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The Congruence Postulate at the Early Stage of Dynamic Econometric Modeling

1. Introduction

Application of mathematical instruments for the description of relationships among different economic phenomena leads to econometric models. Models built for stochastic processes or time series based models are called dynamic models. The general definition of dynamic model is given by Balasko (1984): "dynamic econometric model describes changes in time of observations on economic variable". While the definition presented by Talaga, Zieliński (1986) is as follows: "the model describing relationships among endogenous processes and exogenous (stochastic and non-stochastic) processes" is called a dynamic econometric model.

Different assumptions underlay the formulating of initial model specification in different approaches of dynamic model building. Some approaches referred to causal relationships, the other ones – to the internal structure of processes of interest with omission of causality, and others took both into account.

The concept of dynamic congruent modeling formulated by Professor Zygmunt Zieliński takes into account both assumptions, i.e. causal relationships and the internal structure of given processes. The model is congruent according to Zieliński if the harmonic structure of dependent process Y_t is the same as the harmonic structure of explanatory processes X_{it} (i=1,2,…,k) and the residual process which is independent of explanatory processes. Model built for processes having the white noise properties is always congruent, i.e.

$$\varepsilon_{yt} = \sum_{i=1}^{k} \rho_i \varepsilon_{x_i t} + \varepsilon_t \ . \qquad (1)$$

Model (1) is congruent because harmonic structure of both sides of the equation is identical or in other words the spectra of processes are pararel to the frequency axis.

Let Y_t and X_{it} (i=1,..,k) denote the endogenous process and explanatory processes respectively those univariate time series models describing their internal structure are as follows:

– models describing non-stationary components:

$$Y_t = P_{yt} + S_{yt} + \eta_{yt}, \qquad X_{it} = P_{x_it} + S_{x_it} + \eta_{x_it}, \tag{2}$$

where P_{yt}, P_{x_it} denote polynomial function of time variable t for respective processes, S_{yt}, S_{x_it} – seasonal components with constant or changing in time amplitude of fluctuations for respective processes, η_{yt}, η_{x_it} – stationary autoregressive processes for respective processes;

autoregressive processes:

$$B(u)\eta_{yt} = \varepsilon_{yt}, \qquad A_i(u)\eta_{x_it} = \varepsilon_{x_it}, \tag{3}$$

where $B(u)$, $A_i(u)$ stand for stationary autoregressive backshift operators for which all roots of equations $|B(u)| = 0$ and $|A_i(u)| = 0$ are outside the unit root circle, $\varepsilon_{yt}, \varepsilon_{x_it}$ – white noises for respective processes.

Information about the internal structure of all studied processes and the relationship of white noise components of respective processes (model (1)) allow to build the dynamic congruent econometric model.

The congruent model for real processes Y_t and X_{it} can be recived by substituting for white noises $\varepsilon_{yt}, \varepsilon_{x_it}$ in (1) from models (3) and next for autoregressive processes η_{yt}, η_{x_it} from models (2). After some transformations the dynamic congruent model is received:

$$B(u)Y_t = \sum_{i=1}^{k} A_i^*(u)X_{it} + P_t + S_t + \varepsilon_t \tag{4}$$

where P_t denote polynomial function of time variable t, S_t – seasonal component with constant or changing in time amplitude of fluctuations $A^*_i(u)$ stand for stationary autoregressive backshift operators. In model (4) the residual process ε_t has the same properties as in model (1). This means that the congruence postulate was satisfied. The congruent model (4) includes all components resulting from the internal structure of given processes.

Shortly outlined concept of dynamic congruent modelling points out the necessity of including the information about internal structure of given processes already at the stage of the model specification, such as trend and seasonal components, autoregressive components.

The purpose of this paper is to present the different approaches to the specification of elements of internal structure at early stages of building the dynamic econometric models.

