DYNAMIC ECONOMETRIC MODELS
Vol. 7 – Nicolaus Copernicus University – Toruń – 2006

Mirosław Wójciak, Aleksandra Wójcicka
Katowice University of Economics, Poznań University of Economics

Comparative Analysis of Credit Risk Change Dynamics

1. Introduction

One of the most important types of financial risk is credit risk. This risk can be defined as *"possibility of loss arising from the failure of a counterparty to make a contractual payment"*[1]. Main problem of all credit risk models is a proper evaluation of probability of default (PD). Some models estimate it (e.g. Moody's KMV model), some calculate it on the basis of historical data and in some cases it is simply impossible to obtain it (discrimination analysis).

The main aim of this paper is to compare a traditional conception and new approaches of credit risk modelling. When comparing chosen models authors tried to stress how soon both models showed the deterioration of company economical and financial situation. The research was based on financial data of construction industry companies quoted on Warsaw Stock Exchange (WSE) in 2000-2004.

2. Methods of Credit Risk Evaluation

The paper uses two methods of credit risk evaluation which are: dynamic multidimensional analysis (DMDA) and credit risk evaluation method based on option pricing model - Moody's KMV model (MKMV). The first one is a traditional way of credit risk evaluation, the other – a new approach. Both models are: general models – can be used according to any company; descriptive – they focus only on the level of credit risk and its results; dynamic – data used for their construction are from different time periods. When it comes to DMDA the data is taken quarterly whereas in MKMV – daily and

[1] Jajuga (2004), p. 119.

quarterly. Each model takes different risk measures. The result of DMDA is a classification into various credit risk levels; in MKMV – probability of default. Comparing those two methods a great attention was paid to time horizon in which they indicate the deterioration of company's economical situation. Calculating market assets value and its volatility (MKMV model) and financial ratios (DMDA) the level of appropriate variables from previous period was taken. This approach is closer to reality, as the level of liabilities published in balance sheet is known only from previous (not current) period[2].

A combination of two DMDA techniques was used. One is a Z. Hellwig's linear ordering pattern method (best value) proposed in [1969]. First financial ratios, which describe various areas of analysis (liquidity, profitability, efficiency, debt) were normalised. It means that each nominanty was changed into a stimulant and all variables were standardised as follows:

$$z_{ij}^{t} = \frac{x_{ij}^{t} - \overline{X}_{j}}{S(X_{j})} \tag{1}$$

t – time of data, $\overline{X}_j$ – arithmetic average of *j* variable of all objects in all periods, $S(X_j)$ – standard deviation of variable X_j calculated as explained above.

Table 1. Results of ratios division

Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
Profitability	Debt	Additional ratios	Efficiency	Liquidity	Additional ratios 2
GTPR, NTPR, SPR, CLR, EPR, FSR	SFR, DR	APR, SD	CATR, ADTT	CR, QR, ADTLR	IT, LATR, SEPR, CCR

CR – Current ratio,
QR – Quick ratio,
ADTT – Amount due to turnover,
IT – Inventory turnover,
ADTLR – Amount due to liabilities ratio,
GTPR – Gross turnover profitability ratio,
NTPR – Net turnover profitability ratio,
SPR – Sales profitability ratio,
EPR – Enterprise profitability ratio,
CLR – Costs level ratio,
DR – Debt ratio,
SFR – Self-financing ratio,
FSR – Financial surplus ratio,
CATR – Current assets turnover ratio,
LATR – Liquid assets turnover ratio,
APR – Assets profitability ratio,
SEPR – Shareholders' equity profitability ratio,
CCR – Cost change ratio,
SD – Sales dynamics.

[2] The delay in publishing (which is about 4 weeks) was not taken into consideration.

Next step was to divide diagnostic elements into homogeneous groups according to agglomerate methods of Lance-Williams-Ward. The distance measure used was metrics:

$$d_{ij} = 1 - r_{ij}^2, \tag{2}$$

where r_{ij} is correlation coefficient between *i* and *j* variable.

Ward method was used to measure the distance between particular clusters on each level. The division into homogeneous groups of variables is due to the fact that areas of economical condition include different number of variables. This implicates that some groups may have a bigger influence on synthetic measure of credit risk (SMCR). Also some ratios between one another may be strongly correlated which could lead to untrue results. Table 1 shows the division of ratios.

Later for each separate group of variables synthetic measure (Q_i) was calculated in comparison to the best value (pattern). Because variables were standardised some object-time are below zero and character of variables is not uniform. However in pattern method it does not matter because the character of variables is taken into consideration when the pattern is settled and negative values of normative variables have no influence when using Euclidean distance. The pattern was settled basing on the formula:

$$\begin{cases} z_{0j} = P90(x_{ij}^t), & \text{when } x_{ij} \in S, \\ z_{0j} = P10(x_{ij}^t), & \text{when } x_{ij} \in D. \end{cases} \tag{3}$$

where: S is a stimulant and D is anti-stimulant.

