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The Econometric Models Satisfying the Congruence Postulate – an Overview

1. Non-stationarity – the Key Problem of Dynamic Modeling

The non-stationarity of economic processes should be treated as the key problem of dynamic modeling. Three main approaches to non-stationarity may be distinguished: (1) classical decomposition, (2) integrated processes approach, (3) new decomposition of economic processes as an agreement of trend stationarity versus difference stationarity debate.

(1) Till the early seventies of the 20th century the classical decomposition of economic processes was dominated, i.e. $Y_t = P_t + S_t + C_t + \eta_t$, where P_t – a deterministic trend component, S_t – a deterministic seasonal component, C_t – deterministic business cycle fluctuations, η_t – a stochastic component which is assumed to be stationary. This decomposition assumes that an economic process is non-stationary in mean. In other words, fluctuations of deterministic time function are stationary what matched the prevailing view amongst economists that fluctuations (including business cycle) around deterministic function of time variable are transitory. To eliminate non-stationarity in mean the function of time into the model for levels was introduced or it was subtracted from a given process. Only rarely the transformation of process to its differences or growth rates was used. As a consequence the prevailing strategy in dynamic modeling was "always take levels" (or deviations from deterministic trend), if economic processes were non-stationary.

(2) Since three seminal works, i.e. the publication by Box and Jenkins (1976) which brought the ARIMA modeling into general use, the paper by Granger and Newbold (1974) focusing on the danger of spurious regression for non-stationary processes and mainly paper by Nelson and Plosser (1982) referring to distinguishing the type of non-stationarity for macroeconomic processes,

the alternative decomposition of non-stationary processes was preferred, i.e. $Y_t = \mu_t + \gamma_t + \xi_t + \eta_t$, where μ_t – stochastic trend component, γ_t – stochastic seasonal component, ξ_t – stationary stochastic component. This decomposition assumes that an economic process is non-stationary in variance, which means that random shocks have a permanent effect on the system, i.e. there is no tendency for future values of a process to revert to a trend line (a fall of process today brings about that forecasts will fall in the indefinite future). The non-stationarity in variance was eliminated by calculating the first differences (or in general differences of *d*-order). As a result the prevailing strategy in dynamic modelling was "always take differences" if economic processes were non-stationary[1]. Since that time the non-stationarity of economic processes started to be automatically referred only to non-stationarity in variance[2].

(3) In eighties and first half of nineties of the 20th century the lengthy and vast debate on the trend stationarity versus difference stationarity took place. At the early stage of that debate the hypothesis of difference stationarity outperformed those of trend stationarity. In favour of the former hypothesis spoke the more serious effects of underdifferencing than effects of overdifferencing on estimation and statistical inference and low power of unit root tests which too often did not reject the hypothesis of difference stationarity. At the later stage of that debate some balance between trend stationarity and difference stationarity hypothesis was obtained what resulted from the acceptance of different properties of trend stationary processes and difference stationary processes (in terms of mean, variance, autocorrelation function, mean reverting property and persistence) and general agreement that economic processes can be non-stationary as well in mean as in variance. Therefore since second half of nineties of the 20th century after the debate on the trend stationarity versus difference stationarity (none of approaches gained an advantage) a new decomposition started to be dominating, i.e. $Y_t = P_t + S_t + C_t + \mu_t + \gamma_t + \xi_t + \eta_t$ .This decomposition includes the wide class of non-stationary processes (as well in mean, e.g. production, income, sale, as in variance, e.g. exchange rates, stock indexes).

From that debate resulted some suggestions for econometric modeling. The distinction of trend stationary or difference stationary processes is important especially with regard to economic forecasting because each type of non-stationarity can assume quite different dynamics and hence different forecasts. The unit root tests can be useful as a diagnostic tool in model specification for forecasting purposes when the point is not to find the "true" model but to find

[1] This was suggested by the results of unit root tests which have a low power and therefore preferred the null of non-stationarity in variance. It should be clearly emphasized that difference between strategies "always take levels" and "always difference" does not only refer to the way of removal of non-stationarity but is much deeper and refers to the economic interpretation (Piłatowska, 2003).

