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The Phillips Method of Fractional Integration Parameter Estimation and Aggregation of PLN Exchange Rates

1. Introduction

Let $\{y_t\}$ be a nonstationary series. An integration level was defined by Engle and Granger as the least integer d for which $(1–L)^d y_t$ is stationary. Later this definition was extended to real values of d with use of the gamma function:

$$\Delta^d = (1-L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-1)^k L^k = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(k+1)} L^k \ ,$$

where L is lag operator. There are several methods of a fractional d estimation. Here we apply the Phillips (1995) method. In earlier research, Syczewska (2005a), we have shown that results of fractional parameter estimation with the three methods:

- the generalized rescaled-range Lo procedure (1991),
- the Geweke and Porter-Hudak method (1983), based on a periodogram regression,
- the Robinson's method (1995),

do depend on the aggregation level. Here we intend to compare results of the Phillips' method for the same currencies and series extended to the end of May 2005. We study the Polish zloty exchange rates: daily average exchange rates of the NBP, and their weekly and monthly averages. Data base for daily rates covers period from 4th January 1993 until 31st May 2005. Exchange rates are expressed as numbers of Polish zlotys per unit of foreign currency, or in few cases, per 100 units of a currency. The euro exchange rate series has been extended backwards, based on irrevocable conversion units (see, e.g., Table 3 in

Oręziak (2003), p. 78). Logarithmic returns $r_t = \ln(e_t) - \ln(e_{t-1})$ are computed for all daily rates, and their weekly and monthly averages.

2. Stationarity and Unit Root Tests

As an additional check, we compared results of the stationarity and unit root tests for all currencies. We expect that for exchange rates logarithms the tests should suggest nonstationarity, and stationarity for logarithmic returns.

The Kwiatkowski, Phillips, Schmidt and Shin Test

Let e_t denote errors of regression of a variable with respect to a linear trend or to a constant. Let S_t denote partial sums of e_t. Estimator of a long-run variance is defined with use of Bartlett weights $w(s,l) = 1 - s/(l+1)$ as

$$s^2(l) = T^{-1}\sum_{t=1}^{T} e_t^2 + 2T^{-1}\sum_{s=1}^{l} w(s,l)\sum_{t=s+1}^{T} e_t e_{t-s}$$

The KPSS test statistics is defined as

$$\hat{\eta} = T^{-2}\sum S_t^2 / s^2(l).$$

If a computed value is lower than the appropriate critical value, null hypothesis of trend-stationarity of a series is rejected. We use asymptotic critical values for the KPSS test, from Table 1, p. 166 in Kwiatkowski et al. (1992). Schwert (1989) gives a general rule for number of lags used in the KPSS test regression. According to his formula, for daily exchange rates there were 20-28 lags, depending on length of the series. In the case of weekly data, we have 13-18 lags, and for monthly returns –19-23. The results are as expected and do not differ for the three levels of aggregation.

The DF-GLS Test of Elliott, Rothenberg and Stock

The original augmented Dickey-Fuller test is based on regression

$$\Delta y_t = \delta y_{t-1} + \sum_{j=1}^{k} \gamma_j \Delta y_{t-j} + \varepsilon_t,$$

where number of lagged terms are included to eliminate autocorrelation of ε_t. The ADF test statistics is defined as

$$DF = \hat{\delta} / s_{\hat{\delta}}.$$

For a computed values of DF lower than the critical value, the null hypothesis of non-stationarity has to be rejected. Elliott, Rothenberg and Stock (1996) have modified the ADF test. First, the series $\{y_t\}$ of observations is modified:

$$d(y_t) = \begin{cases} y_t & \text{if } t = 1, \\ y_t - a y_{t-1} & \text{if } t > 1, \end{cases}$$

where a is a constant, equal to $1 - 7/T$ for a model with constant, $1 - 13.5/T$ for a model with constant and trend (T is the number of observations). Let $d(x)$ denote regressors modified in a similar way. Let $\hat{\delta}(a)$ denote the OLS estimates of regression $d(y_t)$ with respect to $d(x_t)$. The test statistics is based on the ADF regression for $y_t^d = y_t - x_t \hat{\delta}(a)$:

$$\Delta y_t^d = \delta y_{t-1}^d + \sum_{j=1}^{k} \gamma_j \Delta y_{t-j}^d + \varepsilon_t .$$

