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Markov-Switching Models for the Prices of Electric Energy on the Energy Stock Market in Poland

1. Introduction

GARCH models, characterised by autoregressive dependencies in the conditional variance equation, are used in modelling financial time series on account of the variance grouping phenomenon they demonstrate. In turn, Markov-switching models, through random process switching to different regimes, allow the differentiation of periods corresponding to different levels of volatility of the endogenous variable (Stawicki, 2004). Hamilton and Susmel (1994) proposed the integration of these two approaches by introducing a random regime switching in the conditional variance equation of the ARCH model (*Markov-switching ARCH, SWARCH)*. A generalised version of the SWARCH model is the GARCH model with a regime switching (*MS-GARCH)*, which makes it possible to provide a more detailed description of the dynamics of the variance process which differs across the regimes (Frömmel, 2004).

In this paper we present theoretically different specifications of Markov-switching models, which take into consideration the autoregressive dependencies both in the conditional mean and in the conditional variance of process. In the empirical section of this paper the Markov-switching models for daily prices of electric energy in Poland are estimated and tested.

2. General Overview of the Markov-Switching Model

Hamilton (1990) proposed a form of the Markov-switching model MS(N)-AR(p) which describes the changes of the mean and the variance of autoregressive economic process across the regimes:

$$y_t = c_{s_t} + \phi_{1,s_t} y_{t-1} + \phi_{2,s_t} y_{t-2} + ... + \phi_{p,s_t} y_{t-p} + \varepsilon_t , \tag{1}$$

nowhere: $\varepsilon_t = \sigma_{s_t} u_t$ for $u_t \sim$ IID(0,1), s_t - denotes a first-order homogeneous Markov chain with N states and transition probabilities matrix $P = \lfloor p_{ij} \rfloor_{N \times N}$, c_{s_t}, ϕ_{1,s_t} - parameters depend on the state variable s_t.

Hamilton (1994) formulates the following relation for the conditional distribution of the state variable s_t:

$$P(s_t = j|\Phi_t) = \frac{f(y_t, s_t = j|\Phi_{t-1})}{f(y_t|\Phi_{t-1})} = \frac{f(y_t|s_t = j, \Phi_{t-1}) \cdot P(s_t = j|\Phi_{t-1})}{\sum_{i=1}^{N} f(y_t|s_t = i, \Phi_{t-1}) \cdot P(s_t = i|\Phi_{t-1})}, \quad (2)$$

where the form of the density function of conditional distribution of the variable y_t depends on the density function $g(\cdot)$ of the postulated distribution of the innovations (Doman, 2005):[1]

$$f(y_t|s_t = j, \Phi_{t-1}) = \frac{1}{\sigma_t(j)} g_j\left(\frac{y_t - \mu_t(j)}{\sigma_t(j)}\right), \quad (3)$$

The generalisation of the Markov-switching model of the form (1) consists in including an additional exogenous variable into a model. In such a case the matrix form of model is following (Kaufmann, 2000):

$$y_t = X_t \beta_{s_t} + \varepsilon_t, \qquad \varepsilon_t \sim \text{IID}\left(0, \sigma_{S_t}\right), \quad (4)$$

where: X_t = (1, y_{t-1},..., y_{t-p}, x_t,..., x_{t-q}) – vector of observable variables, which may contain lagged both endogenous and exogenous variables, $\beta_{s_t}{}^T = (c_{s_t}, \phi_{1,s_t}, \phi_{2,s_t}, ..., \phi_{p,s_t}, \psi_{0,s_t}, \psi_{1,s_t}, ..., \psi_{q,s_t})$ – parameters vector, which depends on the state variable s_t.

3. GARCH Structure for Markov-Switching Models

The econometric literature provides the descriptions of various possible specifications of the conditional variance equation for the MS-GARCH model, which generalize the GARCH (p, q) model by allowing for regimes with different volatility levels (Gray, 1996; Klassen, 2002):

$$h_t = \omega_{s_t} + \sum_{i=1}^{q} \alpha_{i,s_t} \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \beta_{i,s_t} h_{t-i}, \quad (5)$$

[1] $\mu_t(j)$ and $\sigma_t(j)$ are the conditional mean and the conditional standard deviation of the financial process respectively, dependent on the regime j, in which the process is found at a moment t. One can include the autoregressive scheme in the conditional mean equation depending on the properties of the modelled series. In empirical research it is frequently assumed that innovations have the following distributions: normal, Student's t-distribution, GED, skew Student's t-distribution.

where: h_t – the variance of random term conditional on observable information Φ_{t-1} and on the regime path $\tilde{s}_t$, ω_{s_t} – the intercept depends on the state variable s_t, α_{i,s_t} – the ARCH parameters depend on the state variable s_t, β_{i,s_t} – the GARCH parameters depend on the state variable s_t.

