DYNAMIC ECONOMETRIC MODELS
Vol. 7 – Nicolaus Copernicus University – Toruń – 2006

Monika Kośko
The University of Computer Science and Economics in Olsztyn

An Application of Markov-Switching Model to Stock Returns Analysis

1. Introduction

The main characteristics of financial time series is volatility clustering for high and low activity periods. Empirical researches often confirm occurrence of some framework that divides a period into subspaces with different parameters. The way to describe such a relationship is a model with simultaneous switching of explained variable and parameters between subspaces. A Markov-switching models *MS* have this property, because both variable and parameters describe a process dynamics between states. Initially, an econometric dynamic model with Markov type switching was introduced by J. Hamilton (1989, 1994), as a tool which characterizes inner structure of changes between regimes of business cycle fluctuations. In that paper Hamilton considered a two state chain, for expansion and recession respectively, whereas the mean of return rate is specified. The continuation of this research was proposed by Clements and Krolzig (2000), then models with switching in variance or both variance and mean (Turner, Startz, Nelson 1989, Yin 2003), Markov-switching VAR models (Linne 2002, Krolzig 2001) and Markov-switching ARCH models – SWARCH (Hamilton, Susmel 1993).

Yin (2003) in his article used the S&P500 monthly market returns (1970.02−2003.01). The sample period was divided into 4 groups and for each subspace there were models estimated with both mean return and variance as a subject to change in regimes. The results implied that stock market could switch between two states with extremely different means and variances. The "good" state is characterized by about 4,5 times higher mean return and about 2 times lower variance in comparison to the "bad" state. Furthermore the "good" state turned out to be extremely persistent (p_{11}=0.9999) and the "bad" state very transitory ($p_{22} = 0.0004$), which was explained by quick coming back process to the "good" state. The conclusions thereof can be paralleled with early empirical findings about asymmetric volatility of stock markets process, namely that sudden increase in volatility tends to be associated more often with large negative returns. It seems to be unreasonable intuitively, since taking more risks is expected to bring a higher return.

The purpose of Linne's paper (2002) was to examine the contagion effects on several emerging stock markets in Central and Eastern Europe as result of currency crises in the Czech Republic in May 1997, in Asia in Summer 1997 and in Russia in August 1998. Weekly stock returns of seven Central and East European markets were used in this study. The research countries were the Czech Republic, Estonia, Hungary, Poland, Russia, the Slovak Republic and Slovenia. Linne considered a Markov-switching vector autoregressive models *MS(2)-VAR(1)*, in which autoregressive coefficients additionally determined the influence of shocks on particular markets. There were three alternative specifications of model examined, in which either the mean, the variance, or both differed between two regimes. The contagion market results in higher trading activities, higher price volatility and falling stock prices. The paper is an attempt to answer the following questions: are the shifts in returns related to currency crisis and are the stock returns features similar across different markets? The results showed an occurrence of two states, the "calm" state was characterized by low variance and a positive mean; the "crisis" state had a higher variance and a negative mean. The probabilities of remaining in the same state for both regimes were large, and moments in which the high probability of the "crisis" state appeared reflect the crisis episodes during the sample period (it is the most apparent in *MSMV(2)−VAR(1)*). The results implied that switching model is able to capture the stock returns volatility common for all markets. Thereby, the model provided an explanation for the volatility clustering present in the stock price data. The residuals of switching models were tested for the presence of ARCH effect, following the F test suggested by Garcia and Pierre (1996). The results showed that null hypothesis of no ARCH effects cannot be rejected for five of seven stock returns.

The purpose of this paper is introduction to autoregressive Markov-switching model *MS* with different kinds of switching. In empirical research weekly and daily data, from Polish stock market were used. The estimated models are compared with *ARMA(p,q)* model and tested for ARCH effect.

