DYNAMIC ECONOMETRIC MODELS
Vol. 9 – Nicolaus Copernicus University – Toruń – 2009

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Markov Switching Models with Application to Contagion Effect Analysis in the Capital Markets

A b s t r a c t. This article presents the analysis of the contagion effect in the capital markets on the basis of the Markov switching models MS. The research is based on the return of the indexes. There is a distinction of two regimes with different volatility levels, the calm period and the crisis period. Then the analysis of the period's occurrence was conducted, in reference to global financial crisis. Periods with a similar level of volatility occurrence in the same time. This analysis evidences the shocks transmission between financial markets, what confirms an occurrence of the contagion effect.

K e y w o r d s: Markov switching model, contagion effect.

1. Introduction

The aim of the article is an application of the Markov switching model MS to contagion effect analysis in the capital markets. There are different definitions of contagion effect. The most popular definition affirms that shocks transmissions are caused by the herd behavior of investors and this is the most often assumed in the empirical research. There can be found three approaches in an application of the MS models to contagion effect analysis, such as:

- univariate models with the switch in variance MSH (Moore, Wang, 2007);
- multivariate models with the switch in variance MSH-VAR or both in the variance and mean MSMH-VAR (Linne, 2001; Mandilaras, Bird, 2005);
- the GARCH models with the Markov switching MS-GARCH (Edwards, Susmel, 2001).

2. A Contagion Effect Definition

A contagion effect definition the most often concerns the financial markets, but the transmission processes envelope an economic connections too. The

Word Bank assumes three versions of the contagion effect definition[1]: broad, narrow and very narrow definition. According to broad definition the contagion effect is an international shocks transmission or wide-spread spillover effect. The transmission can refer to both good and bad periods and it's not always identified as crisis. However in the crisis it can be more noticeable. In the narrow definition there is an assumption that contagion is a shocks transmission to other countries or the relations between economies except the fundamental connections and common shocks. This definition the most often is reduced to very similar changes in the financial markets that are usually explained by the herd behavior. The very narrow definition assumes that a contagion effect occurs when in the crisis period the correlation between economies is stronger than in the calm period.

According to Fiszeder (2009) the narrow definition of contagion effect is the shocks transmission between countries that cannot be explained fundamentally. These transmissions are real financial, economic and political connections. The most often cause of the contagion in narrow sense is the herd investors behavior. There can be noticed some specific group behavior of investors, what is more distinct in the crisis periods and causes crossing shocks over the financial markets. The understanding these behaviors could help to explain the transmission of the shocks. In the analysis of the contagion effect a transmission channels have the essential meaning. There can be found a three basic transmission channels:

- real channel (international trade);
- financial channel (global diversification of the investment portfolio);
- herd behavior (a copy strategy in the investment);
- international policy.

3. The Markov Switching Model

The Markov switching model AR(p)-MSMH(r) for stochastic process y_t is given by:

$$y_t = \mu(s_t) + \alpha_1(y_{t-1} - \mu(s_{t-1})) + \alpha_2(y_{t-2} - \mu(s_{t-2})) + \ldots\alpha_p(y_{t-p} - \mu(s_{t-p})) + \varepsilon_t , \quad (1)$$

$$\varepsilon_t \sim IID(0, \sigma_\varepsilon^2(s_t)),$$

$$\varepsilon_t = y_t - E[y_t \mid y_{t-1}, s_t],$$

where $\mu(S_t)$ is conditional expected value of y_t $(\mu = \mathrm{E}[y_t \mid y_{t-1}, s_t])$ and $\sigma_\varepsilon^2(s_t)$ is the variance of the disturbance term.

[1] *Contagion of Financial Crises*, World Bank, http://www.worldbank.org.

In the Markov switching models the parameters $\mu(S_t = j)$, $\sigma_\varepsilon^2(S_t = j)$, π_j [2] are the unobserved variable S_t realizations, that has a Markov property[3]. The conditional probabilities $p_{ij}(t)$ create the transition probabilities matrix $\mathbf{P}$ with the $r \times r$ dimension that is given by:

$$\mathbf{P} = \begin{bmatrix} p_{11}(t) & p_{12}(t) & \cdots & p_{1r}(t) \\ p_{21}(t) & p_{22}(t) & \cdots & p_{2r}(t) \\ \vdots & \vdots & \ddots & \vdots \\ p_{r1}(t) & p_{2r}(t) & \cdots & p_{rr}(t) \end{bmatrix}_{r\times r}, \tag{2}$$

where r is the states (regimes) number of the S_t variable process.

