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Imposing Economic Restrictions in a VECM-form Demand System

Demand systems have been used in applied demand analysis for about fifty years. It is usually assumed that both prices of goods under consideration and the total expenditure are exogenous, whereas quantities demanded are endogenous. Equations of such a system are interpreted as (transformed) demand functions. Properties of the functions can be derived from the standard microeconomic theory, resulting in complicated (possibly cross-equation) restrictions linking structural parameters of the model. Such restrictions correspond to the setting of a single utility-maximizing agent. When aggregate data are used, the representative-agent assumption is implicitly introduced into analysis. Imposing and testing of the economic regularity restrictions is therefore vital in order to maintain economic interpretation of the results obtained.

Traditional time-series applications in the vein assume trend-stationarity of all the variables. Modern techniques of dynamic analysis account for non-stationarity of the macroeconomic time series. Exogeneity restrictions are also quite demanding, especially with respect to the total expenditure. An approach addressing all the issues has been developed by Pesaran and Shin (2002). They formulate a standard model used in dynamic macroeconometrics, namely the VECM model, for all the variables. Economic structure corresponding to a demand system is then imposed on cointegrating vectors. In the setting, $I(1)$ non-stationarity of the observed time series is taken into account and no arbitrary exogeneity assumptions are made. Unfortunately, only linear cointegrating relations can be modeled within the standard VECM framework. The requirement limits the scope of the demand system functional forms that can be employed within the VECM approach.

With long-run structure interpreted as corresponding to equations of a demand system, all restrictions mentioned above have to be imposed on elements of the cointegrating vectors. Methods of imposing and testing such constraints (with maximum likelihood inference methods) are reviewed in Boswijk and Doornik (2004). In common macroeconomic applications however, information concerning economic structure of a model is often quite weak. Conversely, in the case discussed here, economic restrictions are quite sophisticated, with cross-equation or highly non-linear constraints. The constraints become more complicated as the number of goods increases. Applications of VECM modeling discussed here differ from the standard ones in degree of complexity of the structure imposed. One of the consequences of the fact is that standard econometric software packages cannot be used.

The paper presents an application of the VECM-demand system approach with the emphasis on imposing economic restrictions. Number of the aggregate goods considered is such that full VECM model is formulated for twelve variables, which is more than in most illustrative applications. A convenient functional form of the demand system is employed, namely the Generalized Addilog model of Bewley (1986). Equality restrictions are imposed and tested with ML methods, and some results concerning inequality constraints are also presented. Since the latter pose serious theoretical problems for the ML-based statistical inference, only some ad-hoc technical results are presented in that case.

The plan of the paper is as follows. Firstly, the Generalized Addilog functional form is presented and a particular formulation of the economic restrictions in the case is discussed. Secondly, a VECM model is briefly introduced, together with some methods of imposing long-run cross-equation linear and non-linear restrictions. Thirdly, the VECM-demand system approach of Pesaran and Shin is described. Finally, empirical application of the methods is presented and summarizing remarks conclude.

1. Properties of Demand Systems and the GADS Functional Form

Let w_t be a n-vector of the observed expenditure shares corresponding to n (aggregate) commodities, p_t denote n-vector of the corresponding price indices, and let μ_t represent corresponding total expenditure. A demand system (with stochastic structure omitted) can be specified as:

$$f(w_t) = f(\omega(p_t, \mu_t; \theta)),\ t = 1,\dots, T \qquad (1)$$

where ω (.) is n-vector of functions representing theoretical expenditure shares, θ is k-vector of the structural parameters. The system represents a set of demand functions in (transformed) share form, where vector function f(.) accounts for possible transformation of the observed and theoretical shares. Demand system functional forms are usually derived in a share form because of the convenience of the economic duality techniques (see e.g. Pollak and Wales (1992)). Shares are sometimes transformed with function $f(.)$ in order to achieve linearity in parameters. Another reason for the transformation is the fact that shares are restricted to the unit interval, whereas additive normally distributed error terms (attaining any real values) are often introduced, with transformation used to reconcile the discrepancy (see e.g. Fry, Fry and McLaren (1996)).

