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Pricing of Weather Options for Berlin Quoted on the Chicago Mercantile Exchange

1. Introduction

At present the global weather derivatives market is developing very fast. Only in the recent period (April 2007 - March 2008) the notional value of all weather contracts reached over 32 billion USD. As a result of such a dynamic increase on this market the problem of appropriate weather options pricing appears more often. Usually in these situations, the Black-Scholes formula is used. Unfortunately, many observers and weather market participants have noticed that this approach cannot be applied because of the different nature of weather underlying[1]. It must be added, that the unique and complex features of weather indices made it impossible until now to create any complete and universal procedure for pricing this class of instruments. For this reason many different approaches of pricing weather derivatives have been proposed. The most popular are: historical burn analysis, index modelling and daily modelling. Among these, the last one seems to have the greatest potential in creating one precise approach of pricing all weather contracts[2]. Therefore in this paper we concentrate on daily modelling as an approach which makes use of models of stochastic processes. Moreover this work we analyse not only daily time series, but also monthly values of weather indices.

[1] See Dischel (1998).
[2] Brix, Jewson and Ziehmann (2005).

The main goal of this work is to present the approach of pricing weather options using the ARFIMA-FIGARCH model[3]. It has been shown that this model neglects seasonally changing autocorrelation, which leads to significant deviations in option pricing. We propose in the paper two pricing models which is the authors' contribution. First the model ARFIMA-FIGARCH used for daily data (see Moreno, 2003 for analysis of temperature) has been extended by including seasonality in variance what turned out to be statistically significant. The second model bases on monthly values of temperature indices, which is a kind of compromise between simple pricing approaches like historical burn analysis and more advanced methods, like econometric daily modelling.

In section 2 we describe all the stochastic models that we used in our research. In this part we propose a small improvement that allows the inclusion of seasonality in variance. Section 3 contains the description of all statistical features of selected time series and the results of estimation for analysed models. In the next section the analysed models are used in an empirical example for pricing Chicago Mercantile Exchange's weather options for Berlin. Section 5 contains conclusions.

2. Models of Stochastic Processes

In the process of air temperature we can usually set apart such distinctive features like seasonality, trend as a result of global warming and urbanization, long memory, which is typical for climate processes (see Hurst, 1951, Kwiatkowski and Osiewalski, 2002), and random fluctuations. Additionally, we can find seasonality in variance. Volatility of air temperature is higher in winter months and smaller in summer months. Hence all the above features ought to be included in modelling of air temperature.

The models proposed in this work are an extended version of the ARFIMA-FIGARCH model and allow the inclusion of seasonality in mean and variance. The ARFIMA model, as a generalization of the ARIMA model, allows the effective description of short and long memory in mean. The FIGARCH model, as a generalization of the GARCH model, allows similar features to be described in variance. The extended model ARFIMA(P,d_1,Q) - FIGARCH (p,d_2,q) can be presented in the following form:

$$\varphi(L)(1-L)^{d_1}(y_t-\mu_t)=\vartheta(L)\varepsilon_t, \qquad \varepsilon_t|\psi_{t-1}\sim D(0,h_t)\ , \tag{1}$$

$$\mu_t=\sum_{j=0}^{r}\gamma_j t^j+\sum_{i=1}^{m/2}(\theta_i\cos\omega_i t+\lambda_i\sin\omega_i t)\ , \tag{2}$$