The null hypothesis of nonstationarity is rejected if a computed DF value is lower than the critical value. In the Elliott, Rothenberg and Stock (1996) paper, Table 1, p. 825, are given simulated critical values for T = 50, 100 and 200 at 1%, 2.5%, 5% and 10%.

The Unit-Root Tests Results

a) The Dickey-Fuller Test

The critical values are equal to –3.430, –2.860 and–2.570 respectively at 1%, 5% and 10%, in case of our daily series (2000-3000 observations). For weekly averages with 650 observations, critical values are –3.430; –2.860 and –2.570. For the monthly data series are equal to –3.507,–2.889 and–2.579. For logarithmic returns, *p*-values according to McKinnon, are close to zero for all three aggregation levels. Table 1 in Syczewska (2005b) shows examples of the ADF test results[1]. The statistics is lower than the 1% critical value and higher than the 5% critical values for the daily data of the Belgian franc. For weekly and monthly returns of the same currency, the computed value is slightly greater than 5% critical value. In case of the Dutch krona, the ADF test statistics is lower than the critical value only for the weekly averages. This would suggest rejection of the null hypothesis, but bear in mind that the ADF test results are vulnerable to effects of structural changes. In case of the Greek drachma and Japanese yen computed test statistics are lower than the critical value, but do not differ for the three aggregation levels. For the rest of the exchange rates, the results of the ADF test are as expected, i.e. the statistics are greater than the critical value which suggests non-stationarity. In all cases, the ADF test statistic for lo returns is lower than the critical value, suggesting stationarity of returns and I(1) behavior for the series.

[1] Full results for the 24 currencies are available from the author.

b) The DF-GLS Test of Elliott, Rothenberg and Stock

For logarithms of the daily exchange rates we use τ version of the test (including a trend) with 10 lags, and for logarithmic returns – version μ, without trend). The critical values are provided in Elliott, Rothenberg and Scott (1996). Table 2 in Syczewska (2005b) shows results of computation. For any of analyzed daily exchange rates, nonstationarity cannot be rejected. For weekly and monthly averages, it is rejected for the Belgian franc. For daily returns, nonstationarity cannot be rejected for Finnish marka, Italian lira and the Czech korona. For weekly returns, nonstationarity is rejected for all currencies but for Finnish marka, Czech Korona and British pound. For monthly returns, nonstationarity cannot be rejected for the Euro, Swiss franc and Swedish krona.

c) The Phillips-Perron Test

Table 3 in Syczewska (2005b) shows results of the Phillips-Perron test – for the Z(t) statistics the MacKinnon p-values are given. Full set of results is available from the author. The Phillips-Perron test statistics do not differ for the three aggregation levels. Currencies of Belgium, Czech Republic, Greece and Hungary show different behavior than other currencies.

3. The Phillips' Method of Fractional Integration Parameter Estimation

The unit root tests (as the ADF, DF-GLS etc.), and stationarity tests (e.g. the KPSS test) try to distinguish between the I(1) and I(0) series. The fractional integration parameter estimation allows for more detailed investigation of a series behavior – long memory, mean-reverting behavior, or lack of them[2]. In the paper Syczewska (2004) we compared estimates of d computed with use of the STATA procedures, *gphudak* for the Geweke and Porter-Hudak method, *lomodrs* for the Lo's method, and *roblpr* for the modified periodogram Robinson's method, provided by Christopher F. Baum and available at the STATA procedures depositaries. Here we use the procedure *modlpr*, by Ch.F. Baum, and by Vince Wiggins from the Stata Corporation. This is an implementation of the fractional integration parameter estimation method, proposed by Phillips (1999), as a correction of the Geweke and Porter-Hudak method. First, a series is detrended, and the estimation method corrected to take into account density under the null hypothesis that $d = 1$. The procedure results contain also the t and z-statistics for d=0 and d=1 hypotheses, respectively.