The Generalized Addilog demand system form, described in detail by Bewley (1986), with $f_i(.)$ being i-th element of $f(.)$, can be written as:

$$f_i(\omega(.)) = \chi_i + \sum_{j=1}^{n} \overline{\pi}_{ij} \ln p_{tj} + \overline{\theta}_i \ln\left(\frac{\mu_t}{P_t^s}\right), \; i = 1,\ldots,n, \tag{2}$$

where P^s represents Stone price index given by:

$$\ln P_t^s = \sum_{j=1}^{n} \overline{w}_j \ln p_{tj} \,, \tag{3}$$

and $\overline{w}$ denotes vector of the average[1] shares. Observed shares are transformed:

$$f_i(w_t) = \overline{w}_i \ln\left(\frac{q_{ti}}{w_t^+}\right), \tag{4}$$

where $\overline{w}$ denotes vector of the average shares (as above), and w^+ is defined as:

$$\ln w_t^+ = \sum_{j=1}^{n} \overline{w}_j \ln w_{tj} \,,$$

with q_i representing observed quantity demanded of i-th aggregate good. Theoretical and observed shares transformed in the way described above can attain any real vales (not restricted to the unit interval).

Functional form of theoretical shares ω (.) must satisfy certain properties in order to be meaningfully interpreted as a demand function (see e.g. Pollak and Wales (1992)). These properties are:

- *adding up*: theoretical shares must sum to one for any p_t and μ_t,
- *homogeneity*: demand functions are homogenous of degree one w.r.t. p_t and μ_t, consequently shares are homogenous of degree zero w.r.t. p_t and μ_t,
- *symmetry:* corresponding Slutsky matrix has to be symmetric…,
- *negativity:* … and negative semi-definite (see e.g. Mas-Collel, Whinston, Green (1995)).

The first property arises solely from the fact that share form is analyzed. The second property reflects "no money illusion" effect and is expected to hold even in aggregate (for heterogeneous consumers). The last two properties reflect

[1] In fact $\overline{w}$ can represent any specified shares. Usually $\overline{w}$ is defined as a sample mean, but is also assumed non-random which is not fully coherent. Sample mean is used as some proxy for a meaningful „typical" situation, which is important because economic characteristics' estimates are calculated at $\overline{w}$.

the representative-agent assumption (though there are some results on symmetry of the aggregate demand function, see Diewert (1980)). Adding-up and symmetry generate cross-equation equality constraints. Negative semi-definiteness of the Slutsky matrix is not often addressed in applied work. This is because whereas the first three properties result in equality parametric constrains, negativity requires complicated inequality constrains that are difficult to deal with in the context of ML inference.

In the GADS model

$$\sum_{j=1}^{n} \bar{\pi}_{ji} = 0,\ \sum_{j=1}^{n} \chi_j = 0,\ \sum_{j=1}^{n} \bar{\theta}_j = 1$$

is required for adding-up property. Homogeneity is satisfied given that:

$$\sum_{j=1}^{n} \bar{\pi}_{ij} = 0 .$$

Symmetry of the Slutsky matrix can be imposed only locally (for certain values of p_t and μ_t). It is satisfied at a point corresponding to $\omega(.) = \bar{w}$ if:

$$\bar{\pi}_{ji} = \bar{\pi}_{ij} .$$

In such approach, shares given by $\bar{w}$ are distinguished and parameters of (2) can be interpreted as economic characteristics evaluated at $\bar{w}$ (see Bewley (1986)). Parameters $\bar{\pi}_{ij}$ correspond to elements of the Slutsky matrix, whereas $\bar{\theta}_i$ represent marginal shares.

One of the shares is determined by values of the remaining n - 1 shares. In order to avoid singularity of the contemporaneous variance-covariance matrix, one equation is dropped from the system, and its parameters are fully determined by parameters of the remaining equations by means of the adding-up restriction. Consequently, only n - 1 equations are actually estimated.

