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Sensitivity Model Analysis of the Floating–strike Lookback Call Option Pricing

1. Introduction

Lookback options are the extremum–dependent options in the class of exotic options. The value of the extremum–dependent options is influenced by the extreme value reached by the underlying instrument in the exercise period of the option.
The buyer of the floating–strike lookback call option is entitled to the purchase of the underlying instrument at the lowest price reached by the underlying instrument in the option exercise time.
Conversely, the buyer of the floating–strike lookback put option is entitled to sell the underlying instrument at the highest price reached by the underlying instrument in the option exercise time.

The article presents the issues connected with the floating–strike lookback options: the types of the options were characterized, the pricing model was described and the influence of selected factors on the pricing of the analysed lookback options was shown. The empirical research covered in the article deals with the impact of volatility, minimum of the underlying asset price, and time of expiration on the pricing of the lookback call options.
The study of the prices was carried out on the examples of the lookback and standard options pricing on EUR as well as the influence of the selected factors on the price of the analysed options was examined. On the basis of the performed simulations the comparative analysis of the prices for the standard and lookback options was demonstrated.

2. Pricing Model of the Floating–strike Lookback Call Option

The floating–strike lookback call options can be of the *in-the-money* type, where the current price of the underlying instrument is higher than the strike price, or of the *at–the–money* type, where the current price of the underlying instrument amounts to the strike price. The floating–strike lookback options can never be *out–of–the–money*.[1] If the current price of the underlying instrument was lower than the strike price, it would become the strike price and the call option would become *at–the–money.*
At the expiration date the payoff function of the floating–strike lookback call option is the following:

$$\max[0, S_T - m_T^S] = S_T - m_T^S \tag{1}$$

where:
S_T – price of the underlying instrument at the expiration date,
m_T^S – the lowest price of the underlying instrument in the option exercise time.
In time $t \in [0; T]$ the price of the option amounts to:[2]

$$C_t^W = e^{-r(T-t)} E^Q (S_T - m_T^S | F_t) = e^{-r(T-t)} E^Q (S_T | F_t) - e^{-r(T-t)} E^Q \left(m_T^S | F_t \right) \equiv$$

$$\equiv e^{rt} E^Q (e^{-rT} S_T | F_t) - e^{-r(T-t)} E^Q \left(\min \left(m_t^S , S_t e^{-M_{t,T}^X} \right) \right) =$$

$$= S_t - e^{-r(T-t)} E^Q \left(\min \left(m_t^S , S_t e^{-M_{T-t}^X} \right) \right) =$$

$$= S_t - e^{-r(T-t)} \left[E^Q \left(\min \left(m_t^S , S_t e^{-M_{T-t}^X} \right) \right) - m_t^S + m_t^S \right] =$$

$$= S_t - e^{-r(T-t)} \left[E^Q \left(\left(S_t e^{-M_{T-t}^X} - m_t^S \right) \mathrm{I}_{[M_{T-t}^X \geq z]} \right) \right] - e^{-r(T-t)} m_t^S =$$

$$= -e^{-r(T-t)} \left[- S_t \int_z^\infty e^{-y} Q \left[M_{T-t}^X \geq y \right] dy \right] = -e^{-r(T-t)} \left[- S_t \int_z^\infty e^{-y} Q \left[X_{T-t} \geq y \right] dy \right] -$$

$$- e^{-r(T-t)} \left[- S_t \int_z^\infty e^{-2r\sigma^{-2} y} Q \left[X_{T-t} \geq y + 2\lambda(T-t) \right] dy \right] =$$

$$= -e^{-r(T-t)} \left[S_t E^Q \left(\left(e^{-X_{T-t}} - e^{-z} \right) \mathrm{I}_{[X_{T-t} \geq z]} \right) \right] -$$

$$- e^{-r(T-t)} \left[\frac{S_t \sigma^2}{2r} E^Q \left(\left(e^{-2r\sigma^{-2} (X_{T-t} - 2\lambda(T-t))} - e^{-2r\sigma^{-2} z} \right) \mathrm{I}_{[X_{T-t} \geq z + 2\lambda(T-t)]} \right) \right] =$$

$$= -e^{-r(T-t)} \left[S_t e^{r(T-t)} E^Q \left(e^{\sigma \widetilde{B}_{T-t} - 0,5\sigma^2 (T-t)} \mathrm{I}_{[X_{T-t} \geq z]} \right) - m_t^S Q[X_{T-t} \geq z] \right] -$$

