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The Stock Market, Elliott's Waves, Cones and Cylinders

Eppur si muove!
Galileo

I shall start with an anecdote. A student is taking an exam with Newton. Why does the earth revolve? I don't remember, I knew it but I've forgotten. Terrible! You've impoverished human kind! Nobody knows it and you had to forget. The author is also in the situation of that student and tries to answer the question why the rotational movement is common in nature. The only universal law of nature is the principle of equilibrium, sometimes interpreted as the law of symmetry. The whole of nature is in a dynamic equilibrium. Movement is the result of the principle of equilibrium. The constant tendency to even up potentials creates movement. The principle of equilibrium individualises particular elements and causes that every two leaves of the same tree, despite great similarities are so very different. All the known laws of nature can be always reduced to the principle of equilibrium. And the first one who gave this principle it's modern form was Newton, since the thought that action is reaction applies not only in mechanics but to the whole of science.

A circle in every point has the nature of a straight line, it is locally identical with a segment. This is clearly visible when we look at a circle with a large radius. But a circle is not a straight line; it has a richer personality. A set of real numbers with the operation of addition is a group, which is isomorphic with the set of positive numbers with the operation of multiplication. This isomorphism allows us to replace a difficult product with an easy sum. Isomorphism is here an exponential function or a function reverse to it – a logarithmic function. These are natural functions of growth. They include, undoubtedly true, the Malthus Law which states that natural development is exponential. The law of natural growth is expressed by a simple differential equation

$y' = py$

stating that an increment in every moment is proportional to the possessed resources. Because of known reasons, exponential development must be only in the short term.

Exponential growth changes into cyclical repetitions. These are truly the fundamentals of science of periodicity. The theory of cybernetic sets enables examining cycles because an object is an isolated entirety, a particle isolated by definition. The fundamental tool of description is an abstract creation called a vector which is an ordinary finite sequence of numbers. Vectors are states of cybernetic systems and the relation they fulfill are laws of science. This means that we generally assume the principle of numeric description of states of a cybernetic system. The description is more accurate the more component parts |a vector has, the more precise the measurement is. Every state is a vector; they are static models. Manifold in the space of states are the laws of science.

Economics is the science of rational management, of good housekeeping, and good management of assets. The fundamental principle of this science is rationality, *homo oeconomicus*. This is an ideal object, a theoretical model. From the principle of rationality, it follows that, together with the passage of time, the interest rate descends to zero. This is connected with the growth of efficiency of capital. Thrift is a natural tendency. In normal conditions, without wars and catastrophes, savings grow. Capital is a commodity. Demand and supply define an interest rate. With the growth of savings, the value of capital decreases (A. Smoluk, 2003). Hence we can derive a thesis that interest rates asymptotically decreases to zero (Fig. 1)

$\lim(p_n) = 0.$

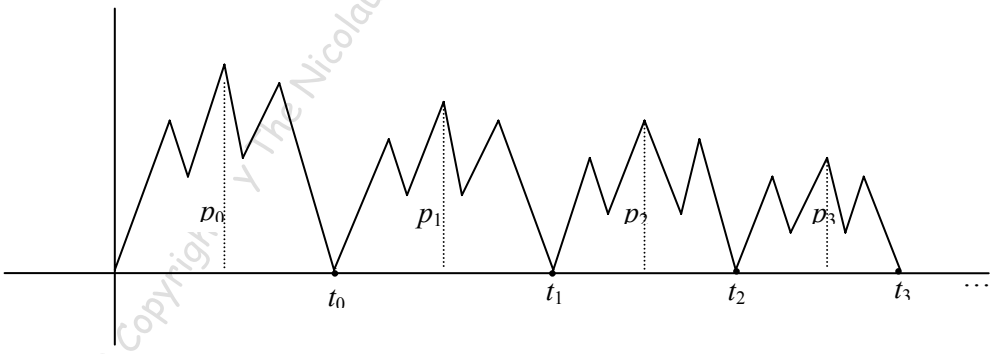

Fig. 1. Evolution of interest rate

An interest rate is defined as a charge for putting aside consumption. Interest rates levels off inflation. An interest rate is also a charge for risk. All these statements are not contradictory to our thesis. The average increment of capital

is zero; average means after including inflation and risk. There are more and more free means because we produce more than we consume. It is better to give than to take. The growth of efficiency increases the natural tendency to accumulate.