2. Trend and Seasonal Component

The oldest example of the approach directed to satisfy the congruence postulate can be found in the work of Jevons from 1862 who wrote[1]:

> *Every kind of periodic fluctuation, whether daily, weekly, monthly, quarterly, or yearly, must be detected and exhibitive, not only as a subject of study in itself, but because we must ascertain and eliminate such periodic variations before we can correctly exhibit those which are irregular or non-periodic, and probably of more interest and importance.*

The necessity of including or eliminating periodical component has been noticed by Jevons on daily trade reports being analysed. The proper analysis of relationships calls for taking into consideration or eliminating periodical components of processes.

Such approach to building the dynamic econometric models was encountered already at the beginning of the XX century when both the guidelines from economic theory and elements of internal structure of given processes were used in model specification. In works of Jevons (1862), Moore (1914), Hooker (1901), Pearson (1919), Schultz (1929), Lange (1931) many information about the structure of studied processes and its use in modelling can be found. For example, the trend component or seasonality was eliminated by including these components into a model or by subtracting them from each process of interest. However this approach resulted only from the researcher's intuition, but not from theoretical fundamentals.

The first works of Moore, from the beginning of XX century, referred to the dynamic analysis of agricultural processes of United States. The American statistician Henry L. Moore used the multivariate correlation and regression to analysis of cereal crops depending on prices during harvest and rainfall level. Using Fourier transformation and periodogram he determined the rainfall cycles of 8 and 33 years length which were treated as completely independent of economic phenomena, therefore purely exogenous processes. He combined the periodicity of 8 years length in cereal crops with the periodicity of the same length in rainfall.

It should be emphasized that Jevons and Moore considered the business cycles only statistically, but did not analyse their economic causes. However they

[1] Jevons (1862), in: Hendry, Morgan (1995).

were the first who searched the “periodic cause” of “periodic effect” understood as business cycle (Morgan, 1990). Their research aiming at the detection of internal structure of economic processes represented the proper direction in satisfying what later was called the congruence postulate.

The direction, concerning the taking into account the internal structure of economic processes in modelling, started by Moore was continued by Schultz (1938). Having analysed the demand and supply equations he introduced polynomial of first order (linear trend) or second order (quadratic trend) because of the lack of statistical information about typical economic factors. The polynomial trend appearing in demand equation (or supply equation) played the substituting role with regard to the all not included variables influencing the demand (or supply). Certainly it would be desirable to include explicitly the factors having impact on demand (or supply) instead of including time variable (or generally polynomial function of time variable) which does not contribute to changes in demand (Lange, 1961). In further studies the inclusion of time variable became a frequent practise, however the time variable was treated only as substituting factor, but not as the component eliminating the non-stationarity in mean. It is worth noting that the inclusion of time variable into model should be regarded as required approach when both the elimination of non-stationarity and identification of model parameters are concerned.

The theoretical fundamentals of taking into account the internal structure of studied processes were given by the Frisch-Waugh theorem (1933) assuming that the inclusion of time variable t into the model eliminates the linear trend from all studied processes. Further Tintner (1952) expanded the Frisch-Waugh theorem to the polynomial trend, and Lovell (1963) – to seasonal component. The Frisch-Waugh, Tintner and Lovell theorems indicate that including the elements of internal structure of processes (trend, seasonality) to the model eliminates those elements from all processes of interest. In that case it should be remembered that the interpretation of parameters refers to stationary processes from which trend and seasonal components were eliminated.

All these actions both led to better explanation of economic phenomena and realized the importance of their internal structure (or time series decomposition) in building an econometric model.

The problem of time series decomposition in polish literature was firstly described by Oskar Lange in his work from 1931 where the methodology of time series decomposition is presented. Lange distinguished four components: trend, seasonal fluctuations, periodical fluctuations (business cycles) and irregular fluctuations and showed the methods of decomposition into these components. For example, to distinguish the periodical fluctuations and he used a periodogram as a basis of choosing the harmonics to evaluate the length and strength of a cycle (Lange, 1931).

The intention of Lange to build the congruent model was displayed in the expression that "each component of a process has its own group of causes". He assumed that causes influencing the time series can be divided into four groups each has statistically relevant effects. The groups are following:

– causes constantly affecting over long time,
– causes affecting periodically in certain periods of year or month, for example in autumn, at the end of month),
– causes influencing the periodic fluctuations,
– causes affecting irregularly without any specific scheme.

