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Joanna Bruzda
Nicolaus Copernicus University in Toruń

Output-Capital Nexus in the Solow and Romer Growth Models. LSTR or ESTR Cointegration?

1. Introduction

The first round of empirical studies on the new growth theory[1] by and large focused on cross-country examinations. These studies were devoted to verify either the augmented Solow-Swan model (Mankiw, Romer, Weil, 1992) or the endogenous growth theory (see, for example, Sala-i-Martin, 1997). Recently criticism has been raised against cross-country econometric studies, resulting from the assumption of equal values of parameters in the growth models under scrutiny for all examined countries. As A. Greiner, W. Semmler and G. Gong (2005), Chapter I, state, such an assumption may be misleading. Firstly, cross-country examinations, by lumping together countries at different stages of development, may miss the thresholds of development. Secondly, these studies rely on imprecise measures of the economic variables involved, and the results are by far nonrobust. Furthermore, different institutional conditions, social infrastructure and preference parameters will make the countries heterogeneous. Due to this recently in verifying growth models more and more often time series techniques are applied (see, for example, Jones, 1995; Lau, Shin, 1997; Lau, 1999; Greiner, Semmler, Gong, 2005; Ha, Howitt, 2006). In particular in Lau, Shin (1997), Lau (1999) and Ha, Howitt (2006) econometric implications of different exogenous, semi-endogenous and endogenous growth models are discussed in the context of cointegration analysis and cointegration techniques are applied to examine these models.

[1] It is widely accepted that the new growth theory started with the publications of Romer's paper explaining persistent economic growth by referring to the role of externalities – see Romer (1986).

In this paper we take a similar viewpoint and use non-linear cointegration techniques to verify the neoclassical Solow-Swan growth model (see Solow, 1956; Swan, 1956) and the endogenous growth model of Romer (see Romer, 1986) for 6 countries: the United States, Great Britain, Japan, Holland, France and Germany. The relaxation of the linearity assumption in cointegration analysis concerns here the dynamics of the adjustment process to the long-term equilibrium relationship. Namely, it is admitted that the adjustment may be either asymmetric – different for positive and negative deviations from the equilibrium – or, alternatively, disproportional – different for large deviations, in case of which the correction of the disequilibrium is stronger, and for small deviations, for which the correction is weaker or there is no correction at all. Simulation analyses in Pippenger, Goering (2000) and Bruzda (2006), (2007) point that standard cointegration tests lack their power in the presence of non-linear adjustments. On the other hand, one may expect that economic fluctuations connected with business cycles may cause the adjustment to be of a non-linear nature. This motivates the use of cointegration tests relaxing the linearity assumption of an adjustment process in verifying growth models.

Further in the text in Section 2 the growth models of Solow and Romer are briefly presented with special emphasis on their econometric implications. In Section 3 the methodological approach used in the empirical examination is described, while in Section 4 empirical results are presented.

2. The Solow-Swan and Romer Growth Models

The difference between the neoclassical Solow-Swan growth model and the endogenous growth model of Romer consists in different specification of production technology resulting from different sources of growth. In the neoclassical model an exogenous technological progress is assumed, which in the case of constant returns to scale fully explains observed persistent growth. On the contrary, in the Romer model the growth rate of an economy is determined endogenously and persistent economic growth is explained by referring to the role of externalities, arising from learning by doing and knowledge spillovers. Due to externalities investment not only affects the stock of physical capital but also increases knowledge, such that returns to capital on the economy-wide level are much larger than at the microeconomic level of an individual firm.

Assuming constant returns to scale the Cobb-Douglas production functions in the Solow-Swan and Romer models may be written, respectively, as:

$$Y_t = AK_t^{\alpha}\left[(1+\gamma)^t L_t\right]^{1-\alpha}\theta_t \qquad (1)$$

and

$$Y_t = AK_t^{\alpha}L_t^{1-\alpha}\kappa_t^{1-\alpha}\theta_t\,, \qquad (2)$$

where K_t and L_t denote aggregate stock of physical capital and aggregate labour in year *t*, Y_t stands for the aggregate output in year *t*, κ_t denotes capital per capita in year *t*, $A\theta_t$ is the total factor productivity, TFP (*A* is a positive constant, while θ_t denotes a stochastic component of TFP with mean zero), α is the capital share in the production functions ($0 < \alpha < 1$), while γ stands for the average growth rate of the level of technology ($\gamma > 0$).