[2] See Piłatowska (2000), pp. 132–135, for discussion of consequences of applying misspecified ARIMA model to a series with fractional d and ARFIMA to an integer d.

The results of the Phillips method, shown in Tables 1-3, contain *d* estimates, errors, and two test statistics for *d*=0 and *d*=1 with their respective p-values. Table 1a shows results for daily exchange rates. For all currencies hypothesis that *d* = 0 is rejected, for most of them the hypothesis that *d*=1 cannot be rejected (it is rejected only for daily rates of Finnish mark, forint, Irish pound and Norwegian krona. For the daily returns H_0: *d* = 1 is rejected. The *d* estimates have varying values: for the Austrian shilling, Deutsche mark, yen are higher than to 0.5, for currencies of Finland, Spain, Portugal, France, Luxembourg and Holland close to 0.4. For Swiss franc and British pound slightly less than 0.4, for forint higher than 0.3. For currencies of Belgium, Canada, Czech Republic and Norwegian currency H0: *d*=0 cannot be rejected. The *d* estimates for the euro returns and for dollar returns are close to 0.2 but are insignificant. Quite untypical value is for returns of historic Irish pound, 0.8, but the hypothesis that *d*= 1 is rejected. In case of weekly averages, *d* parameter estimates are mostly greater than 1, and $H_0 : d = 1$ cannot be rejected.

Table 1a. The results for logarithms of daily exchange rates

Currency	The *d* estimate	Error	t(H0: *d*=0)	P>|t|	z(H0: *d*=1)	P>|z|
ATS	1.043	0.081	12.859	0.000	0.469	0.639
BEF	0.912	0.097	9.390	0.000	–0.949	0.343
CAD	0.922	0.067	13.839	0.000	–0.900	0.368
CHF	1.004	0.076	13.216	0.000	0.045	0.965
CZK	1.021	0.094	10.883	0.000	0.206	0.836
DEM	1.122	0.096	11.727	0.000	1.308	0.191
DKK	1.012	0.088	11.562	0.000	0.139	0.889
ESP	0.963	0.095	10.082	0.000	–0.403	0.687
EURO	1.047	0.105	9.979	0.000	0.541	0.588
FIM	1.180	0.081	14.602	0.000	**1.944**	**0.052**
FRF	1.066	0.069	15.552	0.000	0.709	0.479
GBP	0.999	0.081	12.280	0.000	–0.010	0.992
HUF	1.231	0.111	11.048	0.000	**2.274**	**0.023**
IEP	1.165	0.078	15.028	0.000	**1.769**	**0.077**
ITL	1.047	0.070	14.972	0.000	0.509	0.611
JPY	0.995	0.095	10.433	0.000	–0.053	0.958
LUF	1.046	0.074	14.115	0.000	0.497	0.619
NLG	1.162	0.111	10.485	0.000	**1.735**	**0.083**
NOK	1.036	0.081	12.754	0.000	0.412	0.680
PTE	0.971	0.132	7.345	0.000	–0.316	0.752
SEK	1.080	0.083	12.970	0.000	0.920	0.357
XEU	1.042	0.111	9.390	0.000	0.407	0.684
USD	1.011	0.096	10.487	0.000	0.128	0.898

Source: author's computations.

