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Bayesian Analysis of Polish Inflation Rates Using RCA and GLL Models*

1. Introduction

An extensive discussion of the empirical evidence of changes in the time series properties of inflation was provided in Cecchetti, Hooper, Kasman, Schoenholtz, and Watson (2007). In their paper they used an unobserved component model with stochastic volatility to characterize inflation and AR model with time varying coefficients and stochastic volatility to describe the growth of real GDP. These models were originally used by Stock and Watson (2007) and Nason (2006). Also Koop and Potter (2001) considered a time-varying parameter AR model where the coefficients evolve over time according to a random walk for quarterly change in the US CPI. All mention above authors found strong evidence of randomness of autoregressive parameters for inflation data. In our model-based analysis the mean of inflation is specified by a random coefficient autoregressive (RCA) or generalized linear (GLL) model. Unlike mentioned above papers, in our models the random parameters and the unobserved component follow stationary processes. Using monthly inflation data, our modelling framework and Bayesian estimation, we find remarkable changes in varying mean.

The paper is organized as follows. Section 2 introduces the time-varying parameter (TVP) models and Bayesian estimation. Section 3 presents empirical results for Polish inflation. Section 4 concludes.

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2. The TVP Models and Bayesian Inference

The simple random coefficient autoregressive model RCA(p) for inflation rates has the form:

$$y_t = \phi_0 + \delta_{1t} y_{t-1} + ... + \delta_{pt} y_{t-p} + \varepsilon_t,\ \varepsilon_t \sim N\left(0, \sigma^2\right), \tag{1}$$

$$\delta_{it} = \phi_i + \eta_{it},\ \ \eta_{it} \sim N\left(0, \omega_i^2\right) \text{ for } i = 1, ..., p, \tag{2}$$

where y_t is observation at time t, ε_t is white noise with variance σ^2, η_{it} are white noises with variances ω_i^2 and ε_t and η_{is} are independent for all t and s.

Andel (1976) and Nicholls and Quinn (1982) first investigated the statistical properties of RCA models. Tsay (1987, 2005) and Granger and Teräsvirta (1993) described properties of conditional variance in RCA models.

The second model is so-called generalized linear model (GLL) (Bos, Mahieu and van Dijk, 2000).

$$y_t = \delta_t + \varepsilon_t,\ \varepsilon_t \sim N\left(0, \sigma^2\right), \tag{3}$$

$$\delta_t = \phi_1 \delta_{t-1} + \eta_t,\ \eta_t \sim N\left(0, \omega_1^2\right). \tag{4}$$

The GLL model contains information on the varying local mean of observations. We also assume that the error in equation (3) is independent of the error in equation (4). The unobserved mean component in the GLL model δ_t is an autoregressive process with variance $\omega_1^2 > 0$ and the autoregressive parameter $-1 < \phi_1 < 1$. For $\phi_1 = 1$ and $\omega_1^2 > 0$ we have the well-known model called local level model (see Harvey, 1989; Durbin and Koopman, 2001; Koop, 2003) and it implies that y_t follows an I(1) process. When the disturbances are Gaussian with constant variance, this model displays the same correlation structure as the ARMA(1,1) model (Bos, 2001). Though this model is extremely simple, it is a basic model in many financial market and macroeconomics models (Bos, Mahieu and van Dijk, 2000; Stock and Watson, 2007). This model is supposed to pick up the periods of rising or falling inflation rates levels. The GLL model is also state space models, where (3) is measurement and (4) is transition equation.

Our GLL model is linear and Gaussian. In this case, the Kalman filter equations (Harvey, 1989; Bos, 2001; Koop, 2003) lead to a prediction-error decomposition. This decomposition filters out the prediction error e_t at time t given all previous observations and the corresponding variance F_t. The sampling distribution is:

$$p(y \mid \theta_i, M_i) \propto \prod_{t=1}^{N} F_t^{-0.5} \exp\left(-\frac{e_t^2}{2F_t}\right). \tag{5}$$

In our case, the Kalman filter begins by setting $a_1 = 0$ and $P_1 = 50$, the starting values for the initial state $\delta_1 \sim N(a_1, P_1)$ and θ_i denotes vector of unknown parameters in M_i model and $y = (y_1, ..., y_N)$ is vector of observation.

Combining the sampling distribution (5) with a prior $p(\theta_i \mid M_i)$ we get the kernel of posterior densities of GLL parameters.