Basic characteristics of the demands can be also calculated; total expenditure elasticity and own price elasticity evaluated at $\bar{w}$ are given by:

$$\xi_i = \frac{\bar{\theta}_i}{\bar{w}_i},\quad \xi_{ii} = \frac{\bar{\pi}_i}{\bar{w}_i} - \bar{\theta}_i .$$

2. ML Inference in VECM Model with Restricted Long-run Structure

In the section VECM model is briefly introduced in order to establish the notation, and basic facts concerning estimation of restricted long-run structure are presented. The exposition follows that of Boswijk and Doornik (2004).

Standard cointegrated VECM model for p variables with k lags in the original VAR model and normal errors can be written as:

$$\Delta x_t = \Pi x^*_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Psi g_t + \varepsilon_t, \quad \varepsilon_t \sim iiN(0,\Omega) \tag{5}$$

where:

$$\Pi = \alpha\beta', \ x^*_{t-1} = (x_{t-1}', d_t')', \ rank(\Pi) = r, \ 0 < r < m,$$

and d_t are deterministic terms restricted to the cointegration space, whereas g_t represents unrestricted deterministic components; α and β are assumed to have full column rank equal to r.

In the setting it is assumed that all variables in x_t are $I(1)$ non-stationary, so $I(2)$ and seasonal non-stationarity and cointegration are ruled out for simplicity. Ω is assumed non-singular, initial condition matrix X_0 is assumed non-random.

Elements of α and β represent long-run dynamics of the system: columns of β are interpreted as cointegrating vectors (i.e. parameters of the long-run relations being stationary linear combinations of variables in x_t), whereas elements of α transfer impact of deviations from long-run equilibrium onto current dynamics of the variables. Long-run weak exogeneity of variables in the system results in zero restrictions on the corresponding rows of α. Parameters in α and β are unidentified without further restrictions.

Standard constraints imposing just-identifying restrictions on α and β, together with methods of estimation and asymptotic inference on cointegration rank r, known as "Johansen's procedure" are described e.g. in Johansen (1996). Unfortunately, such constraints are of a purely statistical origin, without any economic interpretation. Pesaran and Shin (2002) advocate "structural" approach, where just-identifying and over-identifying restrictions are imposed in a way that allows for economic interpretation of the long-run structure.

A method of imposing and testing general linear restrictions on α and β is described by Boswijk and Doornik (2004), with α and β formulated as:

$$vec(\beta) = H\phi + h_0 \quad vec(\alpha') = G\psi,$$

where G, H and h_0 are known constant matrices, with ϕ and ψ containing unrestricted parameters and no restrictions linking α and β. Maximum likelihood estimates of ϕ and ψ, provided that identification conditions are satisfied, can be found using a switching-algorithm of Oberhofer-Kmenta type, with analytic conditional estimators of ϕ, ψ and Ω evaluated sequentially at each step:

$$\psi(\phi,\Omega) = \left[G'(\Omega^{-1} \otimes \beta' S_{11}\beta)G\right]^{-1} G'(\Omega^{-1} \otimes \beta' S_{11})\mathrm{vec}(\hat{\Pi}'_{LS})$$

$$\phi(\psi,\Omega) = \left[H'(\alpha'\Omega^{-1}\alpha \otimes S_{11})H\right]^{-1} H'(\alpha'\Omega^{-1}\alpha \otimes S_{11})\left[\mathrm{vec}(\hat{\Pi}'_{LS}) - (\alpha \otimes I)h_0\right]$$

$$\Omega(\phi,\psi) = S_{00} - S_{01}\beta\alpha' - \alpha\beta' S_{10} + \alpha\beta' S_{11}\beta\alpha'$$

where LS subscript denotes OLS estimates obtained from unrestricted version of (5), and S_{11}, S_{00}, S_{01} being certain sample product matrices defined in Johansen's procedure. With α, β being functions of ψ, ϕ and most recent estimates of Ω, ψ, ϕ used at each step, the algorithm converges at ML estimates.

