DYNAMIC ECONOMETRIC MODELS
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Financial Econometrics – 25 Years Later

1. Financial Econometrics – Introduction

It has been a quarter of a century since the day being considered as the birth of the new area of research, called financial econometrics. On one hand, this field is considered as part of econometrics, particularly dynamic econometrics, on the other hand it is part of modern finance. In 1982 Robert Engle published the seminal paper (see Engle, 1982), where the first classical strictly financial econometric model was proposed. This paper presents some general remarks on the development of financial econometrics in last 25 years.

We start, however, with the presentation of some considerations on the notion of "financial econometrics". It seems that the models belonging to financial econometrics are two types of models:

– They verify hypotheses developed in the theory of finance;
– They identify some patterns in financial data.

This explanation corresponds to two standard definitions of financial econometrics, given in the literature. The first one was proposed in the famous monograph of financial econometrics (see Campbell, Lo, MacKinlay, 1997): "raison d'etre of financial econometrics is the empirical implementation and evaluation of financial models". The second one comes from the Journal of Financial Econometrics, defining the profile of this scientific journal in the following way:

„Designed to address substantive statistical issues raised by the tremendous growth of the financial industry. Papers providing or applying new econometric techniques which are particularly well suited to deal with financial data and models fall within the scope of this journal".

Robert Engle defines financial econometrics as "simply the application of econometric tools to financial data", which seems maybe too general and wide statement.

Traditionally, data considered in financial econometrics are time series data. It seems, however, that there are a lot of problems in finance solved by econometric methods, where cross–sectional data (or panel data) are used. This refers, for example to credit risk analysis.

From the historical point of view it is generally understood that financial econometrics contains two types of models:

– Dynamic econometric models, developed before and after 1982, adapted for financial time series;
– Econometric models developed after 1982 solely for the purpose of solving financial problems.

To conclude these introductory remarks, it is worth to mention about two important tendencies, observed particularly in empirical studies.

The first tendency is the integration of financial econometrics with financial mathematics and financial economics in the empirical studies leading to solution of particular financial problems. Sometimes it is referred to as empirical finance, where the theoretical models developed by financial mathematics are interpreted by financial economics and then are applied with the use of the methods typical for financial econometrics.

The second tendency is linked to the extension of classical financial econometric models, where the emphasis was put on the dynamic properties of the studied variable, paying no (or very little) attention to distributional properties, by assuming a normal (univariate or multivariate) distribution. This extension is related to the use of statistical methods to analyze the distribution of variables being components of a univariate or multivariate stochastic process. In the univariate case it means that in addition to level (mean) and volatility of the variable, also skewness and kurtosis are studied.

2. Some Historical Remarks

As we have mentioned, the birth of financial econometrics took place in 1982, with the publication of the seminal article by Robert Engle, where ARCH model, being the volatility model, was proposed. It is worth to mention, that there were some scientific contributions before 1982, which preceded and to some extent inspired financial econometrics contributions. In our opinion, these were the following achievements (given in chronological order):

– work of Tryggve Haavelmo in 1940–ties, where values of an economic process were regarded as realizations of a stochastic process;
– proposal of Mandelbrot (e.g. Mandelbrot, 1963), based on empirical studies, to apply stable distributions in modeling financial returns;

- introducing Error Correction Model by Philips (1957) and later by Sargan (1964);
- work of Box and Jenkins in 1970–ties, where ARIMA models were introduced;
- work of spurious regression by Granger and Newbold (1974) – this concept was signaled at the beginning of twentieth century by Udny Yule;
- unit root tests (see Fuller, 1976 and Dickey, Fuller, 1979, 1981);
- VAR (Vector Autoregressive) models by Sims (1980);
- long memory models;
- cointegration.

The birth of the ARCH model was made possible thanks to the visit of Robert Engle in London School of Economics in 1979. As Engle confirms, the idea of this model emerged in the discussion with David Hendry, Dennis Sargan and Jim Durbin on the hypothesis of Milton Friedman, stating that inflation uncertainty was a central cause of business cycles.

The publication of the ARCH model is the beginning of very large number of studies, in which financial econometric models were developed and used in practice. As the most important contributions in the area of financial econometrics we consider the following models:

- GARCH model, being direct generalization of ARCH (Bollerslev, 1986, Taylor, 1986);
- ARCH–in–mean model (Engle, Lilien, Robins, 1987);
- MGARCH, being a multivariate generalization of the GARCH model (Bollerslev, Engle, Wooldridge, 1988);
- Stochastic volatility model (SV), developer by Taylor (1982);
- Factor ARCH model (Engle, Ng, Rotschild, 1990);
- ACD, used for volatility of ultra high frequency data (Engle, Russell, 1998);
- Dynamic Conditional Correlation (DCC) model, used to estimate changing correlation coefficient between two time series (Engle, 2002).

In addition to the development of the models designed to analyze financial data, the researchers has worked on more general dynamic econometric models that could be applied also in financial problems. This refers particularly to cointegration approach, developed in 1980–ties – e.g. Engle, Granger (1987), Johansen (1988, 1991).

Such a dynamic development of financial econometrics (as well as dynamic econometrics) was highly appreciated by scientists by awarding in 2003 the so–called Nobel prize in economics – the exact name of the award is: The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel. The official prize announcement states that the prize was awarded to:

- Robert F. Engle (New York University), for methods of analyzing economic time series with time–varying volatility (ARCH);
- Clive W.J. Granger (University of California, San Diego), for methods of analyzing economic time series with common trends (cointegration).

This development was made possible due to several external factors, including first of all:

- Development of financial theory and integration with the other financial areas: valuation, risk analysis;
- Development of computer technology;
- Development of numerical procedures;
- Increasing availability of financial data.

