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Modelling Financial Processes with Long Memory in Mean and Variance[1]

1. Introduction

Long memory is typical for hydrology and climatology (see for example Hurst, 1951, Kwiatkowski and Osiewalski, 2002). The results of analyses for financial time series are not so univocal. Long term dependence can concern both conditional mean and conditional variance. In most papers only one of those relationships is analysed, without the other. Meanwhile it has been known that neglecting the ARCH effect decreases efficiency of parameters estimators in the mean equation. The ARCH effect can also influence the values of standard errors. On the other hand incorrect specification of mean equation can influence the results of tests for conditional variance (see Lumsdaine and Ng, 1999). The main purpose of this analysis is to model long memory of financial processes. It will be shown that some financial processes have long memory in mean or variance, which can be described by ARFIMA–FIGARCH model.

The article is laid out in four sections. In Section 2 models, which can capture long run dependence of data, namely ARFIMA and FIGARCH are presented. In Section 3 selected financial time series are analysed. Section 4 concludes.

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2. ARFIMA–FIGARCH Models

The notion of process memory is not univocal in the literature. In this article the notion process with long memory means process for which autocorrelation function $\rho(k)$ decays hyperbolically to zero and series $\sum_{k=-\infty}^{\infty}|\rho(k)|$ does not converge to a finite limit. Two processes with long memory are considered – ARFIMA (see Granger and Joyeux, 1980) and FIGARCH (Baillie, Bollerslev and Mikkelsen, 1996). ARFIMA model, which is a generalisation of ARIMA model, can capture long term dependencies between observations of series. FIGARCH model, which is a generalisation of GARCH model, can capture long–term dependencies between squared observations of series. ARFIMA model describes the conditional expected value, on the other hand, FIGARCH model describes the conditional variance. FIGARCH model is the process with long memory, but only in volatility.

ARFIMA (P,D,Q) model can be written as:

$$\varphi(L)(1-L)^{D}(y_t-\mu_t)=\vartheta(L)\varepsilon_t, \tag{1}$$

where $\varphi(L)=1-\sum_{j=1}^{P}\varphi_j L^j$, $\vartheta(L)=1+\sum_{j=1}^{Q}\vartheta_j L^j$, L denotes the lag or backshift operator ($L^s\varepsilon_t=\varepsilon_{t-s}$), $-1<D<0{,}5$, ε_t is a white noise. The fractional differencing operator $(1-L)^D$ is defined in the following way:

$$(1-L)^D=\sum_{k=0}^{\infty}\binom{D}{k}(-L)^k. \tag{2}$$

The process described by formula (1) is stationary when $D<0{,}5$ and all roots of equation $\varphi(L)=0$ lie outside the unit circle, while it is invertible when $D>-1$ and all roots of equation $\vartheta(L)=0$ lie outside the unit circle. When $D\in(0;\,0{,}5)$, the ARFIMA process is called long memory process (see Kwiatkowski and Osiewalski, 2002). Autocorrelation function $\rho(k)$ decreases very slowly to zero at hyperbolic rate and series $\sum_{k=-\infty}^{\infty}|\rho(k)|$ does not converge to a finite limit. When $D=0$, process in (1) is the ARMA (P,Q) process and autocorrelation function converges to zero at geometric progression (at fast exponential rate). The ARMA process is a process with short memory. When $D\in(-0{,}5;\,0)$, the process has intermediate memory.

FIGARCH(p,d,q) model can be expressed as:

$$\varepsilon_t \big| \psi_{t-1} \sim D(0,h_t), \tag{3}$$

$$\phi(L)(1-L)^d \varepsilon_t^2 = \alpha_0 + [1-\beta(L)]\nu_t , \tag{4}$$

where ψ_{t-1} is the set of information available at time t–1, $D(0,h_t)$ is a particular probability density function with zero mean and variance equal to h_t, $\phi(L) = 1-\sum_{j=1}^{q}\phi_j L^j$, $\beta(L) = \sum_{j=1}^{p}\beta_j L^j$, $\nu_t = \varepsilon_t^2 - h_t$, $0 < d < 1$ and all roots of equation $\phi(L) = 0$ lie outside the unit circle.