However, the estimation of the conditional variance parameter h_{t-1} in equation (5) causes problems of numerical nature, owing to the occurrence of path-dependence which illustrates the entire history of process switching to particular regimes (Cai 1994 and Hamilton, Susmel 1994). This problem will be presented for the MS(2)-GARCH (1,1) model.

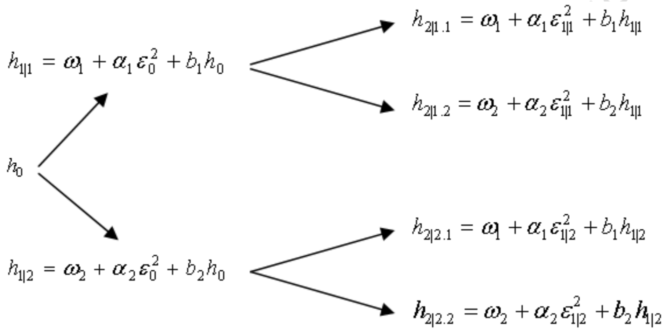

Figure 1. The evolution of conditional variance in the GARCH model with a regime path-dependence

Source: Gray S. F., Modeling the conditional distribution of interest rates as a regime – switching process, *Journal of Financial Economics* 3/1996, p. 35.

Each conditional variance (concerning the model with path-dependence) depends not only on the current regime, but also on the entire history of the process which controls the switches between different volatility regimes, which is reflected by the branching nodes of the above tree.

In the subsequent specification, the conditional variance was made dependent only on the current regime s_t and not the entire path $\tilde{s}_{t-1}$, through having introduced an expected value operator into the expression (5) (Gray, 1996):

$$h_t = \omega_{s_t} + \alpha_{s_t} \varepsilon_{t-1}^2 + \beta_{s_t} E_{t-2}[h_{t-1}], \tag{6}$$

where:

$$\begin{aligned} h_{t-1} &= E[y_{t-1}^2 | \Phi_{t-2}] - (E[y_{t-1} | \Phi_{t-2}])^2 = P(s_{t-1} = 1 | \Phi_{t-2}) \cdot (\mu_1^2 + h_{1t-1}) + \\ &+ P(s_{t-1} = 2 | \Phi_{t-2}) \cdot (\mu_2^2 + h_{2t-1}) - \{P(s_{t-1} = 1 | \Phi_{t-2}) \cdot \mu_1 + P(s_{t-1} = 2 | \Phi_{t-2}) \cdot \mu_2\}^2 \end{aligned} \tag{7}$$

Figure 2 illustrates the evolution of conditional variance in the MS-GARCH model without a regime path-dependence.

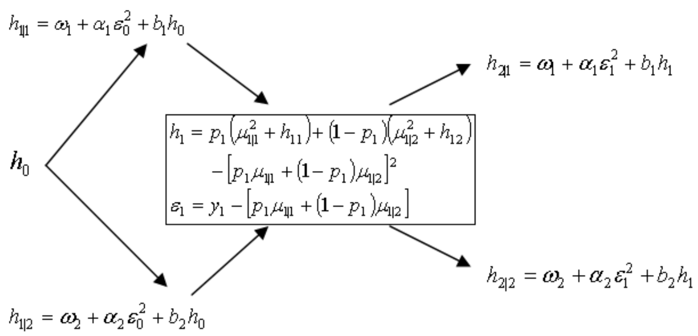

Figure 2. The evolution of conditional variance in the GARCH model without a regime path-dependence

Source: Gray S. F., Modeling the conditional distribution of interest rates as a regime – switching process, *Journal of Financial Economics* 3/1996, p. 36.

In any given period, as a consequence of the conditional variances ($h_{t/i}$ for i=1,2) integration, a conditional variance (h_t) is constructed relative to the set of observed data (Φ_{t-1}), not regimes, as it occurred in the previous specification. The described dependency is illustrated by the tree nodes which combine into a shared node in each subsequent step. The final specification of the conditional variance of the GARCH (1,1) model with a volatile regime can be presented in the form of the following equation:[2]

$$h_t = \omega_{s_t} + \alpha_{s_t} \varepsilon_{t-1}^2 + \beta_{s_t} E_{t-1}[h_{t-1} | s_t], \tag{8}$$

where:

$$\begin{aligned} h_{t-1} &= P(s_{t-1} = 1 | \Phi_{t-1}) \cdot (\mu_1^2 + h_{1t-1}) + P(s_{t-1} = 2 | \Phi_{t-1}) \cdot (\mu_2^2 + h_{2t-1}) - \\ &- \{P(s_{t-1} = 1 | \Phi_{t-1}) \cdot \mu_1 + P(s_{t-1} = 2 | \Phi_{t-1}) \cdot \mu_2\}^2. \end{aligned} \tag{9}$$

The problem of the occurrence of path-dependence in the conditional variance equation is of vital importance when computing multi-periods-ahead forecasts.