Such procedure is much less computationally expensive than direct maximization of the concentrated log-likelihood function w.r.t. ψ and ϕ. Nevertheless, in practice it seems necessary to use some proposal density to draw starting points Ω^0 ψ^0 , ϕ^0 and run the algorithm for several hundred times in some cases.

Estimated asymptotic variance-covariance matrix of the ML estimator can be computed as an inverse of the information matrix calculated at ML estimates according to the formula:

$$V_{as}\begin{pmatrix}\hat{\psi}_{ML}\\ \hat{\phi}_{ML}\end{pmatrix} = \frac{1}{T}\begin{bmatrix} G'\left(\Omega^{-1}\otimes\beta' S_{11}\beta\right)G & G'\left(\Omega^{-1}\alpha\otimes\beta' S_{11}\right)H \\ H'\left(\alpha'\Omega^{-1}\otimes S_{11}\beta\right)G & H'\left(\alpha'\Omega^{-1}\alpha\otimes S_{11}\right)H \end{bmatrix}^{-1}.$$

Maximum value of the concentrated log-likelihood (up to an additive constant) can be obtained from:

$$l_c^*(\phi,\psi) = -\frac{T}{2}\ln\det(\Omega),$$

by replacing Ω with its ML estimate obtained in last step of the algorithm. Standard likelihood-ratio test statistics can be used in order to test various nested formulations (conditionally on r and k), since it follows asymptotic chi-squared distribution with degrees of freedom equal to the number of parametric restrictions – details are provided e.g. in Boswijk and Doornik (2004).

When nonlinear restrictions on α and β are considered, numerical maximization of the above concentrated log-likelihood is required, where:

$$\Omega(\phi,\psi) = S_{00} - S_{01}\beta\alpha' - \alpha\beta' S_{10} + \alpha\beta' S_{11}\beta\alpha',$$

with α and β being non-linear functions of ψ and ϕ.

3. Dynamic Demand Analysis with GADS in VECM Form

VECM-demand system approach of Pesaran and Shin (2002) is based on the assumption that columns of β can be interpreted as corresponding to equations of the system (1). Generally, with unrestricted α, r^2 restrictions on β are necessary to provide identification of the parameters. As an example consider β' = [-I B] that is, consisting of (minus) identity matrix and block of variation-free parameters B. Such a structure generates long run SURE-like structure. In equilibrium $\beta' x^*_t = 0$ which is parallel to the structure generated by (1), (2) and (3) with all the variables on the right-hand side.

Define x_t as:

$$x_t' = \left(f_i(w_t) \quad \ldots \quad f_{n-1}(w_t) \quad \ln p_{t1} \quad \cdots \quad \ln p_{tn} \quad \ln\frac{\mu_t}{P_t^s} \right),$$

with $d_t = 1$, $x^*_t = (x_t' \quad 1)'$ and arrange β' as:

$$\beta' = \begin{pmatrix} -1 & 0 & 0 & 0 & \bar{\pi}_{11} & \cdots & \bar{\pi}_{1n} & \bar{\theta}_1 & \chi_1 \\ 0 & -1 & 0 & 0 & \bar{\pi}_{21} & \cdots & \bar{\pi}_{2n} & \bar{\theta}_2 & \chi_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & & & & \vdots \\ 0 & 0 & \cdots & -1 & \bar{\pi}_{n-1,1} & \cdots & \bar{\pi}_{n-1,n} & \bar{\theta}_{n-1} & \chi_{n-1} \end{pmatrix}.$$

One (n-th) equation is dropped from the system to avoid singularity of Ω. If the true cointegration rank r is equal to $n - 1$ and deterministic components are properly specified, cointegrating vectors can be interpreted as equations of the GADS system. When unrestricted constant is introduced into the VECM model, one row of β containing χ_i parameters is dropped, estimates of the latter can be derived from estimates of Ψ, see Johansen (1996). When elements of H and h_0 are properly arranged, homogeneity and symmetry restrictions described above can be imposed or tested; G is equal to identity matrix or certain columns are dropped, if exogeity restrictions are imposed. Symmetry can be imposed only locally at $\bar{w}$.