[1] The call option is *out–of–the–money* when the current price of the underlying instrument is lower than the strike price.
[2] See: Goldman, Sosin, Gatto (1979), Musiela, Rutkowski (1998).

$$-e^{-r(T-t)}\left[\frac{S_t\sigma^2}{2r}E^Q\left(e^{-2r\sigma^{-2}(X_{T-t}-2\lambda(T-t))}\mathrm{I}_{[X_{T-t}\geq z+2\lambda(T-t)]}\right)\right]+$$
$$+e^{-r(T-t)}\left[e^{-2r\sigma^{-2}z}\frac{S_t\sigma^2}{2r}Q\left[X_{T-t}\geq z+2\lambda(T-t)\right]\right]=$$
$$=-S_tP[X_{T-t}\geq z]+e^{-r(T-t)}m_t^SQ[X_{T-t}\geq z]-\frac{S_t\sigma^2}{2r}P\left[X_{T-t}\geq z+2\lambda(T-t)\right]+$$
$$+e^{-r(T-t)}e^{-2r\sigma^{-2}z}\frac{S_t\sigma^2}{2r}Q\left[X_{T-t}\geq z+2\lambda(T-t)\right]=$$
$$=-S_tN\left(\frac{\ln(m_t^S/S_t)-(r+0{,}5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)+$$
$$+e^{-r(T-t)}m_t^SN\left(\frac{\ln(m_t^S/S_t)-(r-0{,}5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)- \qquad (2)$$
$$-\frac{S_t\sigma^2}{2r}N\left(\frac{\ln(m_t^S/S_t)-(r+0{,}5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)+$$
$$+e^{-r(T-t)}e^{-2r\sigma^{-2}\ln(m_t^S/S_t)}\frac{S_t\sigma^2}{2r}N\left(\frac{\ln(m_t^S/S_t)+(r-0{,}5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)$$

where:
C_t^W – the price of the floating–strike lookback call option,
σ – price volatility of the underlying instrument,
T – time to the option expiry date,
r – risk–free rate of interest,
$m_{t,T}^S\equiv\min\limits_{u\in[t,T]}S_u=S_te^{-M_{t,T}^X}$, $M_{t,T}^X=\max_{u\in[t,T]}\left(X_u-X_t\right)$,
$m_T^S=\min(m_t^S,m_{t,T}^S)=\min\left(m_t^S,S_te^{-M_{t,T}^X}\right)$,
$S_u=S_t\exp\left(\sigma\left(\widetilde{B}_u-\widetilde{B}_t\right)+\left(r-0{,}5\sigma^2\right)(u-t)\right)=S_te^{-(X_u-X_t)}$,
$X_t=-\sigma\widetilde{B}_t+\lambda t$, $\lambda=0{,}5\sigma^2-r$, $\widetilde{B}_t$ is the standard Brownian motion against measure Q.

Finally, the price amounts to[3]:

$$C_t^W=S_tN(d_1)-S_t\frac{\sigma^2}{2r}N(-d_1)-e^{-r(T-t)}m_t^S\left[N(d_2)-\frac{\sigma^2}{2r}e^{a_1}N(-d_3)\right] \qquad (3)$$

where:

[3] See: Hull (1989), 464.

$$d_1 = \frac{\ln\left(\frac{S_t}{m_t^S}\right) + (r + 0{,}5\sigma^2)(T - t)}{\sigma\sqrt{T - t}}, \; d_2 = d_1 - \sigma\sqrt{T - t},$$

$$d_3 = \frac{\ln\left(\frac{S_t}{m_t^S}\right) + (-r + 0{,}5\sigma^2)(T - t)}{\sigma\sqrt{T - t}}, \; a_1 = \frac{-2(r - 0{,}5\sigma^2)\ln\frac{S_t}{m_t^S}}{\sigma^2},$$

$N(d)$ – cumulative probability function of the standardised normal distribution.

3. Empirical Example

The empirical research is concerned with the pricing simulation of the European currency call options. The options are on EUR. One of the options is the floating–strike lookback option (look. op.), the other is the standard option (stand. op.). These are 4–month options. The considered period is between 03.01.2005 and 05.05.2005.
Fig. 1 shows the prices of the discussed options.