For the RCA case the following density can represent the sampling model:

$$p(y \mid \theta_i, M_i) = \prod_{t=1}^{N} f_N^1\left(y_t \mid \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i}, \sigma^2 + \sum_{i=1}^{p} \omega_i^2 y_{t-i}^2\right), \tag{6}$$

where $f_N^1(z \mid c, w)$ denotes univariate normal density with mean c and variance w (see Tsay, 1987, 2005).

3. TVP Models for Monthly Inflation Rates in Poland

The data comes from the National Bank of Poland, concerning the core inflation index CI_t, excluding administratively controlled prices, and ranges from January 1998 until August 2007. Index represent percentage change from the same period of the previous year. We have a sample of 116 monthly observations.

We use Bayes factors for testing the integration of our series $(\ln CI_t)$. Following Koop and van Dijk (2000) we consider model which illustrates connections between the Dickey-Fuller and KPSS tests:

$$\ln CI_t = \tau_t + \rho \ln CI_t + \sum_{i=1}^{p-1} c_i \Delta \ln CI_{t-i} + v_t, \qquad v_t \sim N(0, \sigma_v^2) \tag{7}$$

$$\tau_t = \tau_{t-1} + u_t, \qquad u_t \sim N(0, \sigma_u^2).$$

Using simple transformation $\lambda = \sigma_u^2 / (\sigma_u^2 + \sigma_v^2)$ with uniform prior $p(\lambda) = 1$ over the interaval $[0,1)$ and $p = 2$, we consider four hypotheses[1]:

H_1: $\lambda = 0$ and $|\rho| < 1$. The series is stationary.

H_2: $0 < \lambda < 1$ and $|\rho| < 1$. The series is I(1) plus a stationary component.

H_3: $\lambda = 0$ and $|\rho| = 1$. The series is I(1) and a random walk.

[1] Koop and van Dijk (2000), expression (A.3).

$H_4: \ 0 < \lambda < 1$ and $|\rho| = 1$. The series is I(2).

The results for logarithm of core inflation index are contained in Table 1.

Table 1. Posterior model probabilities for inflation data

	$P(H_1 \mid Data)$	$P(H_2 \mid Data)$	$P(H_3 \mid Data)$	$P(H_4 \mid Data)$
Posterior model probabilities	3.21E-56	9.99E-01	5.56E-59	9.28E-56

The above results provide strong evidence for integration (I(1)). Hypotheses H_2 receives much more probability than others indicating that the data prefer the unit root with stationary component. Our results are consistent with results obtained in Koop and van Dijk (2000) for macroeconomic data. Given that our data exhibit strong evidence for non-stationary I(1) behaviour, first differences of inflation rates are constructed from the core inflation index by taking $y_t = \Delta \ln CI_t$.

In order to compare different model specifications, we have to calculate marginal data densities $p(y \mid M_i)$. Posterior probabilities and marginal data densities for all competitive models are presented in Table 2. All models have equal prior probabilities. The larger the posterior probability, the more preferable is the designated model.

Table 2. Marginal data densities and posterior probabilities of competitive models

	Model					
	RCA(1)	RCA(2)	AR(1)	AR(2)	GLL	AR(1)-GARCH
Marginal data density	1.28E+43	5.36E+41	2.16E+42	4.94E+41	3.67E+41	1.81E+42
Prior probability	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667
Posterior probability	0.7046	0.0295	0.1189	0.0272	0.0202	0.0996

Results in Table 2 inform that RCA specifications seem to be the best models to describe the dynamics of inflation rates. The highest posterior probability for RCA(1) suggests that it is the best specification among the competitive models. This model has posterior probability equal to 0.7046. Second order RCA and AR models are much less probable.

The posterior probabilities of all RCA models are close to 0.73. Another random coefficient model (GLL) with posterior probability 0.0202 has received the lowest rank. Also the GARCH model is the third with posterior probability equal to 0.0996. The Bayes factor, comparing the RCA(1) model to standard AR(1) is 5.92. Using these facts, it can be seen that the Bayes factor provides evidence in favour of the RCA model.

We also investigate the sensitivity of the posterior results with respect to the choice of the prior. Less informative priors for variances of random parameters in RCA models do not lead to substantial change in model ranking.

Since RCA(1) is the best model specification, we investigate the randomness of the autoregression parameter. Koop and van Dijk (2000) use *signal-to-noise* ratio to carry out Bayesian tests for the randomness of parameter in the case of the non-stationary local level (LL) model. *Signal-to-noise* is defined as the ratio between the variance of signal and of the noise (Harvey, 1989). In our case we have: $\mu = \omega_1^2 / \sigma^2$. Since $\mu \in [0, \infty)$, it may be convenient to map the parameter to the interval $[0,1)$ through the simple transformation $\theta = \omega_1^2 / \left(\omega_1^2 + \sigma^2\right)$. For $\theta = 0$ we have the standard AR model.