In Lau, Shin (1997) and Lau (1999) econometric implications of the growth models with production functions (1) and (2) are discussed. Firstly, in the Romer model output and capital in intensive terms (per unit of labour) are I(1) processes with drift, while in the neoclassical model they may be either trend or difference stationary. Besides, the Romer model implies deterministic cointegration, i.e. cointegration without trend in the cointegration vector, between logarithms of output per unit labour, $\ln y_t = \ln \frac{Y_t}{L_t}$, and logarithms of capital per unit labour, $\ln k_t = \ln \frac{K_t}{L_t}$. On the contrary, an econometric consequence of the Solow-Swan model may be either deterministic or stochastic cointegration (i.e. cointegration with a linear deterministic trend) between $\ln y_t$ and $\ln k_t$. Furthermore, if the variables under scrutiny are trend stationary, cointegration does not take place at all. Therefore the case of deterministic cointegration does not allow discriminating between the two growth models. S.-H.P. Lau and C.-Y. Shin (1997) call this an observational equivalence of the neoclassical and endogenous growth models.

3. Methodology

Among tests of the hypothesis of no cointegration against an alternative hypothesis assuming a non-linear adjustment process one can single out tests for threshold cointegration with one or two thresholds under the alternative and tests for smooth transition (STR) cointegration. In what follows we briefly present tests for STR cointegration and subsequently apply them to examine the output-capital nexus. The STR cointegration tests have been suggested in Kapetanios, Shin, Snell (2006) and Bruzda (2006), (2007). They can be viewed as supplementary to two- and three-regime threshold cointegration tests, having two properties worth stressing. Firstly, the tests are more general as they nest threshold cointegration as a limiting case. Secondly, in aggregate quantities smooth behaviour seems to be more likely than sharp and, due to this, the STR cointegration tests might be somewhat more adequate to study macroeconomic and financial phenomena.

In testing for STR cointegration two approaches are utilized: Taylor series approximation to the transition function under scrutiny and a grid search over possible values of nuisance parameters. In the first approach admitting the exponential (disproportionate adjustment) and logistic (asymmetric adjustment) transition functions leads to the following test equations (see Bruzda, 2007):

$$\Delta u_t = \alpha_1 u_{t-1} + \alpha_2 u_{t-1}^2 + \varepsilon_t \,, \tag{3}$$

$$\Delta u_t = \alpha_1 u_{t-1} + \alpha_2 u_{t-1}^2 + \alpha_3 u_{t-1}^3 + \varepsilon_t \,, \tag{4}$$

$$\Delta u_t = \alpha_1 u_{t-1} + \alpha_2 u_{t-1}^2 + \alpha_3 u_{t-1}^3 + \alpha_4 u_{t-1}^4 + \varepsilon_t \,, \tag{5}$$

where u_t stands for the adjustment process. Then testing the hypothesis of no cointegration against a general alternative consists in examining the joint significance of the parameters in the above equations with the help of the standard F statistic in the form:

$$F = \frac{(SSR_0 - SSR_1)/q}{SSR_1/(n-q)} \,, \tag{6}$$

where $SSR_0 = \sum_{t=1}^{n} \Delta u_t^{\,2}$, SSR_1 is the sum of squared residuals of the appropriate test equation and q denotes the number of restrictions. If residuals are autocorrelated, equations (3)–(5) are augmented with lags of the dependent variable similarly to the standard augmented Dickey-Fuller (or Engle-Granger) test.

In Bruzda (2006), (2007) a sequential testing procedure based on equations (3)–(5) is presented, which makes it possible to discriminate between exponential smooth transition (ESTR) cointegration, logistic smooth transition (LSTR) cointegration and linear cointegration. In this procedure the F tests are supplemented with tests for the significance of the last parameter in the test equations. Then, under the assumption of cointegration, the significance of α_4 in equation (5) or α_2 in equation (3) suggests the presence of LSTR cointegration, while the significance of α_3 in equation (4) may be interpreted as a symptom pf ESTR cointegration.