The $\mathbf{P}$ matrix is the stochastic matrix, because its elements satisfy the following conditions: $p_{ij}(t) \geq 0$, $\sum_j p_{ij}(t) = 1$.

The homogeneous Markov chain probabilities $p_{ij}(t)$ describing the one step change between states are constant and time independent.

The Markov switching models MS can generate the skew distribution (when the third central moment significantly differs from zero) and the leptokurtic distribution. In example, the model that can generate the skewness and the leptokurtosis of the distribution is the AR(0)-MSM(2). The AR(0)-MSH(2) model is given by:

$$Y_t - \mu_Y = \varepsilon_t, \quad \varepsilon_t \sim NID\left(0, \sigma_{\varepsilon 1}^2 I(s_t = 1) + \sigma_{\varepsilon 2}^2 I(s_t = 2)\right), \tag{3}$$

where $\sigma_{\varepsilon 1}^2, \sigma_{\varepsilon 2}^2$ the variance of the disturbance term, in the following first and second state $I(s_t = 1)$, $I(s_t = 2)$ are dummies variables.

The excess coefficient of the Y_t process for the Markov switching AR(0)-MSH(2) model can be written as:

$$\frac{E\left[(Y_t - \mu_Y)^4\right]}{E\left[(Y_t - \mu_Y)^2\right]^2} - 3 = \frac{3\pi_1\pi_2\left(\sigma_{\varepsilon 1}^2 - \sigma_{\varepsilon 2}^2\right)^2}{\left(\pi_1\sigma_{\varepsilon 1}^2 + \pi_2\sigma_{\varepsilon 2}^2\right)^2}, \tag{4}$$

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[2] Unconditional probabilities of the Markov chain (ergodic) for two states are received from equations: $\pi_1 = \dfrac{1 - p_{22}}{2 - p_{11} - p_{22}}$, $\pi_2 = \dfrac{1 - p_{11}}{2 - p_{11} - p_{22}}$.

[3] The finite homogeneous Markov chain with the state space $\{1,2,\ldots,r\}$ is the stochastic process where for all $i, j \in \{1,2,\ldots,r\}$ the $\Pr(S_t = j \mid S_{t-1} = i) = p_{ij}(t) =$ $= Pr(S_t = j \mid S_{t-1} = i) = p_{ij}(t)$ equality is fulfilled.

where $E\left[(Y_t - \mu_Y)^2\right]$ is the second central moment and π_1, π_2 are the ergodic probabilities in the following first and second states.

The (4) coefficient significantly differs from zero when $\sigma^2_{\varepsilon 1} \neq \sigma^2_{\varepsilon 2}$ and when $0 < \pi_1 < 1$. Therefore the leptokurtosis is confirmed by the Markov structure which has heteroscedastic disturbance term and different from zero the excess coefficient of the distribution.

4. The Financial Time Series Results

In the empirical analysis the weekly return rates of the main stock exchange indexes were used, such as: RTS (Russian), SAX (Slovakia), HIS (China), PX50 (Czech Republic), BUX (Hungary), CAC40 (France), DAX (Germany), FTSE100 (England). The analyzed series come from the period from September the 1th, 1995 to August 21th, 2009. The price series transformation into return series was achieved by calculating the week dynamics. The time series of the returns were multiplied by 100. Then the ADF test for unit root was applied and its results show that for all time series this test rejects the unit root hypothesis. In the next part of the empirical analysis the MSH models with the variance switch and 1, 2 or 3 states were estimated. The appropriate order of autocorrelation and autoregression in these models were determined by the means of the Durbin and Watson test. For two states models one of the states is interpreted as low volatility periods and the second state as high volatility periods. For the MSH(3) models with three states the additional state is characterized as periods with the moderated volatility. The switching models were checking for the presence of the ARCH effect (Ljung and Box test for the squares of return rates) and for the normality of distribution (Jarque and Bery test). The tests results are presented in the Table 1. The distributions of the all residuals series are normal. In most models the ARCH effect doesn't occur. The log-likelihood ratio analysis indicates that the ratios are higher for all models with three states MSH(3) than ratios of the models with two states MSH(2). Moreover the log-likelihood test for the number of states[4] was applied. The results of this test indicate the choice of the MSH(3) models. The estimation results of the MSH(2) and the MSH(3) models are shown in the Table 1 and in the Table 2 appropriately.