It can be seen that β contains block of $\bar{\pi}_{ij}$ parameters corresponding to elements of the Slutsky matrix. Negativity (imposed locally at $\bar{w}$) requires square block of the parameters with both indices ranging from 1 to $n - 1$ to be negative definite. Such restriction is cross-equation non-linear inequality constraint. As mentioned above, formal treatment of inequality constraints within ML inference poses quite advanced problems. In the paper we describe a method that can be used to impose the restriction. In practice obtained estimates are at a border solution, so its formal status is not discussed here.

L. J. Lau (1978) proposed a method of imposing concavity constraints using Cholesky factorization. Any ($m \times m$) symmetric square matrix A can be factorized as A = L D L*'*, with D being a diagonal matrix with m nonzero elements and L being a lower-diagonal matrix with ones on the diagonal. If A is negative definite, all diagonal elements of D are negative. With such factorization, the mentioned above block of β can be written as a function of m negative elements of D and $(m^2 - m)/2$ variation-free elements of L. This translates complicated non-linear restrictions into a simple constraint on $m = (n - 1)$ parameters. Numerical optimization of the log-likelihood function with respect to free elements of β (with no constraints on α, the latter can be concentrated out) leads to point estimates of the GADS parameters that satisfy (locally) all the regularity conditions. This can be useful if some further application require fully regular point estimates of the Slutsky matrix.

4. Empirical Application: Analysis of the Aggregate UK Data

The dataset analyzed here was used by Deschamps (2003). It consists of 172 quarterly deseasonalized observations covering period of 1955:1 – 1997:4. Six consumption categories are included: food, drinks, footwear and clothing, energy, other non-durables, rent.

$\overline{w}$	*0.28*	*0.17*	*0.12*	*0.11*	*0.17*	*0.14*
Case A: Estimated β with k = 2, r = 5, logl = 10986						
	1	2	3	4	5	6
$f_1(w_1)$	-1	0	0	0	0	
$f_2(w_2)$	0	-1	0	0	0	
$f_3(w_3)$	0	0	-1	0	0	
$f_4(w_4)$	0	0	0	-1	0	
$f_5(w_5)$	0	0	0	0	-1	
$\ln p_1$	0.078	-0.0284	0.007	-0.07	0.035	-0.0215
$\ln p_2$	-0.0284	-0.0809	0.089	0.0239	0.0076	-0.0112
$\ln p_3$	0.007	0.089	-0.0589	0.011	-0.0446	-0.0035
$\ln p_4$	-0.07	0.0239	0.011	0.0243	0.0143	-0.0034
$\ln p_5$	0.035	0.0076	-0.0446	0.0143	-0.0617	0.0494
$\ln p_6$	-0.0215	-0.0112	-0.0035	-0.0034	0.0494	-0.0098
$\ln(\mu/P^s)$	0.3203	0.1741	0.1062	0.1142	0.1335	0.1517
Case B:Estimated β with k = 2, r = 5, logl = 10966						
$\ln p_1$	-0.0004	0.0056	-0.0058	-0.0002	0.0015	-0.0006
$\ln p_2$	0.0056	-0.0755	0.0792	0.0029	-0.0204	0.0083
$\ln p_3$	-0.0058	0.0792	-0.083	-0.0031	0.0214	-0.0087
$\ln p_4$	-0.0002	0.0029	-0.0031	-0.0001	0.0008	-0.0003
$\ln p_5$	0.0015	-0.0204	0.0214	0.0008	-0.0055	0.0022
$\ln p_6$	-0.0006	0.0083	-0.0087	-0.0003	0.0022	-0.0009
$\ln(\mu/P^s)$	0.381	0.2005	0.1132	0.1037	0.0324	0.1692