Among tests based on a grid search over possible values of parameters of the transition function under scrutiny of particular interest are the inf t tests. In testing for LSTR cointegration the following test equation may be utilized (Bruzda, 2007):

$$\Delta u_t = \rho u_{t-1}\left(1 - \frac{b}{1 + e^{-\gamma u_{t-1}}}\right) + \varepsilon_t \,, \; 0 < b < 1 \,, \; \gamma \neq 0 \,, \tag{7}$$

which constitutes a basis for computing the ‘inf’ statistic defined as:

$$\inf_{b,\gamma} t = \inf_{(b,\gamma)\in \mathrm{B}\times\Gamma} \hat{t}_{\rho=0}(b,\gamma) \,. \tag{8}$$

The statistic takes the lowest value over a set of t statistics computed for all possible values of the parameters b and γ. The set B may be defined as equally spaced points between 0 and 1, for example B = {0.01, 0.02, …, 0.99}, while the set Γ comprises both negative and positive values, for example Γ = {-5, -4.95, …, -0.05, 0.05, …, 4.95, 5}. With such a definition of Γ we are able to consider both – the case when the negative regime is more persistent, what takes place if the parameter γ is negative, and the case, in which the positive regime is 'less stationary', what takes place for positive value of γ. In practice in computing statistic (8) a rescaled equation in the following form is utilized:

$$\Delta u_t = \rho u_{t-1}\left[1 - \frac{b}{1+e^{-\gamma\left(\frac{u_{t-1}}{sf}\right)}}\right] + \varepsilon_t, \tag{9}$$

where $sf = \sqrt{\frac{\sum_{t=1}^{n} u_t^2}{n}}$ is the so-called scaling factor – see, for example, Park, Shintani (2005) – and in statistical inference a bootstrap distribution of the test statistic is used.

In the case of the ESTR cointegration test the following rescaled equation is estimated:

$$\Delta u_t = \rho u_{t-1}\left[1 - e^{-\gamma\left(\frac{u_{t-1}}{sf}\right)^2}\right] + \varepsilon_t,\ \gamma > 0. \tag{10}$$

Then, the 'inf' statistic is defined as follows:

$$\inf_\gamma t = \inf_{\gamma\in\Gamma} \hat{t}_{\rho=0}(\gamma) \tag{11}$$

and the set Γ comprises positive values only, for example Γ = {0.05, 0.1, …, 4.95, 5}.

4. Empirical Results

In the empirical examination annual time series of capital and the GDP were analysed. The volumes of capital were taken from Maddison (1995), while population and the GDP come from Maddison's webpage. The data were used to compute logarithms of capital and output per capita. The series span the following periods: 1890–1992 (103 observations) for the US, 1830–1991 (162 observations) for the UK, 1890–1991 (102 observations) for Japan, 1950–1991 (42 observations) for France, 1935–1991 (57 observations) for Germany and 1950–1992 (43 observations) for the Netherland. An initial examination with the ADF test showed that logarithms of output and capital per capita may be treated as I(1) processes. In the examination of long-term relationships the following tests were used: the Engle-Granger procedure, the *F* tests for STR cointegration (supplemented with the tests of significance of the last parameter in the test equations) and the bootstrap 'inf' tests for STR cointegration. Each time two specifications of the long-term relationship were examined: one without trend and second with a linear deterministic trend in the equation for logarithms of output per capita relative to logarithms of capital per capita. The results are given in Tables 1–3. They point at stochastic cointegration for the US and UK, while in the case of France deterministic cointegration is present. This stays in accordance with the econometric implications of the neoclassical Solow-Swan growth model. Tests for non-linear cointegration provide almost the same results as the Engle-Granger tests, however they let also make additional inference on the kind of dynamics of the adjustment processes. Besides, the *F* tests reject the null hypothesis of no cointegration at lower significance levels. In the case of the equation for the United States the *F* tests point at an asymmetric adjustment with a 'more stationary' (less persistent) positive regime (for $u_{t-1} > 0$). This conclusion is supported by the 'inf' tests, which also indicate LSTR cointegration with a negative value of the parameter γ, confirming that the positive regime is less persistent. These observations are supported by estimating 3 models for the adjustment process to the US output-capital relationships: a linear model and two non-linear models – ESTAR and LSTAR. The LSTAR model turned out to be the best in terms of the accuracy of fitting and diagnostic tests. For example, the R^2 coefficient, Akaike criterion and Jarque-Bery statistic for the linear model are: 13.8%, -2.934 are 13.829 with *p*-value equal 0.001, while the appropriate statistics for the LSTAR equation are: 21.7%, -2.946 and 5.316 with *p*-value 0.070. Besides, the relaxation of the linearity assumption removes heteroscedasticity of the error term. On the contrary, according to sequential testing procedure applied to the adjustment process in the equation for the United Kingdom, the best model is a linear one, while in the case of France both the *F* and *t* tests (see *p*-values in Table 2) as well as estimation results point at an ESTAR specification for the equilibrium errors. The appropriate characteristics of a linear equation are the following: 26.7%, -1.850 and 1390.7 (0.000), while in the case of the ESTAR model we have: 36.9%, -1.999 and 983.6 (0.000). So, the ESTAR model was not able to fully explain the lack of normality of the error term in the linear model[2].