[2] It is apparent especially in Polish textbooks in econometrics.

the model which gives more accurate forecasts. And finally the main purpose in econometric modeling is not the choice between the strategy: "always take levels" (TS) or "always difference" (DS), but rather to build the dynamic model which has the required properties. The latter is obtained when the model satisfies the congruence postulate.

The purpose of the paper is to overview the different approaches to dynamic econometric modeling satisfying the congruence postulate which was introduced by Granger (1981).

2. The Congruence Postulate in Different Approaches to Dynamic Modeling

The idea of congruence was introduced by Granger (1981) and it was firstly outlined in the context of frequency domain. In the time frequency (in a linear model) this postulate says that if dependent variable Y_t has some dominant features (strong autocorrelation, seasonality, trend in mean or in variance) then the explanatory variables X_t have to contain similar features to explain Y_t and to satisfy a condition for the model to be satisfactory[3]. The model satisfying this condition is called balanced (Granger, 1992). In the case of unbalanced model the dominant features of dependent process not explained by dominant features of explanatory processes will have to appear in the residual, which will then have undesirable features for estimation and inference.

In different approaches to dynamic modeling the congruence postulate is realized in different way and what is worth noticing the reference to this postulate is not always explicitly.

To approaches realizing the congruence postulate belong: (1) the concept of congruent modeling according to Zieliński (1984), (2) general to specific modeling according to Hendry (2000), (3) cointegration and error correction model (Engle, Granger, 1987), (4) the VAR modeling (Sims, 1980).

The Concept of Congruent Modeling According to Zieliński

The idea of congruence outlined by Granger was the starting point to develop the concept of dynamic congruent model in time domain by Zieliński (1984). A model is congruent according to Zieliński if the harmonic structure of dependent process Y_t is the same as the harmonic structure of explanatory processes X_{it} (i=1,2,…,k) and the residual process. The model for white noises of given processes Y_t and X_{it}, i.e.

$$\varepsilon_{y_t} = \sum_{i=1}^{k} \rho_i \varepsilon_{x_{it}} + \varepsilon_t, \qquad (1)$$

[3] Such defined congruence postulate can be extended into conditional variances (Fiszeder, 2006).

is congruent because the harmonic structure of ε_{y_t} and harmonic structure of $\varepsilon_{x_{it}}$ are identical. Hence the residual process has white noise properties.

The model (1) is a starting point to build the congruent model for observed processes Y_t and X_{it}. First step to do it is to specify the dominant features (internal structure) of given processes. It is realized by building time series models: trend/seasonality models, autoregressive models

$$Y_t = P_{y_t} + S_{y_t} + \eta_{y_t}, \qquad B(u)\eta_{y_t} = \varepsilon_{y_t}, \tag{2}$$

$$X_{it} = P_{x_{it}} + S_{x_{it}} + \eta_{x_{it}}, \quad A_i(u)\eta_{x_{it}} = \varepsilon_{x_{it}}, \; i = 1, 2, \ldots, k, \tag{3}$$

where P_{y_t}, $P_{x_{it}}$ denote polynomial functions of time variable t, S_{y_t}, $S_{x_{it}}$ – seasonal components (constant or changing fluctuation amplitude), η_{y_t}, $\eta_{x_{it}}$ – stationary autoregressive processes for Y_t and X_{it} respectively, $B(u)$, $A_i(u)$ – autoregressive backshift operators with all roots lying outside the unit circle, ε_{y_t}, $\varepsilon_{x_{it}}$ – white noises of respective processes.