2. Building and Estimation of *MS* Model

A fact that parameters of the autoregression can change between regimes as the result of a first-order Markov process is the main characteristic of the Markov-switching model. In this process, current state of a variable depends only on previous state, what may be written as:

$$P(s_t = j \mid s_0 = i_0, s_1 = i_i, \ldots, s_{t-1} = i) = P(s_t = j \mid s_{t-1} = i) = p_{ij}(t) \qquad (1)$$

(it means that the probability of the process at moment *t* relies on the state of this process at moment *t-1* and is defined as a probability of transition from state *i* into *j*). The estimated model is given by:

$$(r_t - \mu_{s_t}) = \gamma(r_{t-1} - \mu_{s_{(t-1)}}) + e_t, \qquad e_t \sim N(0, \sigma^2_{s_t}) \qquad (2)$$

where the subject of change is a mean or a variance, that can be written in the form:

$$\mu_{s_t}, \sigma^2_{s_t} = \begin{cases} \mu_{s_t=1}, \sigma^2_{s_t=1} & dla\ \ s_t = 1, \\ \mu_{s_t=2}, \sigma^2_{s_t=2} & dla\ \ s_t = 2 \end{cases}. \qquad (3)$$

The transition probabilities between a two-state chain are given by:

$$P(s_t = 1 \mid s_{t-1} = 1) = p_{11}$$

$$P(s_t = 2 \mid s_{t-1} = 2) = p_{22}$$

and they form the transition matrix that is characterized by:

$$P = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}. \qquad (4)$$

In this model, the mean (μ), variance (σ_e^2), parameter of autoregression (γ) and the transition probabilities (p_{11}, p_{22}) are subject to estimate. Moreover, they are included in an estimated parameter vector (θ), for the following models with:

− shifts in the mean *MSM(2)*: $\theta = [\,\mu_1, \mu_2, \sigma_e^2, \gamma, p_{11}, p_{22}\,]$,

− shifts in the variance (the heteroskedastic model) *MSV(2)*:
$\theta = [\,\mu_0, \sigma_{e1}^2, \sigma_{e2}^2, \gamma, p_{11}, p_{22}\,]$,

− shifts both the mean and the variance *MSMV(2)*:
$\theta = [\,\mu_1, \mu_2, \sigma_{e1}^2, \sigma_{e2}^2, \gamma, p_{11}, p_{22}\,]$.

The Markov-switching model can be estimated by determining parameter estimates of vector (θ) at maximization of likelihood function given by[1]:

[1] Kim, Nelson (1999), p. 60.

$$L(\theta)=\sum_{t=1}^{T}\ln f(r_t \mid \psi_{t-1}) \tag{5}$$

and $\sum p_i = 1,\ p_i \geq 0$

where:

$$f(r_t \mid \psi_{t-1})=\sum_{s_t=1}^{2}\sum_{s_{(t-1)}=1}^{2} f(r_t, s_t, s_{t-1} \mid \psi_{t-1}), \tag{6}$$

$$f(r_t, s_t, s_{t-1} \mid \psi_{t-1})= f(r_t \mid s_t, s_{t-1}, \psi_{t-1})\cdot P(s_t, s_{t-1} \mid \psi_{t-1}), \tag{7}$$

ψ_{t-1} refers to information up to time *t-1*,

$$\theta=\left[\mu_i, \sigma_{ei}^2, p_i\right]. \tag{8}$$

To start an iteration at time $t=1$, initial values of probabilities given in the following form are assumed:

$$\pi_1 = P(s_0 = 1 \mid \psi_0)=\frac{1-p_{22}}{2-p_{11}-p_{22}}, \tag{9}$$

$$\pi_2 = P(s_0 = 2 \mid \psi_0)=\frac{1-p_{11}}{2-p_{11}-p_{22}}. \tag{10}$$

The expected (average) return to regime *i* is given by:

$$m(i)=\frac{1}{p_{ii}}. \tag{11}$$

The expected duration of regime *i* can be written as:

$$d(i)=\frac{1}{1-p_{ii}}. \tag{12}$$

3. Empirical results

In this paper daily and weekly stock market returns (2000.11.17 −2005.03.11) of the WIG index, TPSA, Prokom and Comarch stocks were analyzed. The Markov-switching models *MSM(2)*, *MSV(2)*, *MSMV(2)* are compared with *ARMA(p,q)* model[2]. Furthermore an Akaike information criterion and the log likelihood function were applied. Estimation results of some selected time series (the WIG and TPSA stocks) are shown in tables 1−4. Considering *AIC* and *LL* criteria, results achieved from *MSV(2)* and *MSMV(2)* models are insignificantly different with *MSV(2)* leading. Empirical results from *MSV(2)* and *MSMV(2)* are better then both *MSM(2)* and the *ARMA(p,q)* models. A determination coefficient R^2 turned out to be higher for *ARMA(p,q)* model for all time series.