The FIGARCH process is not weakly stationary, because the second moment of the unconditional distribution of ε_t is infinite (see Baillie, Bollerslev and Mikkelsen, 1996), however it is strictly stationary. For the FIGARCH(p,d,q) model it is difficult to establish general conditions for positivity of conditional variance, nevertheless it is possible to establish restrictions for models with low orders of p and q. For instance for the FIGARCH$(1,d,1)$ model the following conditions:

$$\beta_1 - d \le \phi_1 \le (2-d)/3 , \quad d\left[\phi_1 - (1-d)/2\right] \le \beta_1\left(\phi_1 - \beta_1 + d\right), \tag{5}$$

are sufficient to ensure the non–negativity of h_t.

Chung (1999) gives other conditions, which also guarantee that conditional variances are non–negative:

$$0 \le \phi_1 \le \beta_1 \le d < 1 . \tag{6}$$

Conditions in (5) and (6) are complementary and none of them is sufficient for the other. There may be parameter values that cannot satisfy either sets of sufficient conditions while still allow conditional variance to be positive.

Chung (1999) noticed that, the value of the parameter α_0 in equation (4) should be zero and demonstrated in simulations[2] that formulation in (4) is problematic and difficult to estimate accurately in practice. He suggests a slightly different version of the FIGARCH(p,d,q) model:

$$\phi(L)(1-L)^d\left(\varepsilon_t^2 - \sigma^2\right) = [1-\beta(L)]\nu_t , \tag{7}$$

for which unfortunately, the problem of estimation is not completely solved, because estimator of the σ^2 parameter is still biased.

[2] The author set the value of α_0 to 1, which is unrealistic and not even theoreticall plausible for returns.

The process in (4) for $d = 0$ is the GARCH process, while for $d = 1$ is IGARCH. However the FIGARCH$(p, 0, q)$ model not always reduces to GARCH(p, q) model. If the GARCH process is weakly stationary, then the influence of the current volatility[3] on forecasts of the variance decreases to zero at exponential rate. For the IGARCH process current volatility has permanent influence for forecast of conditional variance. This effect for FIGARCH model decreases to zero considerably slower than for the GARCH model, namely at hyperbolic rate. However there is a difference between the influence of current volatility for "expectation" of a future conditional variance process, and the influence for the "true" conditional variance process itself (Ding and Granger, 1996). Autocorrelation function of ε_t^2 for GARCH and IGARCH processes decays exponentially to zero, while for the FIGARCH process it decays hyperbolically. Therefore the process in (4) for $0 < d < 1$ has long memory in volatility. On the other hand, despite of permanent influence of current volatility for forecasts of conditional variance, the IGARCH process has short memory in this article's sense.

3. Analysis of Financial Processes

The following financial processes were analysed: stock indices quoted on the Warsaw Stock Exchange (WSE) – MIDWIG, TechWIG, WIG, WIG20, WIRR, foreign stock indices DJIA, Nasdaq Composite, S&P 500 (New York), DAX (Frankfurt), BUX (Budapest), exchange rates – USD/PLN, EUR/PLN, EUR/USD, prices of raw materials – gold, oil, copper. For stock indices quoted on the WSE daily data from 3rd October 1994 (introduction of five–day quotation) or from the introduction of the particular index on the WSE to 28th February 2005 were used in the analysis. For comparison the same period was used for foreign stock indices and raw materials. Properties of currency rates of Polish zloty depend on the system of exchange rate (see Doman and Doman, 2004), that is why for USD/PLN exchange rate two series were analysed. The first series covers the period from introduction of crawling peg (16th May 1995), the second one from introduction of floating exchange rate (12th April 2000) to 28th February 2005. Some of the financial processes have been already analysed (see for example Osiewalski and Pipień, 2000, Fiszeder, 2001, Piontek, 2004, Doman and Doman, 2004), however the results presented in this paper often differ from the findings published in other papers. Firstly, in those analyses only time series from the Polish financial market were used. Secondly, in any of those papers, the ARFIMA model (long memory in mean) was not considered. Thirdly, only in studies by Doman and Doman, and Piontek all three