Consider the Bayes factor comparing $H_0 : \theta = 0$ with $H_1 : \theta \in (0,1)$ which can be calculated using the Savage-Dickey density ratio (see Verdinelli and Wasserman, 1995; Koop and van Dijk, 2000):

$$B_{01} = \frac{p(\theta = 0 \mid Data)}{p(\theta = 0)}, \tag{8}$$

where the numerator is the marginal posterior of θ for the alternative hypothesis and the denominator is the marginal prior for θ evaluated at the point of interest $\theta = 0$. The Bayes factor in favour of the randomness of parameter is less than one. There are two problems with using the Savage-Dickey density ratio in RCA models. Firstly, we need posterior distribution of θ, and secondly we have to evaluate it at zero.

First problem is easily solved, because there exist several different ways of calculating the posterior distribution of θ. For example we can use simply Monte Carlo integration, because θ is a function of σ^2 and ω_1^2. We can also use different parameterization of the RCA model. Note that conditional variance in RCA(1) model in terms of θ and ω_1^2 can be written as:

$$h_t = \omega_1^2 \left(y_{t-1}^2 + (1-\theta)/\theta\right). \tag{9}$$

According to Koop and van Dijk (2000), we use a beta prior for θ, which can take on different shapes depending on the values of the two parameters θ_0 and θ_1. Using this prior we can perform a prior sensitivity analysis on θ to test the robustness of the Bayes factors B_{01}. The posterior of θ (bottom-right corner) and uniform prior distribution $(\theta_0 = \theta_1 = 1)$ are plotted in Figure 1.

Second problem arises with the attempt to evaluate posterior distribution of θ at zero. Due to the difficulties of evaluating (9) at the point 0 due to division by zero, we evaluate it at a point close to zero. Following Koop and van Dijk

(2000) we are testing the hypothesis $\theta^* = 0.000001$. From practical point of view the differences between these two hypotheses are negligible. In our model we can not analytically integrate out all nuisance parameters but we can estimate $p(\theta^* \mid Data)$ by a simulation-based normal approximation (see Verdinelli and Wasserman, 1995):

$$p(\theta^* \mid Data) \approx \frac{|g'(\theta^*)|}{\sqrt{2\pi w}} \exp\left\{-\frac{(g'(\theta^*) - c)^2}{2w}\right\}, \tag{10}$$

where $\tau = g(\theta)$ is a Box-Cox transformation such that τ has nearly a normal distribution, and c and w are the sample mean and variance of the $g(\theta)$. To calculate the Savage-Dickey density ratio, it is necessary to setup MCMC algorithm. In order to provide necessary level of accuracy of normal approximation, for each prior 500000 draws have been simulated.

Table 3. Bayes factors in favour of the hypothesis $\theta = 0$

	θ_0			
θ_1	0.1	0.5	1	2
0.1	1.2502e-15	5.7887e-18	2.4837e-14	1.7937e-10
0.5	3.1561e-16	4.4875e-15	5.3067e-14	2.6716e-10
1	7.1207e-15	7.3525e-13	1.8703e-11	5.4980e-08
2	1.8391e-11	3.7147e-10	8.8766e-09	6.6454e-05

Table 4. Posterior information on Polish inflation rates

	RCA(1)		AR(1)		GLL	
ϕ_0	-0.021 -0.020	(0.022) [-0.057, 0.028]	-0.026 -0.023	(0.0232) [-0.068, 0.025]	-	-
ϕ_1	0.786 0.777	(0.066) [0.683, 0.916]	0.755 0.750	(0.0616) [0.633, 0.866]	0.813 0.789	(0.0622) [0.700, 0.950]
ω^2	0.132 0.113	(0.063) [0.033, 0.266]	-	-	0.048 0.043	(0.011) [0.027, 0,068]
σ^2	0.042 0.039	(0.008) [0.030, 0.056]	0.058 0.060	(0.0080) [0.050, 0.080]	0.008 0.005	(0.006) [0.000, 0.019]
θ	0.692 0.732	(0.140) [0.466, 0.916]	-	-	0.853 0.867	(0.109) [0.650, 1.000]

It can be seen from Table 3 that the Bayes factors, provide clear evidence in favour of the randomness of the autoregressive parameter. Most evidence for randomness of the parameter is found when the prior of θ has a similar location and dispersion to the posterior distribution[2]. The Bayes factor can vary, but in all cases strong evidence of a randomness of autoregressive parameter appears.