Table 1. Estimates of a Markov-switching models of the WIG index
(weekly; 2000.11.17 −2005.03.11)

Parameters		***MSM(2)***		***MSV(2)***		***MSMV(2)***	
μ_1		-0.0013 (0.0234)		0.0021* (0.0015)		0.0036 (0.0052)	
μ_2		0.0063 (0.0254)		−		0.0014 (0.0042)	
γ_1		0.3003* (0.0711)		0.2681 (0.0680)		0.2788* (0.0854)	
σ_{e1}		0.022955		0.014517		0.01761	
σ_{e2}		−		0.026158		0.02719	
p_{11}	p_{21}	0.7260	0.2740	0.9373	0.0627	0.8987	0.1013
p_{21}	p_{22}	0.2884	0.7116	0.0285	0.9715	0.0900	0.9100
m(1)	*m(2)*	1.38	1.41	1.07	1.03	1.11	1.01
d(1)	*d(2)*	3.65	3.47	15.96	35.03	9.88	11.11
AIC		-990.62		-997.58		-992.69	
LL		501.61		505.09		503.73	
R^2		−		0.07192		0.08134	
TR^2 *(ARCH)*		−		9.35		9.09	

ARMA(0,1)					
μ_0	γ_1	β_1	*AIC*	*LL*	R^2
0.002478 (0.002105)	−	0.305660* (0.074199)	-992.68	499.34	0.09416

Note: results come from Ox[3], standards errors are in parentheses, (*) means significance at the 5% level.

Weekly returns (tables 1−2): *MSV(2)* and *MSMV(2)* models identified two significantly different low and high activity regimes for the WIG and TPSA returns. Both of them are characterized by high probability of remaining in regime, for the WIG index in regime *1*: $p_{11}(MSV) = 0.94$, $p_{11}(MSMV) = 0.90$ and regime *2*: $p_{22}(MSV) = 0.97$, $p_{22}(MSMV) = 0.91$. The regime *1* denoted a lower value of standard deviation, so it is named low activity regime and the average times of persisting in regime for both models are 15 and 10 weeks accordingly. In the *2-nd* regime, characterized by a higher value of standard deviation, the process remains for 35 and 11 weeks on average (accordingly for

[2] Engle (1982).
[3] www.doornik.com.

MSV and *MSMV* models). In case of both regimes, system shifts between regimes every week on average (see Fig. 1). The *MSMV* (2) model allows to estimate means in regimes, too. The lower mean relates to the high activity state, what can be explained by negative values of stock return in this regime. However the both means turned out to be insignificant. Furthermore, the probabilities to remain in state 1 for TPSA stock returns are: $p_{11}(MSV) = 0.98$, $p_{11}(MSMV) = 0.98$. State 1 turns out to be a low activity one with a long time of duration in regime, with 43 weeks for *MSV* and 63 weeks for *MSMV* model. The second state is a high activity regime and the system remains on average 27 and 38 weeks in it (for *MSV* and *MSMV* models respectively). The ARCH effect turned out to be insignificant in all weekly stock returns.