Because of the cyclical pulsating of the whole of nature, it seems justified to accept the hypothesis of circular time. Hawkin in his Theory of the Universe, introduces imaginary time (D. Filkin, 1998). Circular time is imaginary time because a circle

$$T = \{\exp(ix) : x \in R\}$$

is a group of complex numbers with a module1. Every function whose domain is circle T, is called a periodical function. The domain of two period functions is torus $T \times T$, and the domain of a multi-period function is a Cartesian product T^n. An interest rate defines a business cycle: from a zero interest rate at the moment t_n, to zero interest rate at t_{n+1}.

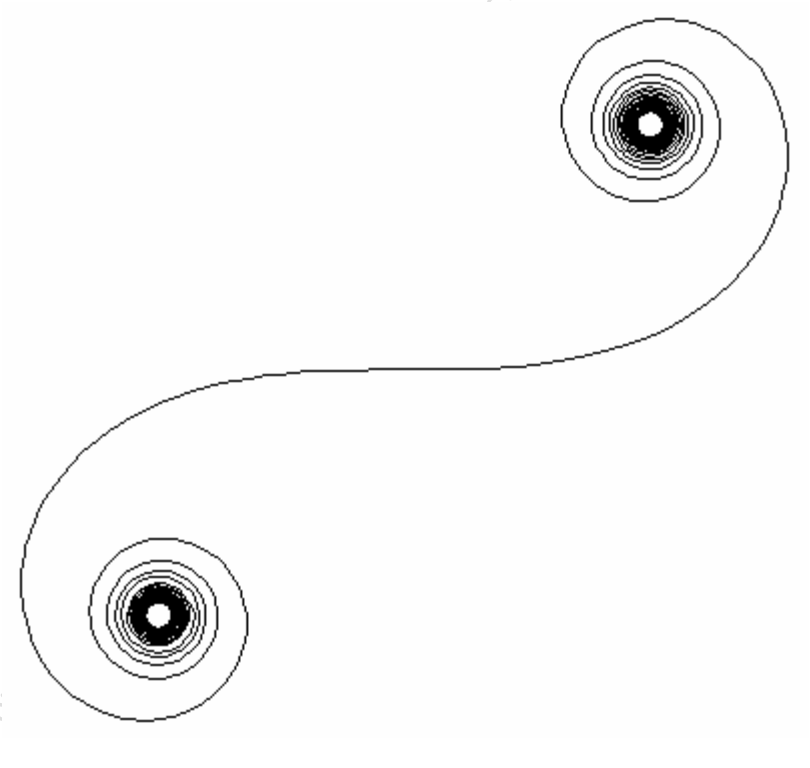

Fig. 2. Kloitoid

A kloitoid (Fig. 2) is a curve winding around two points with a parametrical equation

$$x(t) = \int_0^t \cos \tau^2 \, d\tau, \qquad y(t) = \int_0^t \sin \tau^2 \, d\tau.$$

One of these points is an attractor, and the other a repulsor. If we put this curve on a sphere, taking the distinguished points as poles, and then we identify the poles, then we can obtain a model of the stock market. The surface of the contracted doughnut represented in Fig. 4 is created from a sphere through the identification of the poles with the centre of the sphere. This is a geometrical model of a black hole, therefore a kind of pumping station. A kloitoid, which on the sphere became a spatial spiral, from the north approaches the particular point which is the point joining the north and south poles, and it distances itself from it from the south side. The particular point is the beginning and the end. Here everything commences its life in order that after reaching its destiny to finish its run there.

