in a year	PR	two-month period in a half-year	DWMPR	two-decade period in a two-month period	BISZ	decade in a two-decade period	DD	6
H2323	half-year in a year	PR	two-month period in a half-year	DWMPR	month in a two-month period	MCDW	decade in a month	DTD	6
H3322	four-month period in a year	CZM	four-decade period in a four-month period	CZDCZM	two-decade period in a four-month period	BICZD	decade in a two-decade period	DD	6
H3232	four-month period in a year	CZM	two-month period in a four-month period	DMCZM	two-decade period in a two-month period	BISZ	decade in a two-decade period	DD	6
H3223	four-month period in a year	CZM	two-month period in a four-month period	DMCZM	month in a two-month period	MCDM	decade in a month	DTD	6

Source: Authors' elaboration.