Models with the highest values of probabilities are presented in the Table 3. In MSH(2) models the variance of high volatility state is about two times higher than the variance of the low volatility state.

[4] The log-likelihood test is constructed on the basis of the $LR = -2(L(H_0) - L(H_A))$ statistic that has a chi-square distribution $\chi^2(k)$ and k is a number of additional parameters of the alternative hypothesis model.

Table 1. Estimation results of the MSH(2) models

Series	p_{11}	p_{22}	σ_1	σ_2	μ	LR	Jarque and Bery test (residual)
WIG	0.9695	0.9801	0.0289	0.0626	0.0123 [0.113]	1162.4	3.76 [0.152]
RTS	0.9871	0.9866	0.1149	0.0655	0.0337 [0.006]	795.1	3.18 [0.204]
SAX	0.9435	0.9608	0.0567	0.0327	0.0016 [0.797]	1242.8	6.30 [0.043]
HSI	0.9949	0.9952	0.0290	0.0950	0.0143 [0.032]	1137.9	2.42 [0.298]
PX50	0.9846	0.9965	0.0775	0.0350	0.0111 [0.007]	1249.3	0.57 [0.751]
BUX	0.9557	0.9789	0.0835	0.0387	0.0215 [0.006]	1060.5	4.52 [0.104]]
CAC40	0.9953	0.9931	0.0488	0.0239	0.0134 [0.007]	1291.1	1.49 [0.474]
DAX	0.9778	0.9865	0.0653	0.0306	0.0146 [0.022]	1215.0	1.98 [0.371]
FTSE	0.9262	0.9864	0.0578	0.0249	0.0049 [0.225]	1473.4	1.68 [0.43]
Nikkei	0.9376	0.9961	0.0847	0.0370	-0.0006 [0.915]	1253.9	1.07 [0.586]
SP500	0.9870	0.9789	0.0232	0.0462	0.0091 [0.029]	1430.2	1.49 [0.474]

Note: p-values have been presented in brackets.

Table 2. Estimation results of the MSH(3) models

Series	p_{11}	p_{22}	p_{33}	σ_1	σ_2	σ_3	LR	Jarque and Bery test (residual)
WIG	0.6032	0.9685	0.9721	0.0113	0.0628	0.0288	1164.7	6.50 [0.039]
HSI	0.9949	0.9812	0.9836	0.0287	0.0561	0.0834	1146.0	5.29 [0.071]
BUX	0.9406	0.9827	0.9674	0.1034	0.0354	0.0566	1070.5	10.6 [0.005]
DAX	0.9281	0.9940	0.9835	0.0962	0.0286	0.0462	1230.4	5.95 [0.051]
FTSE	0.9913	0.9937	0.9081	0.0337	0.0177	0.1013	1513.3	7.89 [0.019]
SP500	0.9930	0.9026	0.9642	0.0206	0.0600	0.0321	1441.9	5.24 [0.073]

Note: p-values have been presented in brackets.

The high volatility periods that were pointed out on the basis of MSH models are presented in the Table 3. These periods represent the high variance regime. The common markets (indexes) periods are following (marked in the Table 3):

– 09.1998–12.2000 (the consequences of the Russian crisis);
– 02.2001–12.2002 (the beginning of this crisis is seen in the DAX and SP500 indexes, then in the WIG, RTS and FTSE indexes);
– 01.2003–12.2004 (the beginning of this crisis is seen in the SP500 and FTSE indexes, then in the WIG, DAX and RTS indexes);
– 07.2008–08.2009 (the beginning of this crisis is seen in the SP500 index, then in the RTS, FTSE, WIG, PX500, DAX, SAX and BUX indexes).