4. Numerical Sample

The empirical study has been conducted on daily spot quotations of electric energy on the Polish Energy Stock Market in the period between January 02, 2004 and December 31, 2006. The form of Markov-switching model proposed by Hamilton and Susmel (1994) was estimated:

$$\mathrm{y}_t = \gamma + \varphi \cdot \mathrm{y}_{t-1} + \mathrm{u}_t, \tag{10}$$

[2] The final specification of conditional variance for the GARCH model with a regime –switching takes advantage of filtered probabilities, while in the previous specification the ex-ante probabilities were applied (Klassen, 2002).

whereas for the innovation u_t the SWARCH – L(3,2) specification was used:

$$
\begin{aligned}
&\mathrm{u}_{\mathrm{t}} = \sqrt{g_{s_t}} \cdot \tilde{\mathrm{u}}, \\
&\tilde{\mathrm{u}}_{\mathrm{t}} = h_t \cdot \upsilon_{\mathrm{t}}, \\
&h_t^2 = \alpha_0 + \alpha_1 \tilde{\mathrm{u}}_{\mathrm{t-1}}^2 + \alpha_2 \tilde{\mathrm{u}}_{\mathrm{t-2}}^2 + \xi \cdot d_{t-1} \cdot \tilde{\mathrm{u}}_{\mathrm{t-1}}^2, \\
&d_{t-1} = \begin{cases} 1 \text{ dla } \tilde{\mathrm{u}}_{\mathrm{t\text{-}1}} \le 0 \\ 0 \text{ dla } \tilde{\mathrm{u}}_{\mathrm{t\text{-}1}} > 0, \end{cases}
\end{aligned}
\tag{11}
$$

where state variable s_t follows a first-order homogeneous Markov chain with three regimes.

A likelihood function on the basis of the observations of the variable y_t for $t = 1, 2, ..., T$ can be formulated:

$$
\tilde{L}(y_1, y_2, ..., y_T; \theta) = \sum_{t=1}^{T} \log\left(\sum_{j=1}^{N} f(y_t | s_t = j, \Phi_{t-1}; \theta) P(s_t = j | \Phi_{t-1}; \theta) \right), \tag{12}
$$

The estimation of the parameters of this model has been performed in the Ox package, using the program codes written by Hamilton.

The volatility clustering of the electric energy prices is explained by the fact that the periods of high volatility are followed by periods of low volatility. It is worth to wonder if a period of high volatility will be followed by normal volatility level and, similarly, a period of low volatility will be followed by normal volatility level. So, we used different specifications of Markov switching ARCH model for describing the polish electric energy volatility process. The results in Table 1 show that variances in particular states differ from each other for each estimated switching model. For example, the results obtained for the SWARCH(2,0) model indicate that the variance corresponding to the second regime is over five times higher than the variance which characterises the first regime. It is also worth to observe that the probabilities of remaining in the states of high-, and low-volatility are significantly large, which is reflected by the effect of variance grouping in series of returns of energy prices. Similar results were obtained for the SWARCH-L(3,2) model.

In the case of the first specification of the switching model, in which the modelling of the financial leverage effect was left and a normal distribution of the random parameter was assumed, an atypical estimate of variance in normal-volatility regime was obtained. Moreover, the estimates of switching probabilities for the Markov chain indicate a lower stability of particular regimes in comparison to the remaining models. In the case of SWARCH(3,2) and SWARCH-L(3,2) models, the estimates of parameter vectors are significantly different from each other, what may be connected with some numerical problems (the Hessian matrix is not positive definite, so the conclusion may be that either local maximum has not been found or the estimates are up against boundary condition).

Table 1. Estimates of parameters for the SWARCH model

	SWARCH(3,2)	**SWARCH-L(3,2)**	**SWARCH(2,0)**
const	0.10491258	-0.080703910	0.08988490 (0.70880937)
φ	-0.22516631	-0.14736107	-0.22255117 (-7.0260506)
α_0	15.348751	3.4173519	13.215834 (9.5472570)
α_1	0.24141784	0.0049463894	-
α_2	0.19174416	0.057990611	-
ξ	-	0.90538713	-
ν	-	5.0749257	-
g_1	1	1	1
g_2	0.18257100	4.0064729	5.5615685 (7.8415091)
g_3	2.1873915	13.553465	-
p_{11}	0.57932120	0.99240311	0.9175262 (5.0650989)
p_{22}	0.56702740	0.99388004	0.8019476 (4.6635380)
p_{12}	0.42067880	0.0075968939	-
p_{21}	0.00000000	0.00000000	-
p_{31}	0.40554684	0.0000000066	-
p_{33}	0.59445300	0.98016587	-
Log lik	-3314.7072	-3292.9939	-3344.0826
n	1095	1095	1095

T-statistics in parentheses.