[3] Instead of influence of current volatility often it is said about the effect of a shock to forecasts of the variance.

specifications of the model, namely the GARCH, the IGARCH and the FIGARCH, were analysed (in the later analysis only the WIG index is investigated). Distinguishing between these three parameterisations is important from theoretical point of view, because they describe quite different dynamics of variability. The main difference between my results and those achieved by Doman and Doman concerns stock indices, namely in the later research, for the comparable period, the best model was the GARCH model.

At the beginning for all time series Ng–Perron unit root test (Ng and Perron, 2001) and KPSS stationarity test (Kwiatkowski, Philips, Schmidt and Shin, 1992) were conducted. The summary results are presented in Table 1. The results of Ng–Perron test indicate that all processes are integrated of order one. The same conclusion (except two series) is drawn from the results of KPSS test. Only for the S&P 500 index and USD/PLN currency rate (the first series) the hypothesis about stationarity of the first differences of logarithms was rejected. However the results of the tests should be treated with caution, because all time series had ARCH effect and the sum of parameter's estimates in the GARCH model (without α_0) were often close to one.[4] Decision about strict stationarity for IGARCH processes will be always wrong, because unconditional variance of such process is infinite.

The parameters of ARFIMA–FIGARCH models were estimated for logarithmic returns. Skewed Student–t distribution was used as conditional distribution of ε_t in formula (3) (see Osiewalski and Pipień, 2000). Estimate of D parameter $(0 < D < 0{,}5)$ in ARFIMA model was significant for indices MIDWIG, TechWIG, WIRR and BUX, which indicates long memory of returns. Estimate of d parameter $(0 < d < 1)$ in FIGARCH model was significant for MIDWIG, WIG, WIG20, WIRR, BUX, EUR/PLN, EUR/USD, which indicates existence of long–term dependence in volatility. Skewness parameter ξ in conditional skewed Student–t distribution was significant for MIDWIG, WIG20, WIRR, Nasdaq Composite, S&P 500, DAX, USD/PLN (shorter series), EUR/PLN and oil. Selection of different model specifications (values of P, Q, p, q, forms of mean and variance equations and form of conditional distribution of ε_t) was based on the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) with regard to the results of proper diagnostic tests.

There is no universal method of selecting an appropriate GARCH model. The usual model selection difficulties in linear models are further complicated in GARCH models by the uncountable infinity of functional forms allowed and the choice of an appropriate loss function. Standard model selection criteria, such as the AIC and the BIC, have been widely used in the ARCH literature,

[4] For example Dickey-Fuller tests over-reject in the presence of ARCH effect, when the GARCH process is nearly integrated (see Kim and Schmidt, 1993).

though their statistical properties in the ARCH context are unknown. The most natural loss function may be the one based upon the goals of the particular application. In other analyses I select models based on forecasting performance, however here models are selected on a basis of the AIC and BIC criteria, which seek for a compromise between a number of parameters and the value of likelihood function. The models are presented in Tables 2 and 3.

If the selection was based on the BIC criterion, the specification of the model was simpler, especially for the conditional variance model. According to both criteria the ARFIMA model was found to be superior only for two indices MIDWIG and WIRR. The conditional heteroscedasticity of most series was best represented by the IGARCH model (except copper, EUR/PLN and USD/PL – shorter series, for which it was best represented by the GARCH model and WIRR, BUX, for which the FIGARCH model was the best, but it did not meet nonnegativity constraints of conditional variance). If the selection was based on the AIC criterion, the FIGARCH model was found to be superior, however in most cases estimates of parameters did not meet restrictions for nonnegativity of variance. Comparing the autocorrelation functions calculated for squared simulated values of the GARCH, IGARCH and FIGARCH processes with the autocorrelation functions estimated for squared returns of analysed series, one may notice that in many cases the GARCH and FIGARCH models better explain the short term relations in volatility (for the IGARCH model the values of autocorrelation function for the first tens of lags are significantly overestimated)

Conclusions

ARFIMA and FIGARCH models are characterized in the paper. In empirical part of the article selected financial time series are analysed. For many time series IGARCH model was the best in modelling volatility. However it seems that models selected on the grounds of the Bayesian information criterion are too restrictively parameterised and in many cases they are modelling volatility of empirical returns worse than the GARCH or FIGARCH models.