The pattern used percentile 90 (P90) and 10 (P10) because some ratios had untypical observations that could give misleading classification. Those ratios which exceeded the appropriate levels of percentiles were replaced with the levels of P90 or P10.

Next step was to find for each company the distance c_{i0}^t from the pattern – the Euclidean distance was implemented:

$$c_{i0}^t = \sqrt{\sum_{j=1}^{m} (z_{ij}^t - z_{0j})^2} \quad (i = 1, 2, ..., n;\ t = 1, 2, ..., r;\ j = 1, 2, ..., k), \tag{4}$$

where:

z_{ij}^t – standardised levels of characteristics of *i* object in time *t*,

z_{0j} – levels of characteristics of the pattern from formula (3),

i – number of the object.

Achieved distances allowed to establish sub-synthetic dynamic measures of development Q_{ig}^t:

$$Q_{ig}^{t} = 1 - \frac{c_{ig0}^{t}}{c_{g0}}, \tag{5}$$

where:

$$c_0 = \overline{c}_0 + 2s_0, \tag{6}$$

$$\overline{c}_0 = \frac{1}{n}\frac{1}{r}\sum_{i=1}^{n}\sum_{t=1}^{r} c_{i0}^{t}, \quad \text{and} \quad s_0 = \sqrt{\frac{1}{n}\frac{1}{r}\sum_{i=1}^{n}\sum_{t=1}^{r}(c^{t}{}_{i0} - \overline{c}_0)^2}.$$

After that the average of Q_{ig}^{t} levels obtained for different ratio groups (*g*) (see Table 1) gave us synthetic measure of credit risk (SMCR).

$$SMCR = \frac{1}{6}\sum_{g=1}^{6} Q_{ig}^{t} \tag{7}$$

SMCR is constructed in such a way that it does not exceeds 0. The probability that it will be higher than 1 is insignificant (if it happened it was considered to be 1). The measure value close to 0 is preferable, increasing means financial problems. As the best values, the appropriate values of ratios of all companies of construction industry in 2000-2004 were taken (not only of those analysed ones).

The other method of credit risk evaluation is based on assets volatility (MKMV). On the basis of Black-Scholes-Merton option pricing model one can estimate equity and debt value. It is important because when the company is liquidated the bondholder receives the return when the value of equity is bigger than 0, which means the company value (A) is higher than its liabilities (D). Otherwise the creditor does not get the return because the market value of equity is 0. It means that creditor's return is similar to an income of call option writer on the assets of a company taking the loan.

Assuming that assets value changes can be described by Brownian standard geometric motions one can calculate expected default frequency (EDF) of any debtor. It is the probability that assets value of a company in any time horizon[3] (T) will drop below the critical value (A_{def}) according to the equation:

$$EDF1 = P\left[\varepsilon \le -\frac{\ln\left(\frac{A_0}{A_{def}}\right) + \left(\mu - \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}\right], \tag{8}$$

[3] EDF is mostly estimated in a year horizon (Saunders (2001), p. 37).

where:
A – company's assets value, A_{def} – company's critical assets value below which the company cannot pay back its debts[4], T – time of credit, r – risk-free interest rate, σ_A – company's assets value volatility, μ – average return rate of company's assets.

In the equation (8) A_{def}, D, T, r, μ are directly observable. Company's market assets value (A) and its volatility (σ_A) are not directly observable and they must be estimated. To calculate them one can use following relationship[5]:

$$E = AN(d_1) - De^{-rT}N(d_2), \tag{9}$$

$$\sigma_E E = N(d_1)\sigma_A A, \tag{10}$$

where:

$$d_1 = \frac{\ln\left(\frac{A}{D}\right) + (r + 0{,}5\sigma_A^2)T}{\sigma_A\sqrt{T}}, \quad d_2 = \frac{\ln\left(\frac{A}{D}\right) + (r - 0{,}5\sigma_A^2)T}{\sigma_A\sqrt{T}},$$

E – company's market equity value, D – nominal debt value, σ_E – equity volatility, $N(d_i)$ – normal cumulative distribution function for argument d_i, where i=1, 2.

Having equations (9) and (10) one can calculate (A) and (σ_A) in several iterations. Equations (9) and (10) show that when the debt increases the debt ratio increases as well and company's assets value volatility decreases. Growth of debt ratio will make probability of default (PD). Moreover, company's market assets value volatility will negatively influence it.