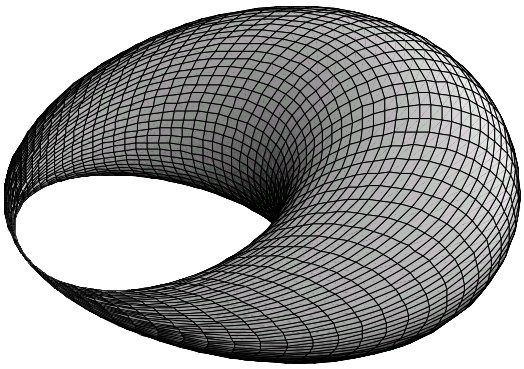

Fig. 3. Collar

A cybernetic system is a certain mathematical structure, which can be identified with an ordered pair

$$((X \times S, f, Y), (X \times S, g, S))$$

of two functions where sets X, S, and Y are one-argument relations, and the functional relations f, g are functions of two variables therefore three-argument relations. Without losing the general meaning, we can always assume that states

are vectors; elements of sets X, S, and Y are therefore vectors. Set X defines the family of input states, S – inner states, and Y – output states. The function

$$f : X \times S \rightarrow Y$$

associates the output state with the input state and the inner state. On the other hand function g steers the work of the system, hence

$$g : X \times S \rightarrow S.$$

The set of acceptable states of a cybernetic system cannot be optional. This set is usually a certain variety, sufficiently regular, describing the nature of the system. In every point of variety there exists tangential space, with the exclusion perhaps of a one particular point. The law of science ruling the system exists in this variety. Such a variety can be a sphere, a torus, a cylinder, a collar (Fig. 3), or a contracted doughnut – anti-collar (Fig. 4). A collar is created from a torus through narrowing – one circle is reduced to a point. A collar is created also from a sphere, when we identify the poles with a point situated outside the sphere – differently than in the contracting of a doughnut, hence the terms collar and anti-collar.

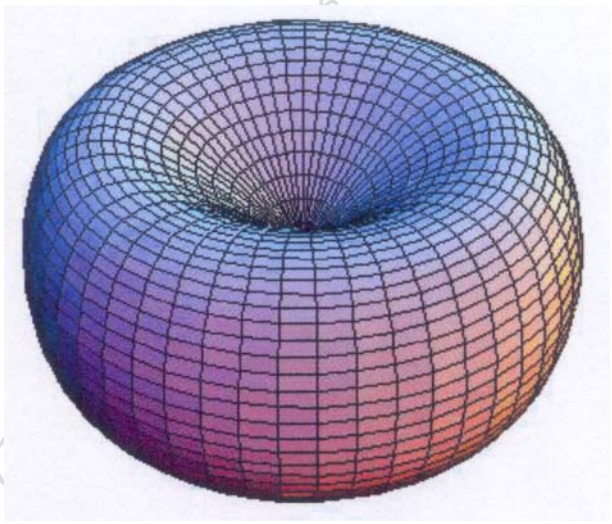

Fig. 4. Anti-collar

Over any given set there can be spread linear space, so that this set becomes a base of the space. Such a statement justifies the comments made above regarding the states of a cybernetic system. A basket of goods is a vector, and a base is constituted by a system of units of commodities. Expansion of the vector

$$x = a_0x_0 + a_1x_1 + ...$$

is a measure of a basket x in the system of units (x_0, x_1, ...). Vectors, i.e. points of linear space, are baskets of goods. Linear functions, i.e. dual vectors, are systems of prices. Assessing the value of a basket – corresponding to a function, by a given system of prices – equivalent to a measure, is a process of integration. A cashier in a shop integrates all day long. Baskets of goods create economic space; it is the space of an infinite dimension since it allows baskets of one commodity, two commodities, many commodities, and you cannot give an upper limit of a number of variables. Economic space is identified with the linear algebra of real sequences with a finite number of terms not equal to zero.