In the next step, substituting for ε_{y_t} and $\varepsilon_{x_{it}}$ in (1) from models (2) and (3) the starting specification[4] of congruent model is obtained:

$$B(u)Y_t = \sum_{i=1}^{k} A_i^*(u) X_{it} + P_t + S_t + \varepsilon_t, \tag{4}$$

where $A_i^*(u) = \rho_i A_i(u)$, $P_t = B(u)P_{y_t} - \sum_{i=1}^{k} A_i^*(u) P_{x_{it}}$, $S_t = B(u)S_{y_t} - \sum_{i=1}^{k} A_i^*(u) S_{x_{it}}$

or equivalently

$$Y_t = \sum_{s=1}^{q_y} \beta_s Y_{t-s} + \sum_{i=1}^{k} \sum_{s=0}^{q_{xi}} \alpha_{i,s}^* X_{i,t-s} + P_t + S_t + \varepsilon_t. \tag{5}$$

It should be noticed that the residual process in the model (5) is the same as in model (3). This means the congruence condition of dominant features (internal structures) of both side of equation is satisfied. To check the congruence the misspecification tests for white noise errors, conditionally homoscedastic errors, normally distributed errors, unconditionally homoscedastic errors, constant parameters are conducted for starting and final model. The specification of model (5) includes four components: lagged dependent process, current and lagged explanatory processes, trend/seasonality component $P_t + S_t$ and residual process,

[4] The estimated initial specification of model (5) has in general excessive insignificant variables which are eliminated by selection methods and at the very end the final congruent model, reduced to significant variables, is obtained. The concept of congruent modelling can be applied to the case of non-stationarity in mean and in variance.

each having a different meaning. Lagged dependent processes should be interpreted as substitute elements which appear in model if: (a) important explanatory variables are omitted, (b) the dependence of Y_t on X_t for different frequency components is not the same, i.e. regression parameter of Y_t on X_t for low frequencies components is different than those for high frequencies components. Current and lagged explanatory processes are treated as economic factors (with causal interpretation), but some of them play the role of substitute factors for omitted variables what leads to the model being balanced. The inclusion of trend/seasonality component to model means that from each process the non-stationarity in mean was eliminated and therefore parameters β_s and α_{is}^{*} refer to the dependence on stationary level.

General to Specific Modeling According to Hendry

In general to specific modeling an empirical model is congruent if it parsimoniously encompasses the local DGP (i.e. the generating process in the space of the variables under analysis) and achieves the pre-assigned criteria: white noise errors, conditionally homoscedastic errors, normally distributed errors, unconditionally homoscedastic errors, constant parameters (see Mizon, 1995; Bontemps and Mizon, 2001).

The empirical analysis commences from the formulation of general unrestricted model GUM, (Hendry, 2000), taking similar form as in the concept of congruent model according to Zieliński (model (5)), which after testing for misspecifications and if none is apparent, is simplified to parsimonious congruent model, each simplification step being checked by diagnostic testing.

The difference in formulation of initial model specification in general to specific modeling and the concept of dynamic congruent model (according to Zieliński) consists in different approach to the recognition of internal structure of given processes. In general to specific modeling usually the linear trend is introduced to eliminate a potential non-stationarity in mean, and the lags are set as a maximal lag length according to available evidence (number of observations and variables), the same for all variables in question, to maintain the congruence. While in the dynamic congruent model (according to Zieliński) the initial model specification is established through the recognition of dominant features (internal structure) separately for given processes. As a result the lag length for different processes (dependent and explanatory) is not the same, and also the degree of polynomial trend may be higher than one.

The next difference in both approaches refers to the selection rules with regard to insignificant variables. In the concept of dynamic congruent model (according to Zieliński) the iterative selection method (*a posteriori*) is preferred, while in general to specific modeling – the automatic selection procedure, *PcGets* (Hendry, Krolzig, 1999). This procedure consists in eliminating insignificant variables by selection tests, both in blocks and individually. Moreover many reduction paths are searched to prevent the algorithm from becoming stuck in a sequence that inadvertently eliminates a variable that matters, and thereby retains other variables as proxies. If several models satisfying the congruence postulate are selected, encompassing tests and model selection criteria resolve the choice.