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Table 1. Unit root and stationarity tests for selected financial processes

Series	Ng–Perron test								KPSS test	
	$\ln y_t$				$\Delta \ln y_t$				$\ln y_t$	$\Delta \ln y_t$
	MZ_ρ	MZ_t	MSB	MP_T	MZ_ρ	MZ_t	MSB	MP_T		
Stock indices of the Warsaw Stock Exchange										
MIDWIG	-0,61	-0,24	0,39	13,13	-308,48*	-12,41*	0,04*	0,09*	2,60*	0,20
TechWIG	-1,09	-0,69	0,63	20,35	-84,55*	-6,48*	0,08*	0,33*	2,50*	0,28
WIG	0,31	0,17	0,55	22,88	-909,31*	-21,32*	0,02*	0,03*	3,18*	0,07
WIG20	-0,84	-0,39	0,47	15,17	-944,75*	-21,73*	0,02*	0,03*	1,54*	0,07
WIRR	1,01	0,91	0,91	58,69	-120,51*	-7,74*	0,06*	0,24*	1,06*	0,24
Foreign stock indices										
DJIA	0,55	0,78	1,41	120,33	-594,59*	-17,24*	0,03*	0,04*	4,22*	0,35
Nasdaq C.	0,06	0,06	0,98	54,84	-653,30*	-18,07*	0,03*	0,04*	2,32*	0,30
S&P500	0,45	0,61	1,36	109,00	-449,25*	-14,99*	0,03*	0,06*	3,29*	0,49*
DAX	-0,06	-0,06	0,95	50,28	-773,43*	-19,66*	0,03*	0,03*	2,17*	0,35
BUX	1,08	1,70	1,58	167,20	-1268,7*	-25,18*	0,02*	0,02*	4,12*	0,24
Exchange rates										
USD/PLN 1	-0,20	-0,26	1,29	83,87	-923,14*	-21,48*	0,02*	0,03*	3,47*	1,07*
USD/PLN 2	2,39	0,86	0,36	17,58	-116,29*	-7,60*	0,07*	0,25*	3,16*	0,32
EUR/PLN	-2,69	-1,16	0,43	9,11	-102,17*	-7,13*	0,07*	0,27*	2,71*	0,22
EUR/USD	0,84	0,56	0,66	33,54	-627,14*	-17,71*	0,03*	0,04*	3,94*	0,24
Raw materials										
Gold	-1,54	-0,85	0,55	15,38	-947,96*	-21,77*	0,02*	0,03*	1,53*	0,44
Oil (fut.)	-0,38	-0,14	0,37	13,00	-1188,9*	-24,38*	0,02*	0,02*	3,70*	0,12
Copper (fut.)	-2,37	-1,02	0,43	9,90	-1057,7*	-23,00*	0,02*	0,02*	1,50*	0,39

* The superscript indicates that the null is rejected at the 5% level.
Source: author's own calculations.