Table 1. Results of the Engle-Granger tests

Country	ADF statistic (augmentation); equation without trend	ADF statistic (augmentation); equation with trend
USA	-2.609 (1)	-3.672* (1)
UK	-1.427 (4)	-5.283 ***(2)
Japan	-2.563 (0)	-2.641 (0)
Germany	-1.471 (1)	-0.858 (1)
France	-3.640** (1)	-3.633* (1)
Holland	-2.354 (0)	-3.020 (0)

'*', '**'and '***' denote rejection of the hypothesis of no cointegration at the 10%, 5% and 1% significance level (the MacKinnon's estimated response surfaces were used to compute the appropriate critical values).

[2] Detailed estimation results are available upon e-mailing the author.

Table 2. Results of the F tests for STR cointegration

Country	Statistic					
	F_4	t (p-value)	F_3	t (p-value)	F_2	t (p-value)
			Equation without trend			
USA	2.951		3.856		3.748	
UK	0.637		0.699		1.009	
Japan	2.122		2.739		3.733	
Germany	0.817		0.691		1.066	
France	11.987***	1.745	14.391	-4.327	9.117***	2.046
		(0.087)		(0.000)		(0.046)
Holland	1.387		1.864		2.827	
			Equation with trend			
USA	4.971**	1.184	6.064**	-0.544	9.012**	-2.029
		(0.239)		(0.588)		(0.045)
UK	7.144***	1.011	9.183***	-0.023	13.864***	0.016
		(0.314)		(0.982)		(0.987)
Japan	2.383		3.205		4.690	
Germany	0.502		0.643		0.483	
France	12.051***	1.988	13.943***	-4.151	9.375***	2.155
		(0.052)		(0.000)		(0.036)
Holland	2.248		3.029		4.455	

The statistics F_2, F_3 and F_4 correspond to test equations (3)–(5); '*', '**' and '***' denote rejection of the joint hypothesis of no cointegration and linearity of the adjustment process at the significance level of 10%, 5% and 1% (simulated critical values were used with the number of replications equal 50 000); augmentation of the test equations is the same as in linear cointegration tests – see Table 1. If the null hypothesis of the F tests is rejected, in the next step tests of the significance of the last parameter in the test equations are performed.

Table 3. Results of the 'inf' tests for STR cointegration

Country	Statistic			
	Equation without trend		Equation with trend	
	LSTR inf t	ESTR inf t	LSTR inf t	ESTR inf t
USA	-2.057	-2.253	-3.364	-2.599
	[-2.883; -3.316]	[-2.979; -3.461]	[-3.361; -4.001]	[-3.708; -4.339]
UK	-1.579	-1.627	-3.725	-3.661
	[-2.933; -3.197]	[-3.189; -3.462]	[-3.187; -3.428]	[-3.309; -3.608]
Japan	-2.702	-2.899	-2.977	-3.099
	[-3.093; -3.493]	[-3.104; -3.721]	[-3.143; -3.633]	[-3.384; -3.827]
Germany	-2.664	-2.934	-2.717	-2.959
	[-3.183; -3.610]	[-3.295; -3.660]	[-3.571; -4.036]	[-3.904; -4.267]
France	-3.205	-3.213	-2.770	-2.805
	[-2.519; -2.857]	[-2.292; -2.621]	[-2.864; -3.382]	[-3.040; -3.577]
Holland	-2.407	-2.290	-3.028	-3.036
	[-2.957; -3.254]	[-3.072; -3.547]	[-3.564; -3.910]	[-3.584; -3.941]

In square brackets bootstrap 10% and 5% critical values are given (the overlapping blocks method was used with the block length set to 10 and the bootstrap samples were generated by equations without imposed restrictions on parameters – see Bruzda (2007); the non-augmented test equations were utilized and the number of bootstrap replications was set to 500); the following sets of admissible values of parameters were applied: for the LSTR cointegration tests $\mathrm{B} = \{0.1, 0.2, ..., 0.9\}$ and $\Gamma = \{-15, -14.85, -14.70, ..., -0.15, 0.15, ..., 14.85, 15\}$, while for the ESTR cointegration tests $\Gamma = \{0.05, 0.1, ..., 10\}$.

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