Table 2. Estimates of a Markov-switching models of TPSA stocks (weekly; 2000.11.17 −2005.03.11)

Parameters		***MSM(2)***		***MSV(2)***		***MSMV(2)***	
μ_1		-0.0047 (0.1262)		0.0008 (0.0024)		0.0029 (0.0043)	
μ_2		0.0034 (0.0590)		-		-0.0056 (0.0085)	
γ_1		0.2365 (0.0739)		0.2329 (0.0676)		0.2242 (0.0692)	
σ_{e1}		0.040332		0.02817		0.02855	
σ_{e2}		-		0.05326		0.05279	
p_{11}	p_{12}	0.7324	0.2676	0.9768	0.0232	0,9842	0,0158
p_{21}	p_{22}	0.2653	0.7347	0.0365	0.9635	0,0265	0,9735
m(1)	*m(2)*	1.37	1.36	1.02	1.04	1,02	1,03
d(1)	*d(2)*	3.74	3.77	43.15	27.39	63,31	37,69
AIC		-752.41		-778.29		-777.18	
LL		382.5		395.44		395.97	
R^2		0.08077		0.05503		0.06436	
TR^2 *(ARCH)*		3.20		3.39		1.65	

ARMA(0,1)					
μ_0	γ_1	β_1	*AIC*	*LL*	R^2
-0.000638 (0.003473)	–	0.246936* (0.059245)	-759.41	382.70	0.05837

Note: results come from Ox, standards errors are in parentheses, (*) means significance at the 5% level.

Daily returns (table 3-4): There were identified two states with low and high activity, for both WIG and TPSA stock returns. The differences between values of standard deviations in daily series are larger then in weekly series. Both considered states are represented by high transition probabilities (p_{11}, p_{22}), therefore inferences of expected duration *d(i)* indicate a long time of staying in states. For the WIG index probability values in both states are $p_{11}(MSV) = p_{11}(MSMV) = 0.989$, $p_{22}(MSV) = p_{22}(MSMV) = 0.994$ (see Fig. 2). In the low activity regime, the considered process remains 88 days on average (for both *MSV* and *MSMV* models), whereas it stays 170 days in the high activity regime on average, which corresponds with appropriate weekly returns conclusions. The transition probabilities of TPSA stock process are: $p_{11}(MSV) = 0.993$, $p_{11}(MSMV) = 1.00$, and $p_{22}(MSV) = 0.9878$, $p_{22}(MSMV) = 0.99981$. The low activity state 1 is characterized by positive mean and the high activity state 2 has a negative mean, which can be a confirmation of empirical research in the stock markets. The average process duration in the 1-st state is 143 days for MSV and 197865 days for *MSMV* model (the high value of *d(1)* is caused by high probability of remaining in the state $p_{11} = 1.00$, which may be an outcome of incorrect assumption of initial values (π_1, π_2), beginning the iteration of the EM algorithm). The average process durations in the 2-st state, which is characterized by 2-times higher standard deviation (the state of enhanced activity) are 82 and 513 of both models respectively. The ARCH effect in both Markov-switching models for daily returns is relevant. The obtained results of estimated parameters for models with shift in the variance and model with shift in both the mean and the variance are similar. Hence the conclusion that additional switching in the mean does not represent an increase in efficiency. Besides all indicated comparison criteria suggest a choice of Markov-switching model with shift in variance *MSV*.

Table 3. Estimates of a Markov-switching models of the WIG index (daily; 2000.11.17 −2005.03.11)

Parameters		***MSM(2)***		***MSV(2)***		***MSMV(2)***	
μ_1		-0.0018 (0.0031)		0.000583* (0.000321)		0.000903* (0.000464)	
μ_2		0.0027 (0.0042)		−		0.000255 (0.000534)	
γ_1		0.0570* (0.0490)		0.06264 (0.0626)		0.06195* (0.03061)	
σ_{e1}		0.011543		0.00779		0.007795	
σ_{e2}		−		0.01329		0.01329	
p_{11}	p_{21}	0.7539	0,2461	0.9886	0.0114	0.9887	0.0113
p_{21}	p_{22}	0.2403	0,7597	0.0059	0.9941	0.0058	0.9942
m(1)	*m(2)*	1.33	1,32	1.01	1.01	1.01	1.01
d(1)	*d(2)*	4.06	4,17	87.54	169.92	88.51	171.28
AIC		-6533.75		-6611.8		-6610.57	
LL		3272.93		3311.95		3312.36	
R^2		−		0.00450		0.00460	
TR^2 *(ARCH)*		−		65.94		65.11	

ARMA(1,0)					
μ_0	γ_1	β_1	*AIC*	*LL*	R^2
0.000436 (0.000358)	0.074599* (0.028731)	−	-6539.71	3272.85	0.00556

Note: results come from Ox, standards errors are in parentheses, (*) means significance at the 5% level.