(shaded entries correspond to restricted parameters)

Case A Estimated income and own price elasticity,						
	FOOD	DRINK	CLOTH	ENE	OTH	RENT
ξ_i	1.13	1.00	0.87	1.04	0.80	1.06
Std err	0.06	0.06	0.11	0.09	0.19	0.08
ξ_{ii}	-0.05	-0.64	-0.59	0.11	-0.50	-0.22
Std err	0.17	0.06	0.24	0.13	0.40	0.05
Case B Estimated income and own price elasticity						
	FOOD	DRINK	CLOTH	ENE	OTH	RENT
ξ_i	1.36	1.18	0.94	0.94	0.19	1.13
ξ_{ii}	-0.38	-0.64	-0.8	-0.1	-0.06	-0.18

Properties of the dataset with respect to cointegration and aggregate demand analysis are subject to detailed analysis in Mazur (2005). Here it can be mentioned that the variables analyzed seem to be $I(1)$ non-stationary with linear trends, and the number of cointegrating relations can be assumed to be 5, which is in accordance with VECM-demand interpretation.

With k = 2 and unrestricted constant in VECM, with just – identified parameters, maximized log-likelihood is 11021. With homogeneity only imposed (5 constrained parameters) it becomes 11004, with symmetry and homogeneity (15 constrained parameters) it is 10986. All the restrictions are therefore rejected asymptotically with LR test (conditionally on selected values of r and k). Balcombe (2004) reports similar results, attributing the rejection to small sample properties of the test procedures. Below are presented results of the estimation with homogeneity and symmetry imposed, with no exogeneity restrictions (Case A) and with additional imposition of negativity (Case B).

It can be seen that imposing negativity results in estimates of own-price elasticities that are more in accordance with economic intuition than in case A. Estimated income elasticities have quite plausible values in case A; imposing negativity seems to influence estimated demand elasticity of the other goods. Estimated values of the characteristics in the restricted case are difficult to interpret, since there is no theory supporting calculation of the standard errors. Sound economic interpretation would require considering other functional forms and broader range of deterministic specifications.

All the calculations were conducted using author's routines written in Ox (Doornik (2002)). It should be noted that with 6 aggregate goods, structure of the restriction matrices G and H is too complicated to be conformable with standard econometric packages. Numerical optimization necessary to impose negativity constraint was conducted with BFGS algorithm being part of the Ox system.

5. Conclusions

The paper discussed problems of imposing and testing economic regularity restrictions in VECM-demand systems. In traditional demand analysis, regularity restrictions were often statistically rejected but nevertheless imposed (see Keuzenkamp and Barten (1995)). In the modern, dynamic approach, similar issues seem to arise. Such a result is perhaps due to small sample distortions, so exact inference techniques should be employed – an example using bootstrap is provided by Balcombe (2004).

Another statistical problem is that of testing inequality constraint (like the negativity constraint arising in demand analysis). ML inference techniques seem unsatisfactory in the case.

Inference in the VECM model is sequential – it is separately decided what lag length should be used, what is the cointegration rank, what specification of the deterministic components is plausible and finally structural and exogeneity restrictions are introduced. It would be desirable to be able to conduct joint inference on interesting aspects of the VECM – demand system. All the statistical problem mentioned above seem to suggest that application of the bayesian inference would be very promising in the area.

Another limitation of the analysis presented here is linearity of the functional form used. In traditional demand analysis, linear forms are abandoned in favor of the complicated non-linear specifications. Unfortunately, non-linear cointegration techniques are not yet well developed. The (quarterly) data used suggest that there is some possibility of seasonal cointegration also.

However, it should be noted that GADS form of demand system seems to be quite an interesting alternative within the VECM approach, dominated by Almost Ideal model of Deaton and Muellbauer (1980). It is of particular interest that it allows for imposition of the negativity constraints when Cholesky factorization is used as proposed by Lau (1978).

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