Lange assumed that each group of causes brings about certain quantitative effects and influences the time series behaviour by producing particular changes. These changes in time series can also be divided into four groups appropriately to causes producing them. Hence each component of time series can be treated as an effect of one group of causes mentioned above. This means that a time series should be regarded as a product of four primal series each being produced by one of the four groups of causes (Lange, 1931). Such understanding of the influence of causes on effects means that in dynamic modelling the structure of causal processes and the explained processes (effects) should be the same, e.g. periodical causes influence the periodical effects.

The conclusions formulated by Lange in the analysis of business cycle can be treated as a intention to build the "balanced model with regard to the dominant structure of causal and explained processes" as later has been formulated by Granger (1981) or to satisfy the congruence of harmonic structure of given economic processes as Zieliński (1984) defined.

Another way to improve the specification of dynamic model was to introduce the dummies in order to describe the outliers. Welfe (1977) presented several methods of using dummies to take into account among others: the changes in mean, changes in parameters (including intercept), changes in variance. However the use of dummies should be treated as an element of the improvement of model specification in order to make the structure of both side of equation the same (or make model congruent)[2].

3. Autoregressive Component

The autoregressive component is taken into account by including the lagged variables into the model. The lags can play the role of causal factor or balancing factor (making model congruent).

Such approach was encountered already at the early stage of dynamic modelling. For example, building model of crop prices Moore[3] observed a simulta-

[2] See Zieliński (1984).
[3] See Moore (1914)

neous relationship for some agricultural goods (crops, potatoes, hay), e.g. the growth rate of crop cereals $\Delta Q / Q_{t-1}$ affecting the growth rate of crop prices $\Delta P / P_{t-1}$, and for cotton – lagged relationship, e.g. the $\Delta P / P_{t-2}$ affecting the $\Delta Q / Q_{t-1}$.

The inclusion of crop prices was interpreted as expected current price (Epstein, 1987) influencing the behaviour of cotton producers who made decision about future production (t+1) on the basis of expected price equal to the price form current period t.

It is worth noting that the idea of building model for growth rates, but not for levels, came from Moore's intuition that the transformation of series was necessary to eliminate the effects of climate shocks on crop cereals and their prices, i.e. changes in variance. This transformation nowadays is treated as differencing filter being applied to remove the non-stationarity in variance what is regarded as one of conditions for correct econometric modelling.

Moore was absolutely convinced that the explanation of periodic fluctuations and finding the laws describing them have been the "fundamental problem of economic dynamics "(Moore, 1914)[4]. Due to that Moore should be treated as a precursor of analysing the internal structure of economic processes in the present meaning. To Moore's successors continuing the research on economic fluctuations can be counted among others: Henry Schultz (Moore's student) and Holbrook Working (see Epstein, 1987; Lange, 1961).

The achievements of Schultz are worth to mention because they refer to the identification of data features and setting the lag length. Analysing the demand-supply-price relationships Schultz was the first who used the lagged price. However, it was possible only for agricultural production for which annual cycle of fluctuations has been observed, but not for industrial production for which the cycles of shorter length, i.e. 1 month, 3 month, half a year, have been more frequently occurred. For lack of economic (casual) factors he introduced the time variable into model (in the form of linear or quadratic trend). All these actions can be treated as first attempts to make model congruent.

In seventies of XX century new trends in econometrics appeared as a result of the criticism of previous applied approach, i.e. modelling economic stochastic processes (based on time series) as if they were random variables. Examples of such criticism can be found amongst others in Hendry[5] who claimed that econometricians should apply the modern techniques and robust testing. This means that statistically discovered relationships should be put higher than the theory.

[4] See Morgan (1990).
[5] See Hendry (2000).

To modern concepts of dynamic modelling taking into account the structure (dominant features) of economic processes belong: general to specific modelling, integration and co-integration analysis, VAR modelling, VECM, ARIMA, GARCH.

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