The laws of science are universal. The statements of physics have their equivalents in economics. The thesis about thermal death of the Universe, where the temperature will level off, and in economics is interpreted as the statement about an interest rate decreasing to zero. It is a similar case with the third law of Newton which suggests that matter in the universe vibrates around the centre of gravity. In economics this means that temporary states of equilibrium vibrate around an unknown ideal state. Hence appears the logarithmic spiral. This spiral is so often encountered in nature because a growing organism retaining similarity to the previous states generates such a spiral. This is beautifully visible in the shell of a snail (Fig. 5).

Fig. 5. Sectio aurea

Reynold's Law of Similarities regards the flow of viscous liquids: the relation of the force of inertia to the force of viscosity is constant and is called the Reynold's number depending obviously on a given speed. There exist such border speeds, depending on the type of liquid, at which laminar movement changes into turbulent flow. At high speeds, the flow becomes a vortex of particles of liquid around the centre of gravity. This means that the forces of viscosity in such extreme conditions stop functioning; only the force of inertia counts. The laws of science are general. Also Reynold's law of flows can be interpreted beyond hydrodynamics (J. Juzwiszyn, 2003). It can be interpreted in economics and sociology. Speed is a capital and viscosity is social links. A large amount of capital breaks professional, friendly and even family links. The temptation of getting rich easily usually wins in confrontation with ethical and moral limitations. The examples of this are so numerous both in Poland and the rest of the world that there is no need to quote them. There are similar reasons between the vibration of shares in the stock market. The desire for quick profit is omnipotent.

Every law of science without theory is an isolate statement. Independently from the way of discovering regularity, whether empirically or through deduction, a scientific regularity in order to deserve such a description should constitute a crucial point in a formal theory. One of the criteria of scientific cognition is beauty. They discovered distributions in physics, and mathematicians gave this term a correct sense. Distribution is a linear functional of a special kind, distribution is an integral. Mathematical knowledge becomes enriched and developed through absorbing discoveries of concrete sciences. Mathematicisation is necessary because it enables reasoning, simplifies notations, and allows to discover the issue. From the abstract viewpoint arithmetical sequences can be identified with geometrical sequences. This is because the additive group of real numbers is isomorphic with a multiplicative group of real positive numbers. A natural logarithm, and any other logarithm, is such an isomorphism. Without mathematics, science loses its beauty. Truth is in concrete interpretations, and utility is identical with good. We learn to foresee and to live better.

Economic metrology is a theory measuring the qualities characterising a population: standard of living, state of health, education. What is poverty and how to measure it? How to interpret the line of permanent snow in terms of economics? Such an economic model would become an answer to the question what is poverty. What is inflation and how to measure inflation? Everybody knows what is number π. It is the ratio of the circumference of a circle to its diameter. This figure can be determined empirically by measuring the circumference of a container and its diameter with a tailor's measure. If the measurements were correct, a quotient of these numbers will be approximately equal to 31/10. This figure is a known law of science. It is a similar case with another constant, a universal number e. This is a capital which we will obtain from a bank after a year when at the beginning we deposit one dollar at the interest rate of one, and, assuming that the bank applies the procedure of permanent

capitalisation. Permanent capitalisation means adding interest in increasingly shorter periods, it is therefore

$$e = \lim(1 + 1/n)^n = 2,\ 718...,$$

where $1/n$ signifies a decreasing interest rate, and n is a growing number of periods of adding up interest.

What are Elliot's waves? What are waves? There exist magnetic waves, electric waves, light waves, gravitational waves; the sea waves, the air vibrates, there is a cyclical change of seasons. Elliot waves can be identified with fluctuations of financial markets. In order to create a consistent theory of financial waves, we should begin from an additive group of real numbers. From a geometrical viewpoint it is a straight line, an idealisation of a stretched thread, the path of a ray of light, and all kinds of objects shaped like a stick. An additive group of real numbers is a structure with three operations: a zero argument, a one argument, and a two argument. These operations are connected with the nature of a straight line: we differentiate a neutral element – zero, a symmetrical reflection in zero – negation of a number, and a sum. A circle is created from the straight line and is undoubtedly the most beautiful curve possessing numerous models in nature.