Table 1b. Results for logarithmic daily returns

Currency	The *d* estimate	Error	t(H0: *d*=0)	P>|t|	z(H0: *d*=1)	P>|z|
ATS	0.517	0.102	5.065	0.000	–5.218	0.000
BEF	0.012	0.125	0.095	0.925	–10.675	0.000
CAD	0.047	0.089	0.520	0.605	–11.027	0.000
CHF	0.370	0.084	4.416	0.000	–7.290	0.000
CZK	0.122	0.118	1.030	0.309	–8.660	0.000
DEM	0.567	0.094	6.047	0.000	–4.627	0.000
DKK	0.260	0.084	3.111	0.003	–8.553	0.000
ESP	0.420	0.093	4.509	0.000	–6.266	0.000
EURO	0.257	0.084	3.061	0.003	–8.590	0.000
FIM	0.476	0.118	4.025	0.000	–5.661	0.000
FRF	0.438	0.099	4.405	0.000	–6.010	0.000
GBP	0.332	0.098	3.379	0.001	–7.729	0.000
HUF	0.306	0.143	2.149	0.038	–6.841	0.000
IEP	0.794	0.089	8.927	0.000	–2.199	0.028
ITL	0.399	0.100	3.988	0.000	–6.498	0.000
JPY	0.502	0.084	6.003	0.000	–5.764	0.000
LUF	0.480	0.098	4.888	0.000	–5.615	0.000
NLG	0.417	0.082	5.110	0.000	–6.228	0.000
NOK	0.100	0.101	0.991	0.326	–10.403	0.000
PTE	0.452	0.095	4.772	0.000	–5.916	0.000
SEK	0.237	0.102	2.329	0.024	–8.829	0.000
XEU	–0.050	0.144	–0.344	0.733	–10.089	0.000
USD	0.291	0.091	3.183	0.002	–8.200	0.000

Source: author's computation.

Table 2a. Results for weekly averages

Currency	The *d* estimate	Error	t(H0: *d*=0)	P>|t|	z(H0: *d*=1)	P>|z|
ATS	1.134	0.195	5.815	0.000	0.954	0.340
BEF	1.039	0.185	5.633	0.000	0.281	0.779
CAD	1.025	0.123	8.342	0.000	0.194	0.846
CHF	1.051	0.120	8.747	0.000	0.401	0.688
CZK	0.870	0.167	5.194	0.000	–0.862	0.389
DEM	1.190	0.146	8.135	0.000	1.359	0.174
DKK	1.065	0.127	8.401	0.000	0.510	0.610
ESP	1.029	0.173	5.953	0.000	0.204	0.839
EURO	1.047	0.129	8.144	0.000	0.368	0.713
FIM	1.063	0.113	9.393	0.000	0.452	0.651
FRF	1.110	0.116	9.575	0.000	0.785	0.432
GBP	1.103	0.143	7.693	0.000	0.800	0.424
HUF	1.114	0.188	5.911	0.000	0.752	0.452
IEP	1.185	0.123	9.655	0.000	1.323	0.186
ITL	0.960	0.126	7.599	0.000	–0.289	0.772
JPY	1.009	0.161	6.262	0.000	0.069	0.945
LUF	1.121	0.130	8.643	0.000	0.862	0.389
NLG	1.211	0.138	8.804	0.000	1.510	0.131
NOK	1.178	0.120	9.818	0.000	1.388	0.165

Currency	The *d* estimate	Error	t(H0: *d*=0)	P>|t|	z(H0: *d*=1)	P>|z|
PTE	1.133	0.204	5.543	0.000	0.950	0.342
SEK	1.123	0.125	8.950	0.000	0.957	0.339
XEU	1.060	0.146	7.246	0.000	0.385	0.700
USD	1.047	0.135	7.772	0.000	0.368	0.713

Source: author's computation.