[2] For $\theta_0 = 0.5$ and $\theta_1 = 0.1$ the beta distribution is U-shaped with mean 0.83, and std. dev. 0.29.

In Figure 1 we present the priors on the parameters of the RCA(1) models that are used (solid lines). We use proper priors which are expected to be only weakly informative compared to the information in the likelihood. The prior distribution of the autoregression parameter ϕ_1 is normal distribution with zero mean and variance equal to one. For constant ϕ_0, we also have normal prior which is also of zero mean and variance one. The variances ω_1^2 of random parameters and residual variance σ^2 have exponential distribution with $\lambda_{\omega_1^2} = 1$ and $\lambda_{\sigma^2} = 10$. We use uniform prior $p(\theta)$ over the interval $[0,1)$. It is seen that the posteriors are much more concentrated than the priors.

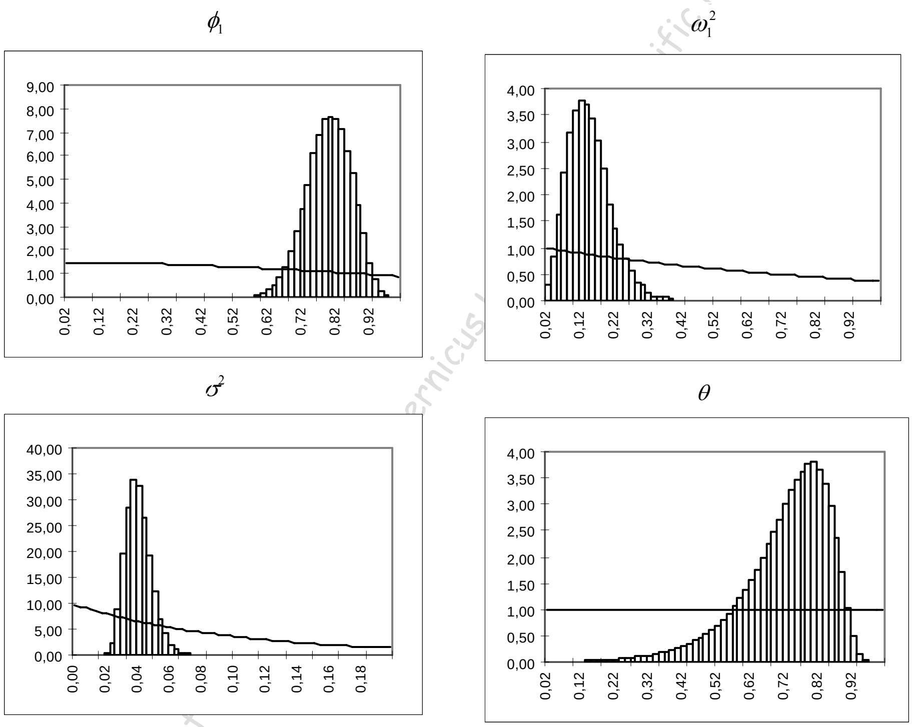

Figure 1. Priors (solid line) and posterior histograms of RCA(1) parameters

In Table 4 the results of the estimations are presented. Here we consider only three models. For each model, i.e. the RCA(1), the second best AR(1) and the worst GLL parameter, the mean, standard deviation (in parentheses), median (in the second line) and the bounds of the 95% HPD region (in square brackets) are reported.

Concerning the posteriors for the selected models the following remarks can be made. From Table 4 it is seen that the posteriors of the autoregressive parameter ϕ_1 are tight. The posterior mean is positive and very close to the posterior median, implying a symmetric posterior distribution. This corresponds with the findings of strong autocorrelation in the series.

For the RCA(1) model, the variances of the observation disturbance and random parameter have a posterior median not very close to the mean, which corresponds to positive skewness of the posterior densities (see top-right and bottom-left panel of Figure 1).