Table 3. The high volatility periods pointed out on the basis of MSH models

WIG	RTS	SAX	HSI	PX50	BUX	CAC40	DAX	FTSE	Nikkei	SP500
11.95-12.00	10.95-04.01	01.96-02.96	04.97-05.02	06.98-05.99	01.96-04.96	01.97-05.03	07.97-12.97	09.97-11.97	03.02	09.98-06.00
09.01-02.02	11.01-01.02	11.96-02.97	07.07-08.09	09.08-08.09	12.96-03.97	07.07-08.09	07.98-03.00	10.98-02.99	10.08-05.09	02.01-12.01
07.03-01.04	10.03-12.03	12.97-01.99			10.97-01.98		02.01-05.01	08.01-11.01		05.02-04.03
04.06-09.06		04.99-01.00			05.98-01.00		08.01-05.03	06.02-09.00		01.08-08.09
03.07-03.08		03.00-12.00			09.00-01.01		09.03-11.03	01.03-05.03		
09.08-08.09	07.08-08.09	03.01-07.01			11.05-12-05		10.08-08.09	09.08-05.09		
		05.02-07.02			05.06-09.06					
		11.02-05.03			10.08-08.09					
		01.04-03.04								
		09.04-04.05								
		10.08-08.09								

The first period, shaded in the Table 3, corresponds with the Russian crisis and its consequences can be notice in the 1998 year. This crisis has sunk into a memory of worldwide economic recession. Its beginnings were noticeable firstly in Czech Republic in March 1997 in the form of the banking crisis, and in markets of south-east Asia. The analysis of these periods shows that the Russian crisis periods coincide with the high volatility periods for the most index series. The next distinguished common markets periods of high volatility might be qualified as a derivative of economic crisis begun in the half of 2008 year in USA. These periods weren't identified only in the case of the HIS and CAC40 indexes. The contagion effect on financial markets in the crisis periods is seen more clearly. The high volatility periods occurrence usually at the beginning of the financial crisis. Then the shocks are transmitted between markets. The analysis evidences these transmissions between financial markets, what confirms an occurrence of the contagion effect.

5. Summary

The main aim of this article was an application of the Markov switching models to identification and analysis the contagion effect in the capital markets. The research allowed to distinguish two regimes with different volatility levels, the calm period (low volatility) and the crisis period (high volatility). Basing on this classification the identification of some interdependence pattern between markets was created. In the empirical part of the article the narrow contagion effect definition was assumed. According to this definition the cause of the shocks transmission are the herd behavior of investors mainly and the fundamental economic factors are not taken under attention. In these researches there were distinguished the high volatility periods (on the indicated capital markets) and the occurrence of these periods was analyzed. The conclusion is that in the case of the Russian crisis and the crisis which begun in the 2008 year, the patterns of the high volatility periods were very similar. The capital markets are connected with themselves what is noticed clearly in investors behavior during the beginning of the crisis period. The high volatility periods usually occur with the price decrease and they coincide or come across on themselves. The more detailed analysis using the switching structure could be carried out on the basis of the multivariate Markov switching models for two series for example, what would allow appointing the periods of common volatility.

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Przełącznikowy model typu Markowa w badaniu efektu zarażania na rynkach kapitałowych

Z a r y s t r e ś c i. Artykuł stanowi próbę analizy efektu zarażania na rynkach kapitałowych z wykorzystaniem przełącznikowego modelu typu Markowa MS. Badanie przeprowadzono w oparciu o indeksy giełdowe wybranych krajów. Wyznaczono stany o niskiej i wysokiej zmienności dla poszczególnych szeregów oraz przeprowadzono analizę ich występowania w odniesieniu do globalnych kryzysów finansowych. Stwierdzono występowanie wspólnych okresów zmienności dla badanych indeksy. Okresy te wskazują na przenoszenie szoków pomiędzy rynkami, potwierdzając występowanie efektu zarażania na tych rynkach.

S ł o w a k l u c z o w e: przełącznikowy model typu Markowa, efekt zarażania.