3. Comparison of the Credit Risk Change Dynamics

The comparative analysis was carried out on companies of construction industry. The choice is supported by the fact that in 2000-2004 there were great differences in financial and economical conditions of companies in this sector (9 of them became either bankrupt or had to settle down with creditors). Analysed companies are from the same sector and all of them had to fulfil many conditions to be quoted on WSE nevertheless the magnitude of their enterprise may vary. Results of the comparative analysis are presented for Mostostal Zabrze. This company in analysed time had severe financial problems which resulted first in additional settlements with creditors and finally in bankruptcy.

[4] According to model authors A_{def} consists of [short-term liabilities + 0.5 long-term liabilities].

[5] Hull (2003), p. 622.

Diagram 1 of Mostostal Zabrze shows PD and sub-synthetic measure for variable groups 1.2 and 5, which represent profitability, debt and liquidity. It can be noticed that sub-synthetic measure for group number 1 (profitability) a bit sooner (about one quarter in comparison to MKMV model) shows the deterioration of economical and financial condition of a company. One can see that Q_1 in different quarters fluctuates much which tells about big changes of profitability levels between the quarters. That fluctuation can be eliminated when calculating profitability ratios as moving average taking e.g. two quarters as smoothing. In analysed time sub-synthetic measure for group number 2 shows increasing tendency. It means that form quarter to quarter debt of a company grew steadily. The shape of this variable is similar to PD but it is late of about two quarters. Similar shape is due to a fact that the credit risk level depends on self-finance abilities of a company. When this index is on a low level then the distance to default (DD) gets smaller so the PD gets higher. Bigger differentiation of EDF is caused by changes of stock price, which reflects currently additional information about the company.

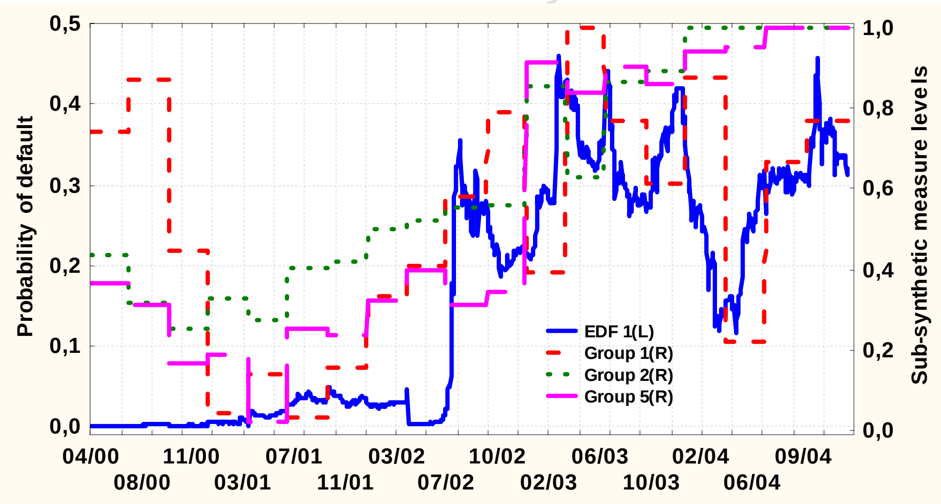

Diagram 1. Probability of default (left axis) and sub-synthetic measures for homogenous variable groups (right axis) for Mostostal Zabrze

Sub-synthetic measure reflecting company's financial liquidity does not show any time shift relatively to estimated probability. Its changes are analogous to changes of PD but of smaller amplitude. Only in the very last period PD was decreasing while sub-synthetic measure reached its maximum.

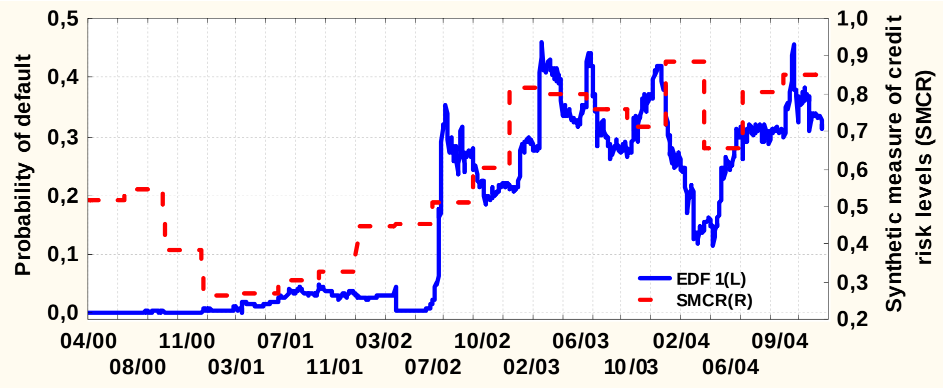

Diagram 2. Probability of default (left axis) and synthetic measure of credit risk (right axis) for Mostostal Zabrze.