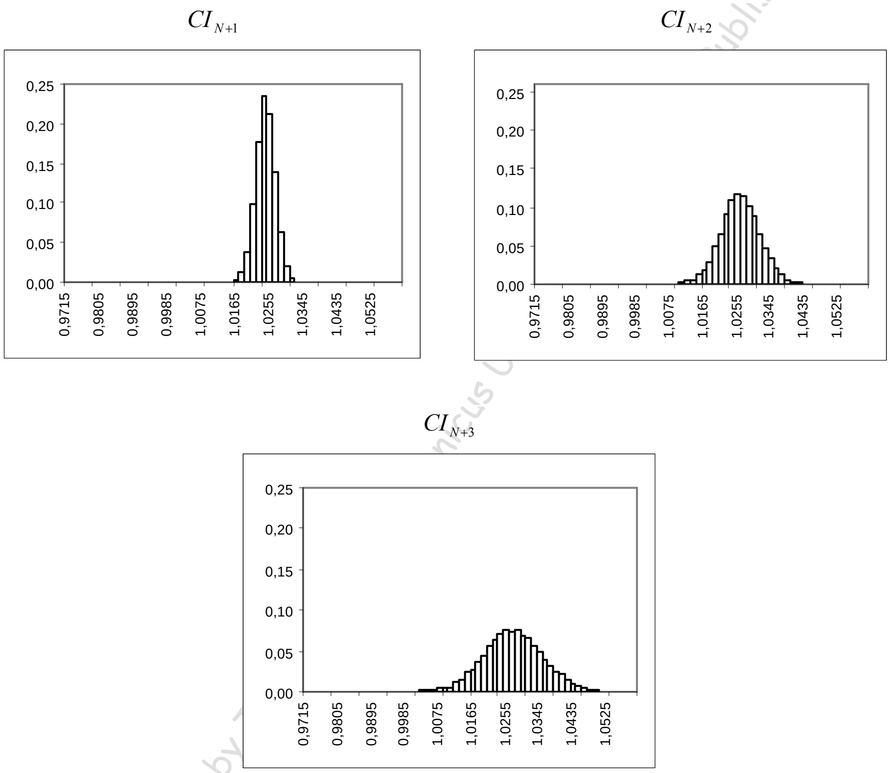

Figure 2. Histograms of predictive distributions of the core inflation index over the period IX-XI 2007 computed using RCA(1)

The variance of the random parameter ω_1^2 takes on values of around 0.13 with rather moderate standard deviation. Note that $\omega_1^2 = 0$ is not in HPD interval for all the models, where the parameter is time-varying. The posterior of the θ parameter (related to the signal-to-noise ratio) points at its large values, indicating randomness of the autocorrelation parameter. Similar effect is found for the parameter θ in GLL model. The posterior distribution of θ for the RCA(1) model has mean 0.69, which rises to 0.85 for the GLL model. The negative skewness of the posterior of θ can be observed at bottom-right panel of Figure 1. In GLL model, parameter θ is more tightly estimated. Nevertheless, the GLL model seems to be much less supported by the data than other, even constant parameter models (see Table 2).

Now we also examine the usefulness of the RCA model for short forecast horizons. We make predictive distributions of monthly inflation for three horizons $h = 1, 2, 3$. Predictive distributions of core inflation index have been calculated using Monte Carlo integration. Figure 2 and Table 5 presents histograms and quantiles of the predictive densities generated by the RCA(1) model.

Table 5. Quintiles of predictive densities for Polish core inflation over the period IX-XI 2007 (RCA(1) model)

Quantiles	CI_{N+1}	CI_{N+2}	CI_{N+3}
Quantile 5	1.020414	1.018342	1.015657
First quartile	1.022925	1.023183	1.022765
Median	1.024729	1.026660	1.027860
Third quartile	1.026518	1.030258	1.033303
Quantile 95	1.029121	1.035947	1.042627
True value	1.026106	1.030070	1.034225

Figure 2 depicts the shape of the predictive densities of core inflation index computed with the best competitive model, namely RCA(1). Even for short horizons, it is seen that the spread of the density changes considerably. In our case the predictive medians underestimate the true values of core inflation. The true values are close to third quartile. Not surprisingly, the best results are found for the one-month horizon. Note that conditional mean for AR and RCA models is the same but AR has smaller variance of disturbances. It seems that, for AR model predictive densities would have the same location. but forecast intervals were tighter. Therefore we can expect that standard AR model will perform worse. The results for the RCA(1) model are more realistic.

4. Conclusions

In this paper we have discussed and implemented Bayesian estimation for stationary random coefficient autoregressive models. Our research suggests that the RCA models, where the autoregressive parameters change smoothly, can have higher rank than other competitive models: standard AR and AR-GARCH. Using Bayesian model comparison we can formally test parameter stability. Sensitivity analysis with Savage-Dickey density ratio confirms and extends these findings. For GLL model the results are ambiguous. This model is the worst according to the marginal likelihood, but has the largest *signal-to-noise* ratio. We think that further research is needed.

Forecasting results are not very satisfactory. We find that the width of the forecast intervals derived from RCA models appears to be more correct, though true values of predicted core inflation were close to third quartile. Nevertheless, the RCA appears to have improved the prediction results of core inflation index.

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