[3] This model is used by practitioners on the financial market and it exists in popular application "SWS 6.0" made by *Speedwell Weather Derivatives Ltd.*

$$\phi(L)(1-L)^{d_2}\varepsilon_t^2 = \alpha_0 + \sum_{i=1}^{m/2}(\kappa_i \cos\omega_i t + \tau_i \sin\omega_i t) + [1-\beta(L)]\nu_t\,, \qquad (3)$$

where L is the backshift operator ($L^s\varepsilon_t = \varepsilon_{t-s}$), $\varphi(L) = 1 - \sum_{j=1}^{P}\varphi_j L^j$, $\vartheta(L) = 1 + \sum_{j=1}^{Q}\vartheta_j L^j$, $\phi(L) = 1 - \sum_{j=1}^{q}\phi_j L^j$, $\beta(L) = \sum_{j=1}^{p}\beta_j L^j$, $\nu_t = \varepsilon_t^2 - h_t$, all roots of polynomials $\varphi(L) = 0$ and $\phi(L) = 0$ are beyond unit circle, $-1 < d_1 < 0.5$, $0 < d_2 < 1$, $\omega_i = \frac{2\pi i}{m}$, $m = 365$.

In order to obtain positive variance h_t without imposing additional restrictions on the parameters in the equation for variance, a logarithmic formula of variance can be applied ($\nu_t = \varepsilon_t^2 - \ln h_t$). The seasonal component in equation (2) can be presented in a different than harmonic way:

$$\mu_t = \sum_{j=0}^{r}\gamma_j t^j + \sum_{k=1}^{12} c_k m_{kt}\,, \qquad (4)$$

where m_{kt} are dummy variables that mean successive months (eg. $m_{1t} = 1$ for January and $m_{1t} = 0$ for other months). In a similar way we can describe a seasonal component for variance:

$$\phi(L)(1-L)^{d_2}\varepsilon_t^2 = \alpha_0 + \sum_{k=1}^{12}\delta_k m_{kt} + [1-\beta(L)]\nu_t\,. \qquad (5)$$

3. Modelling Time Series of Air Temperature and Weather Indices

Historical data for Berlin were obtained from Deutscher Wetterdienst and they were collected over the period from January 1, 1948 to December 31, 2004 (20808 daily observations). Distributions of daily values of temperatures are different in specific months (periods) of a year. Research in this area has shown that in winter months there is left asymmetry while in summer months asymmetry is positive. The hypothesis about normality of distribution, using a variety of tests, has been rejected for specific months and also for the entire year[4]. The results of statistical tests indicate the existence of the following properties of daily average temperature: increasing linear trend, seasonality both in mean and variance, autocorrelation (higher in winter months), long memory in mean and the ARCH effect.

[4] Owing to the limited size of this paper some results of performed tests and estimations had to be omitted.

Apart from the daily average of air temperature, also monthly temperature indices (HDD and CAT) were analysed. These indices can be calculated using the following formulae:

$$\mathrm{HDD} = \sum_{t=1}^{m} \max(0, 18°C - y_t), \tag{6}$$

$$\mathrm{CAT} = \sum_{t=1}^{m} y_t, \tag{7}$$

where y_t - average daily temperature, m - number of days in any given period.

The hypothesis about normality of distribution was rejected only for the HDD index for the months of March and December. In the current research on pricing weather contracts, only the distribution of the cumulated monthly indices (HDD and others) is analysed. In this paper we treat the monthly time series of these indices as a realization of stochastic processes and explore their features. The performed tests indicate the existence of the following features in monthly indices HDD and CAT: linear trend, seasonality in mean and in variance, long memory in mean (weaker than for daily time series).

The model described in equations (1-3) was selected to explain the average daily temperature. The long memory has been described by ARFIMA model (d_1 = 0.1269 (0.0283)). The short-term dependences in the time series of temperature have been explained by autoregressive and moving average parts with lags equal to two ($P = 2$ and $Q = 2$). In the equation for conditional variance the parameter d_2 turned out to be insignificant, hence the process of volatility has no long memory. The GARCH(1,1) model with annual periodical fluctuations and normal conditional distribution turned out to be sufficient to describe variance of temperature.

For monthly values of HDD and CAT two models with different parameterisation of seasonality were considered. If the model consisted of a harmonic seasonal component and linear trend, the error term for the HDD index was best described by the AR(1) model. For the seasonal model with dummy variables and linear trend (equation 4), the error term was best described by ARFIMA(0,d_1,0).