Table 2. Results of estimation for the ARFIMA–FIGARCH class of models selected by AIC criterion

Series	$\phi_0 \times 10^{-4}$	ϕ_1	ϕ_2	θ_1	d_1	$\alpha_0 \times 10^{-4}$	α_1 or φ_1	α_2	β_1	d_2	ν	ξ
MIDWIG	1,1796 (4,7709)	–	–	–	0,1046 (0,0188)	4,9806 (2,8887)	0,1029 (0,0746)	–	0,5800 (0,0947)	0,5911 (0,0651)	9,3522 (2,2249)	-0,0840 (0,0345
TechWIG	2.2738 (4.3247)	0.0903 (0.0282)	–	–	–	0,0124 (0,0090)	0.0732 (0.0137)	–	–	–	11.2753 (3.2175)	–
WIG	5.8458 (4,3244)	–	–	0.1572 (0.0204)	–	0,0257 (0,0119)	0.1018 (0.0154)	–	–	–	7.3670 (1.0141)	–
WIG20	7,7597 (3,4501	0,0698 (0,0200)	–	–	–	0,0614 (0,0181)	0,0970 (0,0127)	–	0,8901 (0,0139)	–	8,2265 (1,3395)	0,0665 (0,0284)
WIRR	-5,6879 (6,3638)	0,0440 (0,0319)	-0,1000 (0,0238)	–	0,1332 (0,0262)	0,0103 (0,0042)	0,1879 (0,0348)	-0,1199 (0,0376)	–	–	7,2820 (0,9252)	-01262 (0,0284)
DJIA	6,3572 (1,7399)	–	–	–	–	0,0098 (0,0034)	0,0665 (0,0094)	–	0,9259 (0,0101)	–	9,6183 (1,5514)	-0,0629 (0,0310)
Nasdaq	10,7963 (2,1535)	–	–	–	–	0,0184 (0,0073)	0,0345 (0,0192)	0,0812 (0,0290)	–	–	13,4195 (2,9514)	-0,1321 (0,0381)
S&P 500	7,6261 (1,6025)	–	–	–	–	0,0058 (0,0023)	0,0329 (00180)	0,0439 (0,0218)	–	–	8,1124 (1,1634)	-0,0650 (0,0300)
DAX	6,6957 (2,1694)	–	–	–	–	0,0158 (0,0065)	0,0004 (0,0161)	0,1135 (0,0230)	–	–	16,5858 (3,9796)	-0,1085 (0,0290)
BUX	10,9156 (2,723)	–	–	0,0673 (0,0326)	–	0,0651 (0,0172)	0,1980 (0,0253)	-01005 (0,0268)	–	–	4,9737 (0,5171)	–
USD/PLN 1	1,6305 (0,9251)	–	–	–	–	0,0123 (0,0028)	0,2493 (0,0382)	-0,1061 (0,0495)	–	–	5,3949 (0,5065)	–
USD/PLN 2	-3.8988 (1.9585)	–	–	–	–	0,0821 (0,0253)	0.1489 (0.0320)	–	0.6947 (0.0646)	–	10.2065 (3.1741)	0.1101 (0.0426)
EUR/PLN	-1,3877 (1,6334)	-0,1106 (0,0317)	–	–	–	1,0134 (0,4522)	0,2929 (0,1340)	–	0,5739 (0,1529)	0,4352 (0,0825)	6,2186 (1,1227)	0,1145 (0,0439)
EUR/USD	2,8011 (1,6978)	–	–	-0,0598 (0,0260)	–	0,5774 (0,1677)	0,2724 (0,0573)	–	0,6526 (0,0848)	0,3914 (0,0921)	10,7057 (3,5385)	–
Gold	-1,3035 (0,9528)	–	–	-0,0463 (0,0187)	–	0,0022 (0,0011)	0,0851 (0,0139)	–	0,9148 (0,0088)	–	3,5742 (0,2912)	–
Oil (fut.)	5,4732 (3,5341)	–	–	–	–	0,0189 (0,0089)	0,0377 (0,0089)	–	–	–	6,361 (0,853)	-0,0596 (0,0277)
Copper (fut.)	0,5315 (2,3480)	-0,0768 (0,0186)	–	–	–	0,0307 (0,0123)	0,0237 (0,0060)	–	0,9627 (0,0097)	–	5,0453 (0,4949)	–

Standard errors are reported in parentheses, ν and ξ are the parameters of skewed Student–t distribution.
Source: author’s own calculations.