Diagram 2 presents estimated PD and synthetic measure of credit risk (SMCR). The shape of both variables is similar. In 2nd and 3rd quarter 2000 higher level of SMCR was caused by low profitability of sales. In and initial part of PD increase (March 2001 – July 2002) the growth of SMCR exceeded the growth of PD. Only just after the sudden growth of PD in 3rd quarter 2002 PD became a leading variable. Signing an annex to an agreement prolonging the time of a current credit (in 2nd quarter of 2002) by the board of the company caused this sudden growth of PD (to 30%). That was the first signal of company's financial problems. It resulted in tremendous drop in company's stock price. On the day of initiating treaty proceeding (07 March 2003) the level of PD was almost 40%, and the level of SMCR – 0.81. Both variables indicated high credit risk on the day of announcing bankruptcy with a possibility of a treaty (18 August 2004) PD was 32%, and SMCR – 0.8.

Table 2. Quartiles of EDF with reference to SMCR with and without time shift

SMCR	Probability of default					
	No time-shift		1 quarter time-shift		2 quarters time-shift y	
	Q25	Q75	Q25	Q75	Q25	Q75
<0-0.1)	---	---	---	---	---	---
<0.1-0.2)	---	---	---	---	---	---
<0.2-0.3)	0.000	0.002	0.000	0.002	0.000	0.002
<0.3-0.4)	0.000	0.026	0.000	0.022	0.000	0.020
<0.4-0.5)	0.001	0.092	0.001	0.089	0.001	0.092
<0.5-0.6)	0.001	0.167	0.001	0.175	0.000	0.175
<0.6-0.7)	0.001	0.279	0.001	0.270	0.001	0.218
<0.7-0.8)	0.207	0.466	0.169	0.402	0.182	0.370
<0.8-0.9)	0.074	0.440	0.053	0.433	0.010	0.257
<0.9-1.0)	0.353	0.727	0.316	0.681	0.103	0.690
>1.0	0.496	0.636	0.524	0.690	0.466	0.620

Source: own calculations.

In this paper the analysis showing what were the ranges of average PD of all the companies of construction sector while SMCR reached values presented in table 2. This analysis has three variants: without any time shift of SMCR, time shift of one and two quarters ahead which allowed to state whether MKMV model reacts earlier than SMCR when credit risk grows. When SMCR is low (preferable) the attention was paid to higher quartile due to the fact that it was supposed to state with certain probability that some levels of EDF will not be exceeded. It concludes that the lower the higher quartiles of average PD are. the better the (sub-)synthetic measure marks out companies of low credit risk level. When it comes to high levels of synthetic measures low quartiles were the main concern checking how strong is model discrimination of companies in good and bad economical condition (high levels of low quartiles are preferable).

In case when SMCR was less than 0.6 then at least 75% of companies had the average risk value lower than 16.7% for the variant without time shift (with time sift 17.5%). But when SMCR was more than 0.7 – at least 75% had credit risk higher than 20.7% (with time shift: 16.9% and 18.2%). Zone of indecision for SMCR is between 0.5 and 0.7.

It can be noticed that indecisive zones in different variants are almost the same. However, when SMCR is low higher quantiles are lower for average EDF results without time shift and in case of high levels of SMCR lower quantiles are higher. The conclusion is that variant without time shift defines PD better than the others. One comes to similar conclusions when comparing correlation coefficient between average EDF and SMCR in all variants (table 3).

Table 3. Correlation coefficient between average EDF and SMCR in all variants.

	SMCR	SMCR +1quarter	SMCR + 2quarters
EDF	0.60	0.58	0.53

Source: own calculations.

Comparing achieved results for companies with financial problems we can state that profitability ratios are the first to deteriorate. Long-term low profitability leads to a financial liquidity decrease, which results in the debt growth. Levels of EDF indicate about 2 quarters sooner that the situation of a company would get worse. Watching EDF distribution in time we can notice that when bad information about company appear a heavy drop of stocks takes place and due to that EDF grows. After some time (usually about a month) the stock price starts growing again and analogously EDF drops (adjustment effect). It confirms the thesis of over-vulnerability of investors in case of bad information. In such situations EDF often rose of 10% within just few sessions. Fluctuations of SMCR in following quarters are smaller.

4. Summary

Conclusions of the research carried out in this paper show that both methods can supplement each other. Further researches should concentrate on fitting SMCR into tighter quantile intervals of EDF. It will enable a more precise credit risk estimation of companies that are not present on WSE. To achieve that a vector of financial indicators. which is used for creating SMCR. should be modified.

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