For the CAT index, for any parameterisation of seasonality, linear trend and long memory were observed. In both cases, the ARFIMA model (0,d_1,0) turned out to be the best. For all indices unconditional variance of random term in all models was variable. The ARCH effect was not present.

The final parameterisation (values P, Q, p, q, r and the number of parts of trigonometric polynomials) were selected by the Schwarz criterion, taking into account results of proper diagnostic tests. The analysed models were verified in terms of the average mean and variance using Monte Carlo simulations. The results are presented in Table 1.

Table 1. Results of models' verification for mean and variance based on Monte Carlo simulations[5]

Parameters	Type of model					
	Model 1 (daily)		Model 2 (monthly)		Model 3 (monthly)	
	January HDD	June CAT	January HDD	June CAT	January HDD	June CAT
Mean	546.01	515.33	529.87	548.16	546.29	526.73
Variance	6746.95	4640.91	7243.87	1655.75	7483.06	1688.18
Δ for mean	2.14	11.00	14.00	21.82*	2.42	0.39
Δ for variance	818.45	3045.30*	321.53	60.14	82.34	92.57

Model 2 includes harmonic structure of seasonality, whereas in model 3 this component consists of dummy variables. The asterisk indicates that the null hypothesis about the equality of the expected value (variance) of the index assuming that the analysed model is true with expected value (variance) for population was rejected at the 5% level. Test statistics Δ for mean (variance) were calculated as absolute values of the differences between means (variances) calculated for generated data and sample.

Mean and variance values for the January HDD index were 555.39 and 7727.93 respectively, while for the June CAT index these parameters were 515.29 and 1536.38. Comparisons of the examined models have also been performed using other criteria, like adjusted determination coefficient or root mean squared error (Table 2).

Table 2. Evaluation of quality of models

Type of model	January - HDD		June - CAT	
	RMSE	Adj. R^2	RMSE	Adj. R^2
Model 1(daily)	11.07	0.997	11.48	0.997
Model 2 (monthly with harmonic seasonality)	55.04	0.922	57.85	0.929
Model 3 (monthly with dummy variables)	52.46	0.928	63.42	0.914

The obtained results show that despite the overestimation of variance during summer months, the model constructed for daily observations of temperature has the best ability to describe monthly values of HDD and CAT indices. The effect of the variance overestimation is caused by omitting seasonal variability of autocorrelation of temperature (see Figure 1).

[5] For the selected model 100 000 time series of temperature with length 30 or 31 observations were generated and afterwards indices were calculated. For models of indices, the values of indices were generated directly. The complete procedure of verification is given in Caballero, Jewson and Brix (2002).

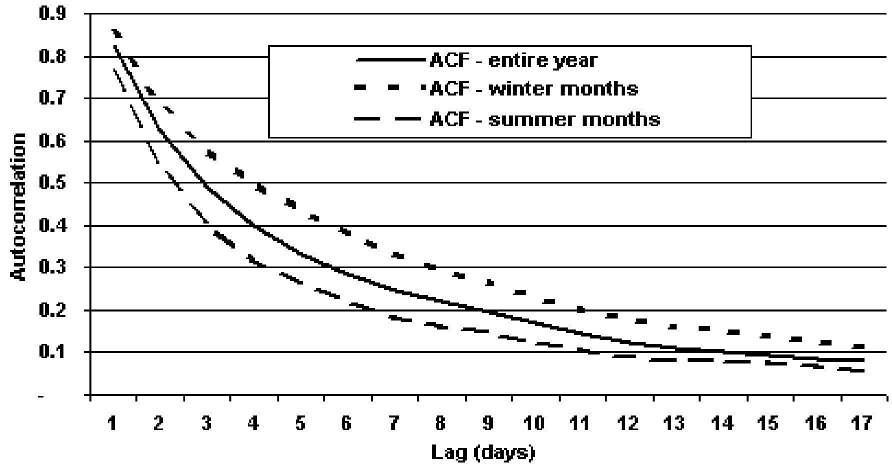

Figure 1. Autocorrelation function for average daily temperature in different periods of the year.