Analysing polygonal functions which are graphs of market share prices, Elliot noticed their similarity with the waves of the sea and the Fibonacci sequence. These observations evolved into empirical science on the basis of intuition, which is called The Elliot Theory of Waves. In this theory, the central role is played by a golden number

$$\chi = \frac{\sqrt{5} - 1}{2}$$

which is encountered in nature because the Fibonacci sequence is a law of science.

A segment of a line can be naturally divided in a golden relation using a compass and a ruler. This has been famous since antiquity as a golden cut – *sectio aurea*. The construction is extremely simple and beautiful. On any given straight line we mark the segment. In this way two points A and B are obtained. That interval AB is divided in half and, on a perpendicular to the given straight line – marked in the point A – we measure from point A into any direction half the segment AB, thus we obtain point C. Through the points C and B we draw a straight line; on this line from point C in the direction of point B we mark also half the segment AB. On the straight line CB we obtain point S.

Statement. If $AB = 2AC$, then $BS = \chi AB$.

After a period of growth there is always a breakdown because the whole universe pulsates because of reasons unknown to us. The function, whose graph is a broken line, is called a polygonal function. A polygonal function is a broken line with a finite number of points at which there is no derivative. Time sequences are represented with such functions. When on a plane appears a point

we consider it a miracle. This is the beginning of birth, an unusual event, and a catastrophe. A point is a place of the junction of a plane with a surface lying above the plane. Lowering the surface we can obtain on the plane contour lines. From a point anything can develop. A point is a comet in the sky, for centuries considered to be an omen of unusual events. It was a point that started writing on the wall during Balthasar's feast. If we raise the dimension of space and enter the external world then mysterious objects look different. Since we usually see only projections, shadows of real objects. A theory of expansion which is the basis of theory of catastrophies created by Thom takes away blindfolds from our eyes which impose one specific point of view. What does an object look like in four dimensional space, whose projections we can see on a plane? Elliot's Waves are projections on the plane of a spatial curve.

We need to look for regularities ruling financial markets, in three dimensional space. A polygonal function – a time sequence of market share prices – has to be raised by one dimension, up to a three dimensional space. We shall obtain then a line wound up on a cone. The three dimensions in the stock market are time, price and number of shares. A projective plan or a plane on which we project a spatial curve is a plane cutting through an axis of a cone and parallel to the axis of the price of shares. It is easiest to assume that the axis of a cone is a line of time, and a projective plane is marked by price and time. There is a spiral on the cone.

The projection of a spiral are Elliot's Waves. Raising the dimension by one and transfer to three dimensional space makes out of intuitive knowledge about Elliot's Waves, a scientific theory.

The current state of the stock market vibrates around the point of an equilibrium which is situated on the axis of the cone. This means that the cone, from a purely geometrical term, becomes the central surface of economics. There are points of ideal balance on the axis of the cone; its surface (Fig. 6)

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{c^2}=0,$$

is defined by parameters a and c, and woven by a spiral line with an equation

$$x = k\exp(mt)\cos t,$$

$$y = k\exp(mt)\sin t,$$

$$z = \frac{c}{a}k\exp(mt),$$

where $t \in R$, and constants k and m define the spiral line. The orthogonal projection of a spiral line on the projective plane $z = 0$ is a logarithmic spiral

$$r = k\exp(mt).$$

Naturally the angle of the opening of a cone is not constant, a cone also changes. We can assume however that in a certain period it is a single cone. The size of the angle of opening of this cone tells us about the stability of the economy. The smaller the angle, the more stable the economy is. Also the top of a cone tells us something about the economy. If it is in the front, then the economy becomes stable, we are approaching the point of equilibrium, fluctuations are getting smaller. If it is beyond us, then the system is getting deregulated and the fluctuation is getting bigger.