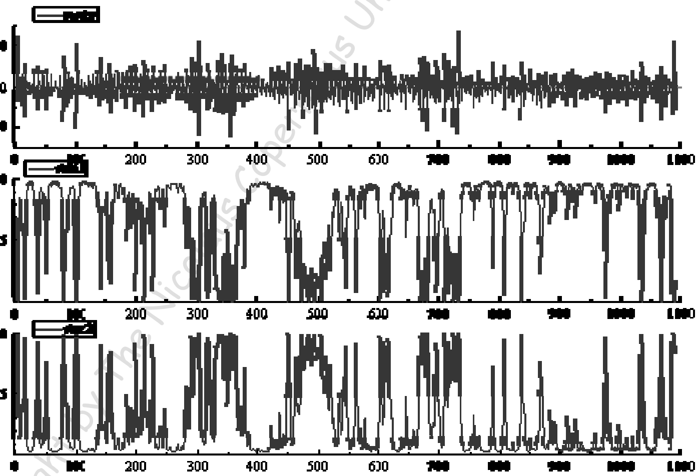

Figure 3. Returns and smoothed probabilities of obtaining low- and high-volatility states for the SWARCH(2,0) model for energy prices

An additional products of the estimation of Markov switching model parameters are the filter and smoothed probabilities. Basing on them one may indicate with a defined probability that the electric energy return at the moment *t* was generated in the unobserved high or low volatility regime. One may also estimate the moment of switching the process from low volatility regime to high volatility (the following dependency must be satisfied: $P(s_t = 1|\Phi_T;\theta) > 0{,}5$ and $P(s_{t+1} = 1|\Phi_T;\theta) < 0{,}5$) and vice versa.

Figure 3 plots the estimated smoothed probabilities of being in the low and high volatility regime, as defined in (2).

On the basis of the computed residuals it was possible to verify the hypotheses concerning the dynamic specification of particular models. The values of particular testing statistics are presented in Table 2.

Table 2. Diagnostics for dynamic specification of Markov switching model

Test	SWARCH(3,2)	SWARCH-L(3,2)	**SWARCH(2,0)**
Ljung-Box test	311.9	331.9	**313.5**
Jarque-Bera test	174.3	163.1	**186.7**
ARCH effect test	84.6	85.5	**94.1**
AIC	-3325.7072	-3305.9939	**-3350.0826**
SC	**-3353.1972**	**-3338.4839**	**-3365.0781**

The results presented in Table 2 confirm that the residuals of given models do not have white noise properties, i.e. they are correlated and conditionally heteroscedastic, and their distributions significantly differ from the normal distribution.

The transition probability values are the starting point for determination of the expected duration of regime *i* (Hamilton, Susmel, 1994):

$$d(i) = \frac{1}{1 - p_{ii}}, \tag{13}$$

The analysis of the information included in Table 1 leads to the finding that state 1 (low volatility level) is expected to last on average for 12 days, while state 2 (high volatility level) typically – for 5 days.

4. Conclusions

To conclude, attention ought to be paid to the following properties of the GARCH model with a regime-switching:

- the structure of the ARCH and GARCH model allows to take into consideration the effect of conditional heteroscedasticity which differs significantly across the regimes.
- higher flexibility of the model with regard to the persistence of shocks in volatility level[3],

[3] Not all the disturbances which occur on the market have permanent influence on the level of prices. Owing to a suitable construction of these models, the stability of this kind of disturbances may be reduced by means of switching to a regime with a lower-

Analyzing the results of the empirical study which concerned the modeling of market prices of electric energy the following conclusions may be formulated. The specification of equation for both the conditional mean and conditional variance should be improved. The series of returns of electric energy prices are subject to periodic fluctuations due to the specific character of electric energy as a commodity traded on the stock exchange. The autoregressive structure of first order considered in models in question turned out to be insufficient for the description of the underlying dependencies. As a consequence, in further research the higher-order autocorrelation should be taken into account and the ARCH structure should be replaced by the GARCH structure with a volatile regime.

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volatility level, which in turn results in a particular piece of information being quickly delivered “outside the market”.