Table 2. Estimates of a Markov-switching models of TPSA stock
(weekly; 2000.11.17 −2005.03.11)

Parameters		***MSM(2)***		***MSV(2)***		***MSMV(2)***	
μ_1		-0.0032 (0.0039)		0.000319 (0.00059)		0.0009 (0.0007)	
μ_2		0.0033 (0.0066)		-		-0.0012 (0.0012)	
γ_1		0.0121 (0.0373)		0.008081 (0.03065)		0.0044 (0.0304)	
σ_{e1}		0.02211		0.01631		0.01618	
σ_{e2}		-		0.02938		0.02758	
p_{11}	p_{21}	0.7268	0.9761	0.9930	0.0070	1,00	0,00
p_{21}	p_{22}	0.3088	0.999	0.0122	0.9878	0,001948	0,9981
m(1)	*m(2)*	1.38	1.00	1.01	1.02	1,00	1,00
d(1)	*d(2)*	3.66	1000.0	143.17	82.11	197865,0	513,35
AIC		-5142.84		-5281.23		-5283.11	
LL		2577.47		2646.67		2648.63	
R^2		0.02351		0.00045		0.00211	
TR^2 *(ARCH)*		123.01		113.85		116.15	

ARMA(1.0)					
μ_0	γ_1	β_1	*AIC*	*LL*	R^2
-0.0001153 (0.000680)	0.02064 (0.02479)	–	-5148.97	2577.49	0.00043

Note: results come from Ox, standards errors are in parentheses, (*) means significance at the 5% level.

4. Conclusions

The purpose of this paper was to show the application of autoregressive Markov-switching model *MS* in stock market returns analysis, then research of the properties of this model and its comparison with *ARMA(p,q)* model – one of the most popular in time series analysis. The empirical results indicate the *MSV* model, both for weekly and daily data, as the most proper one for description of different parameters structure and relationships between them. All examined states were extremely persistent, what can be a proof for the occurrence of some structure with different parameters (the mean and the variance of residuals), that depends on current change in process of explained variable. No ARCH effect in weekly returns implies better properties of *MS* models for lower frequency data, in which the ARCH effect is weaker. In the daily data models, the ARCH effect was not eliminated. Therefore, it is said that *MS* models do not explain the volatility clustering present in the daily stock returns data, what can be preface to further research aiming at construction a Markov-switching ARCH model (SWARCH).

Appendix

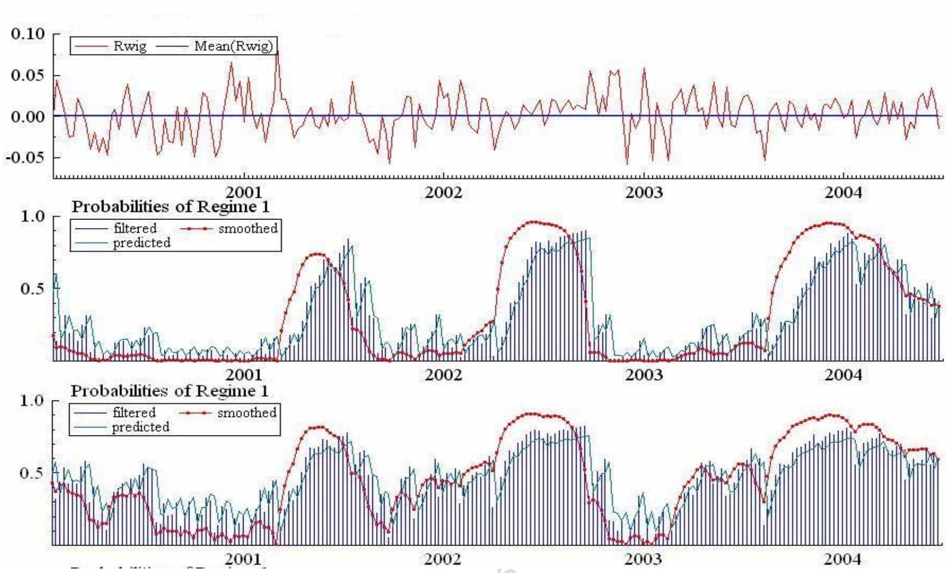

Fig. 1. The WIG returns and probabilities of remaining in regimes for *MSV(2)* and *MSMV(2)* models (weekly; 2000.11.17 −2005.03.11)
Note: results come from Ox.