Cointegration – Error Correction Model

The cointegration idea formulated by Engle, Granger (1987) assumes that a combination of processes nonstationary in variance, each integrated of order one, $I(1)$ and trend in mean is not apparent, is stationary, i.e. integrated of zero order[5], $I(0)$. This means that exists k-dimensional vector θ such that a linear combination $Z_t = Y_t - \theta' X_t$, where X_t – k-dimensional vector of explanatory processes, is stationary. Cointegrating vector θ eliminates a stochastic trend (non-stationarity in variance) and at the same time it is a vector measuring relationship between Y_t and X_{it} on stationary level.

The relationship for cointegrated processes can be expressed in the form of error correction model (Engle, Granger, 1987):

$$\Delta Y_t = \sum_{j=1}^{p} \alpha_j \Delta Y_{t-j} + \sum_{i=1}^{k} \sum_{j=0}^{q_i} \beta_{ij} \Delta X_{i,t-j} + \delta \left(Y_{t-1} - \sum\nolimits_{i=1}^{k} \theta_i X_{i,t-1}\right) + \eta_t, \quad (6)$$

where $EC_{t-1} = Y_{t-1} - \sum\nolimits_{i=1}^{k} \theta_i X_{i,t-1}$ is stationary error correction term representing the deviation of Y_t from the long-run equilibrium[6]. The error correction coefficient δ measures the speed of convergence to equilibrium. Parameters θ_i are long-run coefficients for the response of Y_t to a unit change in X_{it}. The remaining coefficients α_j and β_{ij} relate to the short-run dynamics of the model's convergence to equilibrium.

In fact model (6) satisfies the congruence postulate, however it is realized in different way than in previous approaches. The error correction model is built for first differences, i.e. for processes transformed by the difference filter which eliminates wide band of low frequencies referring to long-run components and as a result it eliminates non-stationarity in variance, but also in mean. Therefore coefficients β_{ij} are the measure of relationship for stationary differences of processes. Into model (6) the lagged differences of dependent variable and explanatory variables and also error correction term EC_{t-1} are necessary to be in-

[5] The first note concerning the cointegration appeared earlier in Granger (1981), in which the idea of congruence postulate was outlined.

[6] It should be remembered that despite the stationarity of linear combination of nonstationary processes still exists the danger of spurious regression effects because of the omission of autoregressive structure of Y_t and X_{it} in cointegrating relation. See results of simulation with regard to spurious regression for independent and dependent autoregressive processes, e.g. Granger, Hung, Jeon (1998); Piłatowska (2003).

cluded, otherwise the autocorrelation of residuals will appear as a result of applying the difference filter. Hence the error correction term not only measures the speed of convergence to equilibrium but also enables to maintain the congruence of model.

The VAR Modeling

The vector autoregregression model VAR(p) has the form:

$$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_k Y_{t-p} + \varepsilon_t, \tag{7}$$

where Y_t is an K x 1 vector of jointly determined variables, the A_i are fixed (K x K) coefficient matrixes of coefficients, ε_t is K-dimensional white noise, that is, $E(\varepsilon_t) = 0$, $E(\varepsilon_t \varepsilon_t') = \Sigma$, for all t, $E(\varepsilon_t \varepsilon_s') = 0$ for $t \neq s$. The covariance matrix Σ is assumed to be nonsingular if not otherwise stated.

The model (7) can be extended by the deterministic term, $A_0 D_t$ (intercept, deterministic trend, seasonal dummies), where D_t denotes a vector of deterministic variables, A_0 – a coefficient matrix. In such a way the non-stationarity in mean can be taken into account. In the case of non-stationarity in variance the model (7) is written for differences of variables in interest. Some explanatory variables can also be added into model (7). In the case of cointegration the VAR model is a starting point to build vector error correction model, VECM.