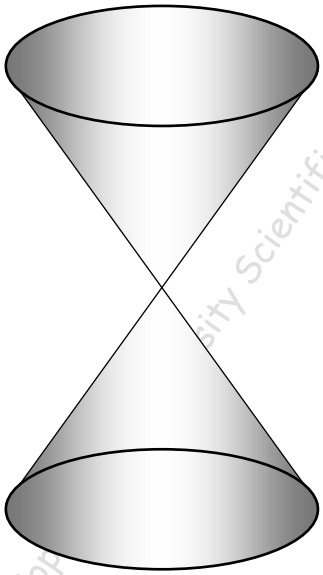

Fig. 6.

An unstable system is convenient for the stock market players because only in such a situation can they do big business. Therefore as you can see, the spatial expansion of Elliot's Waves can be productive. A catastrophical view of the stock market contributes to its better understanding and explains the source of chaos which you observe there. Chaos in the stock market is generated by a spiral. And so in order to see what becomes of one point you need to develop it and move to a higher dimension.

A circle is a quotient group of a straight line of real numbers by the discrete group of total integer. Such property is the essence of a circle. Somewhat oversimplifying we can say that a straight line and a circle create everything. The answer to the question whether development has to be cyclical seems to be straightforward. A circle is also a symbol of infinity. The sun rises everyday, the seasons repeat themselves; in biology cyclicality is visible in constant births and deaths. There is no doubt that a cyclical following is the simplest model of everlasting existence. It is a eternity in a finite and limited circle. A set of states

of the stock market is a two dimensional variety deriving from a torus, that is a Krakow bagel, through narrowing at one point. Limiting and balancing conditions are imposed on constituents. Transfer to dynamics, that is introducing trajectory in a set of states, is a completely natural activity. Trajectories are continuous functions.

The simplest and most proper definition of a state of equilibrium in an economy is the identification of the equilibrium with full employment. In economy and life there is always equilibrium. At first, these two statements seem contradictory. After all, in the majority of countries there is significant unemployment, a cybernetic system cannot exist without equilibrium; equilibrium is an attribute of existence. Naturally there exist different levels of equilibrium, perhaps it would be even better to talk about conditional equilibrium. Such equilibrium is characterised by a lower level of produced goods and services then in conditions of full employment. We accept the law of Say, which says that production creates sales, therefore there are no crises of overproduction. How should we understand this? It is assumed that economic subjects behave rationally and that they have full information about both their future plans and future plans and needs of all their business partners. This is undoubtedly an ideal assumption. In reality such information is hidden for the reason of competition, hence the role of the state in the economy is so important. A central planner is an independent arbiter, impartial and incorruptible, it is he who collects all the information, sums it up and announces what transpires from the plans of particular producers.

Everything is revolving, everything is subject to a whirl, because everything is going to equilibrium.

$$f(t) = \sum_{n=0}^{\infty} (a_n \cos nt + b_n \sin nt).$$

Harmony

$a_n \cos nt + b_n \sin nt$

exists in cyclical returns. A revolving movement dominates in the cosmos, the world pulsates therefore it is alive. We can see in the sky spiral nebulae, galaxies have this exact shape and a sunflower is also arranged spirally and performs cyclical movements following the sun. During low pressure, the air vibrates in spirals and during high pressure also, but in the opposite direction. A tap running strongly is an example of a machine generating chaos, the flowing stream of water whirls and breaks. The Earth revolves around its own axis, and the Moon around the Earth, and all that system runs around the Sun.

We can multiply examples from the world of physics and biology. In economics, price oscillates around the point of equilibrium and the famous Elliot Waves are expressing this oscillation as a projection of a spiral wound up on a cone. The Fibonacci sequence, the golden number, and a spiral on a cone,

together model the functioning of such an unusual institution as the stock market, since fortune turns like a wheel.

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