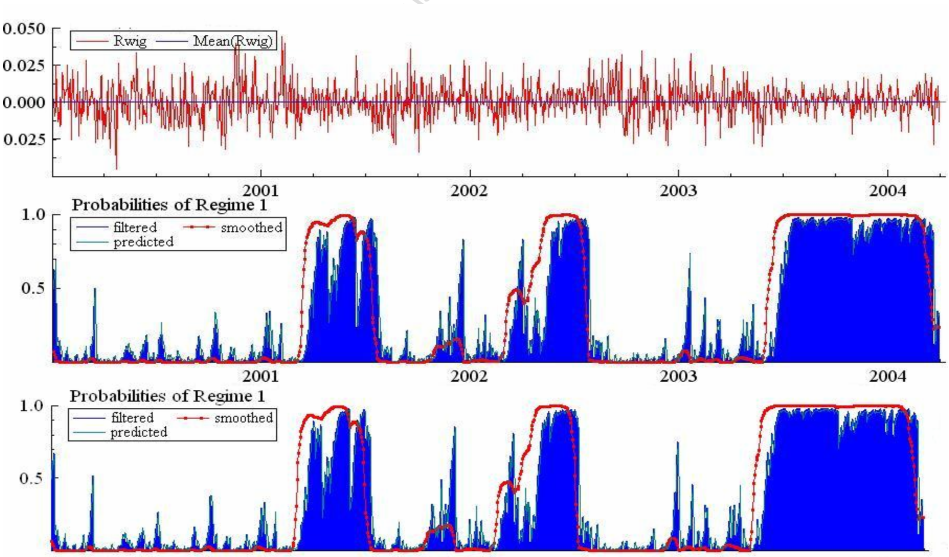

Fig. 2. The WIG returns and probabilities of remaining in regimes for *MSV(2)* and *MSMV(2)* models (daily; 2000.11.17 −2005.03.11)
Note: results come from Ox.

References

Brzeszczyński, J., Kalm R. (2002), *Ekonometryczne modele rynków finansowych, (Econometric models of financial markets)*, WIG-Press, Warszawa.

Engle, R. F. (1982), Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica*, vol. 50.

Garcia, R., Pierre, P. (1996), An Analysis of the Real Interest Rate under Regime Shift, *Review of Economics and Statistics,* vol. 78.

Hamilton, J. D. (1989), A New Approach to the Economic Analysis of Nonstationary Time Series and the Busines Cycle, *Econometrica ,*vol. 57.

Kim, C. J., Nelson, C.R. (1999), *State-Space Models with Regime Switching*, The MIT Press, London.

Koskinen, L., Pukkila, T. (1995), An Application of the Vector Autoregressive Model with a Markov Regime to Inflation Rates, *źródła internetowe*.

Krolzig, H. M. (1998*), Econometric Modelling of Markov_Switching Vector Autoregressions using MSVAR for Ox*, Institute for Economics and Statistics, Oxford.

Linne, T. (2002), A Markov Switching Model of Stock Returns: An Application to the Emerging Markets in Central and Eastern Europe, *East European Transition and EU Englargement A Quantitative Approach*, Berlin.

Podgórska M. (2002), *Łańcuch Markowa w teorii i zastosowaniach, (A Markov chain theory and application),* SGH, Warszawa.

Stawicki, J. (2004), *Wykorzystanie łańcuchów Markowa w analizie rynków kapitałowych*, *(A Markov chain application to capital market analysis)*, Wydawnictwo UMK, Toruń.

Yin, P. (2003), Markov Switching in the Stock Market, *Economics,* vol. 413, www.missouri.edu/~econprm/ec413f02/pyin.