Table 3. Results of estimation for the ARFIMA–FIGARCH class of models selected by BIC criterion

Series	$\phi_0 \times 10^{-4}$	ϕ_1	ϕ_2	θ_1	d_1	$\alpha_0 \times 10^{-4}$	α_1 or φ_1	α_2	β_1	d_2	ν	ξ
MIDWIG	3.3546 (4.5631)	–	–	–	0.1067 (0.0180)	0,0106 (0.0051)	0.1099 (0.0195)	–	–	–	9.6564 (2.0026)	–
TechWIG	2.2738 (4.3247)	0.0903 (0.0282)	–	–	–	0,0124 (0,0090)	0.0732 (0.0137)	–	–	–	11.2753 (3.2175)	–
WIG	5.8458 (4,3244)	–	–	0.1572 (0.0204)	–	0,0257 (0,0119)	0.1018 (0.0154)	–	–	–	7.3670 (1.0141)	–
WIG20	5.0402 (3.0103)	0.0684 (0.0202)	–	–	–	0,0367 (0,0180)	0.0999 (0.0166)	–	–	–	7,5076 (1.0599)	–
WIRR	-6,0511 (6,5688)	0,0438 (0,0335)	-0,0984 (0,0241)	–	0,1355 (0,0262)	11,5654 (7,4217)	0.9982 (0.0012)	–	0.9671 (0.0090)	0.1657 (0.0368)	8.2008 (1.2203)	-0.1256 (0.0291)
DJIA	7,2419 (1.7321)	–	–	–	–	0,0092 (0,0052)	0,0710 (0,0114)	–	–	–	8,4418 (1,2836)	–
Nasdaq	9,6381 (2,2827)	–	–	–	–	0,0112 (0,0051)	0,0861 (0,0134)	–	–	–	14,5024 (3,0713)	-0,1436 (0,0285)
S&P 500	7,6266 (1,5944)	–	–	–	–	0,0018 (0,0009)	0,0670 (0,0094)	–	–	–	8,1882 (0,9806)	–
DAX	6,6957 (2,1694)	–	–	–	–	0,0158 (0,0065)	0,0004 (0,0161)	0,1135 (0,0230)	–	–	16,5858 (3,9796)	-0,1085 (0,0290)
BUX	11,1109 (2,5642)	–	–	0,0811 (0,0216)	–	1,6540 (0,3367)	0,9441 (0,0268)	–	0,9138 (0,0377)	0,1650 (0,0395)	5,9145 (0,5339)	–
USD/PLN 1	1,3811 (0,9263)	–	–	–	–	0,0065 (0,0027)	0,1944 (0,0243)	–	–	–	5,3222 (0,3743)	–
USD/PLN 2	-3.8988 (1.9585)	–	–	–	–	0,0821 (0,0253)	0.1489 (0.0320)	–	0.6947 (0.0646)	–	10.2065 (3.1741)	0.1101 (0.0426)
EUR/PLN	-2,9125 (1,4680)	-0,1127 (0,0288)	–	–	–	0,0168 (0,0057)	0,0858 (0,0176)	–	0,8770 (0,0241)	–	6,8464 (1,1768)	–
USD/EUR	-3,7143 (1,7913)	–	–	-0,0598 (0,0235)	–	0,0007 (0,0008)	0,0316 (0,0091)	–	–	–	10,9976 (3,4399)	–
Gold	-1,2508 (0,9751)	–	–	–	–	0,0034 (0,0013)	0,0749 (0,0101)	–	–	–	4,0938 (0,2548)	–
Oil (fut.)	6,8663 (3,6167)	–	–	–	–	0,0158 (0,0076)	0,0395 (0,0079)	–	–	–	5,9744 (0,7066)	–
Copper (fut.)	0,5315 (2,3480)	-0,0768 (0,0186)	–	–	–	0,0307 (0,0123)	0,0237 (0,0060)	–	0,9627 (0,0097)	–	5,0453 (0,4949)	–

Standard errors are reported in parentheses, ν and ξ are the parameters of skewed Student–t distribution.
Source: author's own calculations.