4. Pricing Weather Options

The models presented above have been applied in pricing two distinct monthly option contracts (call) with cap value[6]. Underlying for these options were indices: HDD for January 2004 and CAT for June 2004. Specification of these contracts is shown in Table 3.

Table 3. Specification of examined CME weather options for Berlin

Name	Contract 1	Contract 2
Type of option	Call	Call
Index	HDD Berlin	CAT Berlin
Period of life	January 2004	June 2004
Strike value	600 HDD	550 HDD
Tick value	10 000 GBP	10 000 GBP
Maximum payout (cap)	500 000 GBP	500 000 GBP

In order to compare pricing approaches, three methods mentioned above: historical burn analysis (HBA), index modelling (IM) and daily modelling (DM) were used. The last method includes all analysed models. The results are given in Table 4.

In the case of January's contracts a clearly lower evaluation was obtained using the model with harmonic seasonality (monthly model 2), for which the mean from the simulation turned out to be lower by 14 points than the mean from the sample.

[6] An option contract with cap value can be obtained by buying a call option and selling the call option with the same expiration date but with different strike prices.

Table 4. Valuations of weather options using different approaches

Valuation methods	Price of contract 1 [GBP]	Price of contract 2 [GBP]	Relative price of contract 1 as a percentage of maximum payout [% limit]	Relative price of contract 2 as a percentage of maximum payout [% limit]
HBA	86 550	45 150	17.31	9.03
IM	99 550	39 450	19.91	7.89
DM – model 1	84 900	96 650	16.98	19.33
MM – model 2	67 250	133 600	13.45	26.72
MM – model 3	92 900	67 150	18.58	13.43

The different results were obtained in pricing June's contract for the CAT index. For the first two methods the obtained valuations were below 10% of the maximum payout. The methods based on time series modelling estimated the price of this contract between 13.43% and 26.72% of the maximum payout. The valuation received on the basis of daily modelling is strongly overestimated because of the clearly overestimated level of variance for June (Table 1). The second model clearly overestimated the mean value and that is why the price of the option was overvalued. The most reliable pricing seems to be the one obtained from the last model (13.43%). Unfortunately, in this case the estimation's error will be higher, because the analysed model provided an inferior fit to the data. Besides, the model for monthly values does not take into account the variable number of days in a month and it cannot be used directly to estimate the price of weather contracts for all lengths of time.

Daily modelling of temperature allows the better usage of historical data. If one wants to price a monthly weather contract using historical burn analysis or index modelling approaches one can use the data only from the expiration period. For example for 10 years of data the estimation will be based on 10 historical values. Application of daily modelling allows the use of 3652 observations in the pricing process. Therefore this approach is potentially better. Only potentially, because it assumes that the model is correct and able to describe all properties of weather time series. Hence, the risk of using any model may be an important factor in derivative pricing[7]. The bayesian model pooling gives possibility to formally include specification uncertainty in statistical inference (see for example Osiewalski, 2001).

5. Conclusions

The models constructed for monthly observations of indices do not explain volatility of temperature indices in a sufficient way. The model constructed for daily observations of temperature is the best in describing temperature indices. This model describes most of the important features. Despite the application of

[7] See, for example, Jajuga (2007).

such an advanced model, it is not able to describe the real temperature process completely, because it neglects seasonal variability of autocorrelation, which causes over- or underestimations of variance in some periods.

Caballero and Jewson (2003) suggest as an alternative using the SAROMA model. This model considers seasonality in the autocorrelation function, but the number of parameters that need to be used in this model, makes the estimation process difficult. Besides, the SAROMA model requires many observations in order to avoid a spurious explanation of data. In addition, this model omits long memory. The other solution could be to include in the ARFIMA-FIGARCH model a set of time-varying parameters of long memory in mean (and perhaps also in variance). Unfortunately, the specification of this model will be more complex than the one presented in this paper.

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