The VAR model with deterministic term realizes the congruence postulate by aiming at such a model specification that the residual process has required properties. The number of lags in the VAR model results from the setting of maximal lag length according to available evidence (number of observations and variables) which further is reduced by the means of appropriate tests. While in the multivariate congruent model (according to Zieliński) the number of lags is separately established by the detection of autoregression order for all variables in interest. Moreover the multivariate congruent model enables to contemporaneous relationships among variables, while the VAR does not give such possibility.

3. Final Remarks

The congruence postulate, aimed at building the model which takes into account the dominant features of endogenous and explanatory processes, is an idea which links all presented concepts of econometric modeling. The attractiveness of this idea consists in benefits resulting for the estimation and statistical inference (e.g. avoiding the danger of spurious regression) and for forecasting because the models satisfying the congruence postulate give in general better forecasts. However it should be emphasized that the models will differ with regard to economic interpretation and forecasting behaviour, especially models for levels and models for differences. Hence the idea of congruence does not solve the dichotomy between model selection and forecasting – it still remains. Therefore it is sensible to accept the coexistence of models with different specification rather than search for the only one true model.

References

Bontemps, C., Mizon, G. E. (2001), Congruence and Encompassing, in: Stigum, B. (red.), *Studies in Economic Methodology*, Cambridge, Mass., MIT Press.

Engle, R. F., Granger, C. W. J. (1987), Co-integration and Error Correction Representation: Estimation and Testing, *Econometrica*, 55, 251–276.

Fiszeder, P. (2006), Consequences of Congruence for GARCH Modelling, in: Zieliński, Z. (ed.), *Dynamic Econometric Models*, Wydawnictwo UMK, Toruń, 143–149.

Ganger, C. W. J. (1981), Some Properties of Time Series Data and their Use in Econometric Model Specification, *Journal of Econometrics*, 16, 121–130.

Granger, C. W. J. (1992), Where Are the Controversies in Econometric Methodology?, w: Granger, C. W. J. (red.), *Modelling Economic Series*, Clarendonpress, Oxford.

Granger, C. W. J., Hyung, N., Jeon, Y. (1992), Spurious Regressions with Stationary Series, Discussion Paper 98-25, University of California, San Diego.

Granger, C. W. J., Newbold, P. (1974), Spurious Regression in Econometrics, *Journal of Econometrics*, 2, 111–120.

Hendry, D. F. (2000), *Econometrics: Alchemy or Science?*, Oxford University Press, Oxford.

Hendry, D. F., Krolzig, H.-M. (1999), Improving on 'Data Mining Reconsidered' by K. D. Hoover and S.J. Peres, *Econometrics Journal*, vol. 2.

Mizon, G. E. (1995), Progressive Modeling of Macroeconomic Time series: the LSE Methodology, in: Hoover, K. D. (red.), *Macroeconomics: Developments, Tensions and Prospects*, Dordrecht: Kluver Acadmic Press.

Nelson, C. R., Plosser, C. I. (1982), Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications, *Journal of Monetary Economics*, 10, 139–162.

Piłatowska, M. (2003), *Modelowanie niestacjonarnych procesów ekonomicznych. Studium metodologiczne* (Modeling of Non-stationary Economic Processes. Methodological Study), Wydawnictwo UMK, Toruń.

Sims, C. A. (1980), Macroeconomics and Reality, *Econometrica*, 48, 1–48.

Zieliński, Z. (1984), Zmienność w czasie strukturalnych parametrów modelu ekonometrycznego (Time Variability of Structural Parameters in Econometric Model), *Przegląd Statystyczny (Statistical Survey)*, z. 1/2, 135–148.

Zieliński, Z. (2002), Analiza wybranych koncepcji modelowania dynamicznego w ekonometrii (Analysis of Different Concepts of Dynamic Modeling), in: Zieliński, Z., *Analiza ekonomicznych procesów stochastycznych. Pisma wybrane* (Analysis of Economic Stochastic Processes), Wydawnictwo UMK, Toruń.