Table 2b. The results for weekly logarithmic returns

Currency	The *d* estimate	Error	t(H0: *d*=0)	P>|t|	z(H0: *d*=1)	P>|z|
ATS	**–0.070**	**0.185**	**–0.377**	**0.710**	–7.645	0.000
BEF	**0.099**	**0.190**	**0.523**	**0.606**	–6.436	0.000
CAD	**–0.0004**	**0.145**	**–0.003**	**0.998**	–7.800	0.000
CHF	0.552	0.154	3.592	0.001	–3.497	0.000
CZK	**0.027**	**0.191**	**0.150**	**0.883**	–6.427	0.000
DEM	0.330	0.117	2.807	0.011	–4.790	0.000
DKK	0.556	0.155	3.586	0.001	–3.462	0.001
ESP	**–0.121**	**0.155**	**–0.780**	**0.444**	–8.007	0.000
EURO	0.574	0.182	3.158	0.004	–3.318	0.001
FIM	**–0.018**	**0.238**	**–0.075**	**0.941**	–7.273	0.000
FRF	0.559	0.158	3.533	0.002	–3.153	0.002
GBP	0.665	0.136	4.899	0.000	–2.610	0.009
HUF	**0.112**	**0.228**	**0.488**	**0.631**	–5.878	0.000
IEP	0.466	0.217	2.145	0.044	–3.813	0.000
ITL	**0.082**	**0.196**	**0.420**	**0.679**	–6.559	0.000
JPY	0.484	0.122	3.975	0.001	–4.026	0.000
LUF	**–0.042**	**0.174**	**–0.241**	**0.812**	–7.445	0.000
NLG	0.585	0.272	2.154	0.043	–2.966	0.003
NOK	0.587	0.132	4.451	0.000	–3.223	0.001
PTE	**–0.116**	**0.188**	**–0.614**	**0.546**	–7.972	0.000
SEK	0.599	0.211	2.843	0.009	–3.124	0.002
XEU	0.576	0.174	3.313	0.004	–2.724	0.006
USD	0.356	0.128	2.783	0.010	–5.023	0.000

Source: author's computation.

Table 3a. Estimates for monthly averages

Currency	The *d* estimate	Error	t(H0:*d*=0)	P>|t|	z(H0:*d*=1)	P>|z|
ATS	1.125	0.192	5.869	0.000	0.617	0.537
BEF	1.041	0.292	3.559	0.005	0.201	0.840
CAD	1.323	0.284	4.656	0.001	***1.746***	***0.081***
CHF	1.061	0.194	5.469	0.000	0.331	0.741
CZK	0.871	0.359	2.429	0.041	–0.567	0.571
DEM	1.059	0.203	5.214	0.000	0.290	0.772
DKK	1.263	0.188	6.706	0.000	1.419	0.156
ESP	1.396	0.349	4.005	0.002	**1.953**	**0.051**
EURO	1.262	0.213	5.919	0.000	1.415	0.157
FIM	1.266	0.236	5.366	0.000	1.311	0.190
FRF	1.197	0.196	6.119	0.000	0.974	0.330
GBP	1.294	0.224	5.773	0.000	1.589	0.112

Currency	The *d* estimate	Error	t(H0:*d*=0)	P>|t|	z(H0:*d*=1)	P>|z|
HUF	0.853	0.220	3.874	0.005	–0.650	0.516
IEP	1.343	0.314	4.275	0.002	***1.690***	***0.091***
ITL	1.188	0.232	5.114	0.000	0.926	0.354
JPY	1.037	0.252	4.123	0.001	0.200	0.841
LUF	1.201	0.175	6.869	0.000	0.993	0.321
NLG	1.066	0.187	5.690	0.000	0.327	0.744
NOK	1.184	0.168	7.040	0.000	0.995	0.320
PTE	1.347	0.268	5.027	0.001	***1.712***	***0.087***
SEK	1.247	0.156	7.974	0.000	1.333	0.183
XEU	1.023	0.267	3.836	0.005	0.100	0.921
USD	1.354	0.198	6.831	0.000	**1.911**	**0.056**

Source: author's computation.