It seems that all mentioned factors are still main driving forces of the development of financial econometrics.

The most important applications of financial econometric models are by no doubt the analysis of the level and the volatility of financial time series, including: stock prices, exchange rates, interest rates, commodity prices and real estate prices. The results of such analysis are directly applied in financial problems, such as: valuation of financial instruments and other assets, market risk analysis, credit risk analysis, investment portfolio management, term structure of interest rates. Models with varying volatility are mainly applied to estimate risk.

To conclude with historical remarks, it should be added that financial econometrics has been developed also in Poland after 1990. It has made possible to some extent because of the development of the Polish financial market (including Warsaw Stock Exchange). The methods of financial econometrics have been extensively studied in Toruń University (group headed by Professor Zieliński). As far as Bayesian approach is concerned, one should mention the group of Cracow University of Economics headed by Professor Osiewalski.

3. Financial Econometrics – Recent Developments and Challenges

Financial econometrics is going through the period of very extensive development. The most important areas that were recently developed and that will face a lot of interest of researchers and practitioners in practice, in our opinion, are:

- Multivariate models;
- Models of continuous time stochastic processes;
- Market microstructure analysis;
- Ultra high frequency data analysis;

– Interest rate modeling;
– Credit risk modeling.

Multivariate Models

Many financial problems are of a multivariate nature. This refers, for example, to portfolio of investments, credit portfolio etc. Therefore, it requires a multivariate approach, where, as a base, a multivariate stochastic process is used. The analysis of methods proposed by researchers and (to some extent) used by practitioners shows four possible directions:

1. Natural generalization of univariate ARIMA–ARCH model

Here we have the general multivariate discrete time model. It can be given in the following simplifying form:

$$X_t = \mu_t + \sum_t^{0.5} Z_t \quad (1)$$

$$\mu_t = E\left(X_t \middle| X_{t-1},...\right) \quad (2)$$

$$\sum_t = E\left(X_t X_t^T \middle| X_{t-1},...\right) \quad (3)$$

The main problem with such type of models lies in the large number of parameters to be estimated, especially when the number of dimensions is above

2. Conditional correlation models

This approach originated from Engle paper (Engle, 2002) consists in separate estimation of correlation coefficient using conditional modeling (like GARCH model for volatility).

3. Copula based approach

The key feature of copula analysis is that it gives the decomposition of the multivariate distribution into two components. The first component is marginal distributions. The second component – called copula function – is the function linking these marginal distributions to form a multivariate distribution. The copula function reflects the structure of the dependence between the components of the multivariate random vector. Therefore the analysis of multivariate distribution function can be performed by „separating" univariate a distributions from the relation between these distributions. Here the dependence parameters and scale parameters are "separated".

This idea is reflected in the following formula:

$$F(x_1,...,x_n) = C(F_1(x_1),...,F_n(x_n)) \quad (4)$$

where:

F – the multivariate distribution function;

F_i – the distribution function of the i–th marginal distribution;

C – copula function.

Continuous Time Models

Most of the classical financial econometric models are base on the stochastic process in discrete time. However, finance theory is strongly based on stochastic processes in continuous time (like Geometric Brownian Motion, being the main process used in option pricing). These processes have very often clear financial interpretation, they reflect the properties of financial data and they are the models used by financial economics.

Market Microstructure

Market microstructure is defined as (see O'Hara, 1995) "the study of the process and outcomes of exchanging assets under a specific set of rules. While much of economics abstracts from the mechanics of trading, microstructure theory focuses on how specific trading mechanisms affect the price formation process".

The most important areas of market microstructure are the following ones:

– Price formation and price discovery – how demands are translated into prices and volumes;
– Market structure and design – how trading scheme affects price formation;
– Information and disclosure – how information is observed by market participants;
– Transaction costs and timing costs – how transaction costs influence investment returns and execution methods

Ultra High Frequency Data Analysis

Ultra high frequency data analysis refers to data occurred more often than daily. The features of such data are different than other time series data, therefore they require special models. Very popular are duration models, attempting to model time between transactions. Due to this feature, it reflects the measurement of liquidity on the financial market.

Interest Rate Modeling

Interest rates are one of the main time series modeled by financial econometrics. However, modeling interest rates may be a challenging task, since:

– There are many possible types of interest rates that can be of modeling interest, including spot rates, forward rates, yields to maturity. So, one has to consider the whole term structure of interest rates;
– There is advanced financial theory of interest rates, which means that econometric models of interest rates should be consistent with this theory;
– Some theoretical rates are not observable, which means that they should be extracted from existing data.

Credit Risk Modeling

Credit risk is risk that the other party will not make contractual payments. There are types of credit risk models, in all of them financial econometric models are applied:

– Structural models (modeling firm value using option based approach – Merton model);
– Empirical models (deriving function of default based on past data for different entities – scoring models, etc.);
– Reduced form models (modeling intensity process leading to probability of default – intensity models).

The development of a particular discipline creates also challenges. This is certainly true also for financial econometrics. There are challenges of theoretical and practical nature. Theoretical challenges arise due to the complicated nature of studied problems. It was characterized by Robert Engle in his Nobel lecture, where he pointed out two such challenges (Engle, 2004): multivariate modeling and modeling transactional data, being high frequency data. According to him:

"There appear to be two important frontiers of research that are receiving a great deal of attention and have important promise for applications. These are high frequency volatility models and high dimension multivariate models"

As far as practical challenges are concerned, we think that two of them are of particular importance, namely:

– Financial econometric models should approach more closely real world problems and hypotheses formulated by theory of finance;
– More transparent and understandable mathematical tools for end users on the financial market.

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