Table 3b. Estimates for monthly logarithmic returns

Currency	The *d* estimate	Error	t(H0:*d*=0)	P>|t|	z(H0:*d*=1)	P>|z|
ATS	–0.099	0.377	–0.263	0.798	–5.420	0.000
BEF	0.308	0.344	0.895	0.392	–3.413	0.001
CAD	0.010	0.310	0.034	0.974	–5.346	0.000
CHF	0.019	0.205	0.091	0.929	–5.301	0.000
CZK	0.189	0.337	0.559	0.591	–3.579	0.000
DEM	–0.031	0.283	–0.108	0.916	–5.082	0.000
DKK	0.207	0.207	0.999	0.338	–4.284	0.000
ESP	–0.089	0.308	–0.288	0.779	–5.369	0.000
EURO	0.247	0.213	1.156	0.270	–4.069	0.000
FIM	0.181	0.364	0.498	0.629	–4.037	0.000
FRF	–0.121	0.312	–0.388	0.706	–5.528	0.000
GBP	0.120	0.149	0.808	0.435	–4.753	0.000
HUF	0.141	0.199	0.712	0.497	–3.787	0.000
IEP	0.066	0.547	0.121	0.907	–4.606	0.000
ITL	0.082	0.351	0.232	0.821	–4.529	0.000
JPY	–0.044	0.126	–0.353	0.731	–5.642	0.000
LUF	–0.195	0.389	–0.500	0.628	–5.891	0.000
NLG	–0.208	0.305	–0.683	0.510	–5.959	0.000
NOK	**0.533**	**0.179**	**2.976**	**0.012**	–2.522	0.012
PTE	–0.175	0.384	–0.456	0.658	–5.794	0.000
SEK	0.574	0.347	1.654	0.124	–2.302	0.021
XEU	0.429	0.236	1.822	0.106	–2.517	0.012
USD	–0.163	0.191	–0.855	0.409	–6.284	0.000

Source: author's computations.

The *d* estimates for weekly returns take negative values for ATS, CAD, ESP, FIM and LUF, but are insignificant. Also for Belgian franc, Czech korona and the forint they are insignificant. For other currencies, the estimates are significant and take rather high values, reaching 0.5, and even 0.7 for the British pound. For the US dollar, the estimate is significant but lower, close to 0.35. For all weekly returns, H0: *d*=1 is rejected.

The *d* estimates are for monthly averages higher than 1, but the null hypothesis *d* =1 mostly cannot be rejected. It can be rejected only for the Canadian dollar, escudo, Irish pound and US dollar.

In case of the monthly logarithmic returns, estimates are negative and insignificant for 9 currencies, positive and insignificant for 14 currencies. Only for the Norwegian krona, the estimate is significant and higher than 0.5. Figures 1 and 2 show the estimates for averages, and for returns, respectively. The estimates are quite similar for average rates. In case of the Canadian and US dollar, and euro, estimates increase with higher levels of aggregation. In case of forint and the Czech korona, estimates decrease with increase of aggregation. For the Swiss franc and yen, estimates do not differ.

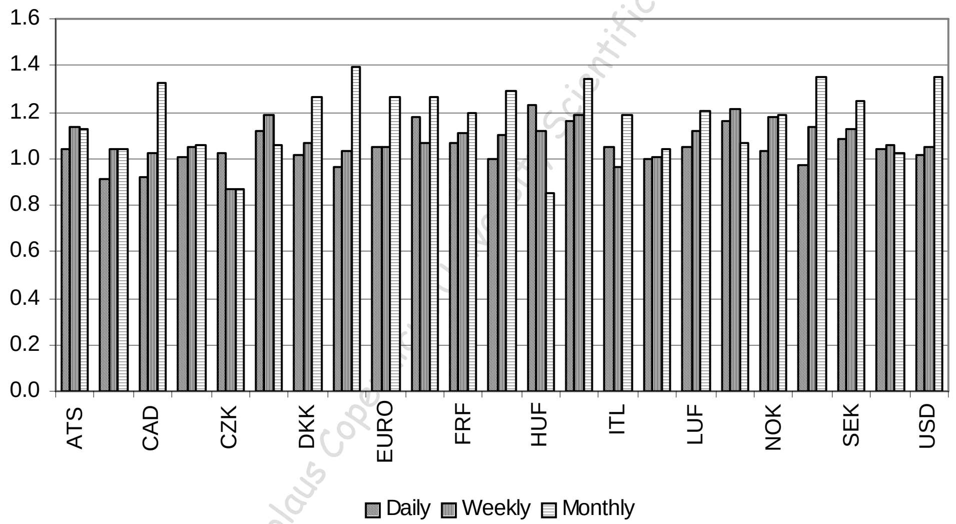

Fig. 1. Fractional integration parameter estimates for average exchange rates
Source: author's computations

Fractional integration parameter estimates for logarithmic returns differ between aggregation levels and between currencies. For most currencies estimates are positive, for a few – negative, between –0,2 do 0,8. For one currency – e.g., the Austrian shilling – estimates may be positive and significant for daily returns and insignificant for weekly and monthly returns. In case of Norwegian currency, *d* estimate is insignificant for daily returns, and for weekly and monthly returns – significant, with values higher than 0.5.

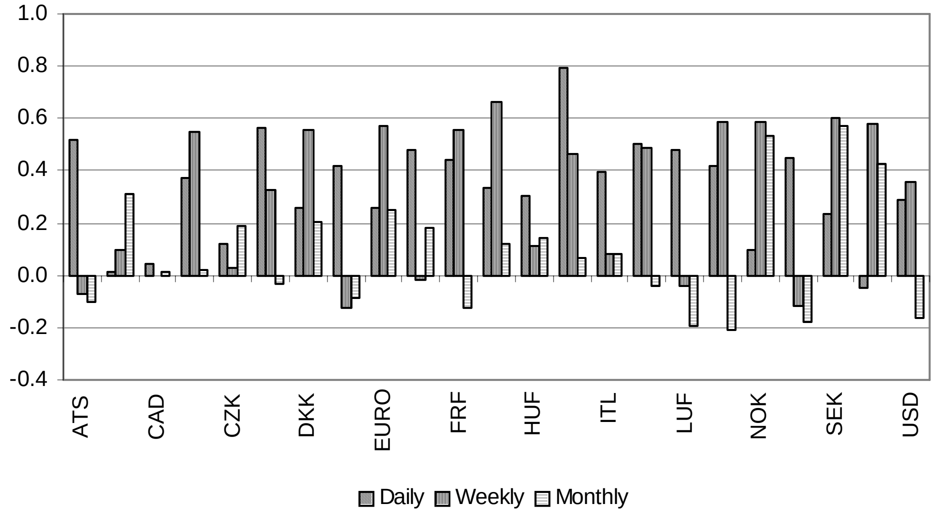

Fig. 2. Integration parameter estimates for logarithmic returns
Source: author's computations.

In case of the yen, estimates for averages do not differ much, but for daily and weekly returns the estimates are significant and greater than 0.5, in case of monthly returns – insignificant. For euro, the *d* estimates are close to 0.25 for daily and monthly returns, higher than 0.5 for weekly returns. For the US dollar, in spite of its high correlation with other currencies, the estimates for daily and weekly returns are positive, equal to 0.29 and 0.36, the estimate for monthly returns is negative.

4. Summary

Fractional integration parameter can be treated as an indicator of a series behavior – long or short memory, mean-reverting, stationarity etc. For d=1, this is an I(1) process, nonstationary, with infinite variance. If d>1, shock effects increase with time. For $0.5 \le d < 1$, the series is nonstationary but mean-reverting in long term (see Hosking (1981)). For $0<d<0.5$, the series is stationary and mean-reverting. If $d = 0$, this is a stationary process, mean-reverting, with finite variance.

Fractional integration parameter estimation can be performed with use of several methods. In earlier research the GPH and Robinson methods have been applied to the same set of exchange rates (monthly, weekly and daily data), and results of estimation do depend on aggregation levels. The Phillips method results, presented here, confirm our conjecture that the estimates differ between aggregation levels and between currencies.

Fractional integration parameter estimate can be used as indicator of model specification (e.g., to chose between ARMA, ARIMA and ARFIMA models), hence in the process of its estimation special attention should be paid to properties of a particular method applied.

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