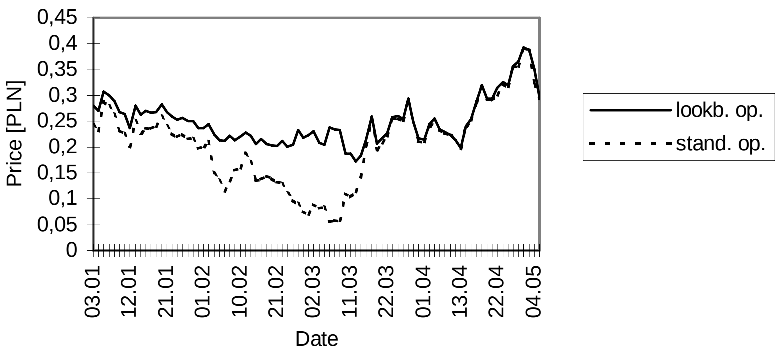

Fig. 1. The price of the floating–strike lookback call option and of the standard option, on EUR. The considered period: 03.01.2005 – 05.05.2005.
Source: Author's calculations.

The price analysis shows that the floating–strike lookback call option is more expensive than the standard option. A slump in the price of the underlying instrument considerably influenced the fall in the price of the standard call option as well as the increase in the price of the floating–strike lookback call option. The approaching expiration date narrowed the gap between the prices of the considered options.

Fig. 2 shows the influence of the expiration date on the price of the floating–strike lookback call option and standard call option.

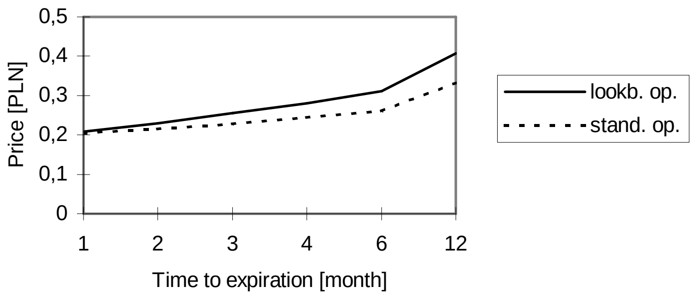

Fig. 2. The influence of the time to the option expiry date on the pricing of the floating–strike lookback call option and standard call option
Source: Author's calculations.

Price analysis proves, that for every expiration date, the prices of the floating–strike lookback call option are higher than for the standard call option. In the case the options have a longer–term expiry date, the differences between the prices of the floating–strike lookback call option and standard call option are greater. The approaching expiry date narrows down the differences in the price.

Fig. 3 illustrates the influence of the minimum price of the underlying instrument on the prices of the floating–strike lookback call option.

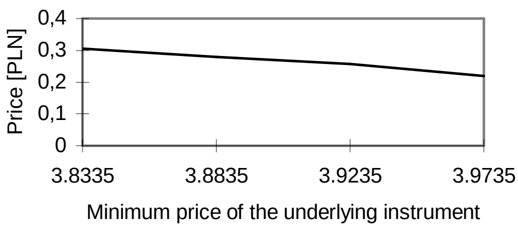

Fig. 3. The impact of the minimum price of the underlying instrument on the price of the floating–strike lookback call option
Source: Author's calculations.

The increase in the minimum price of the underlying instrument results in the drop in the price of the floating–strike lookback call option.

Fig. 4 demonstrates the impact of the volatility on the prices of the lookback and standard options.

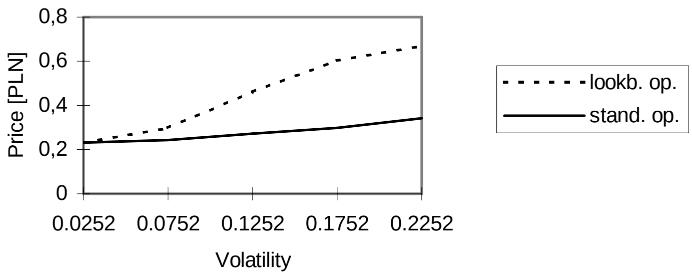

Fig. 4. The influence of the volatility on the prices of the lookback and standard call options
Source: Author's calculations.

Price analysis confirms that the increase in the volatility results in the rise of the lookback and standard option. However, in the case of the volatility growth, the prices of the lookback options tend to rise more dramatically. In the case of the volatility fall, the differences in the prices of the described options tend to get smaller.

Fig. 5 and 6 show the impact of time to expiry and volatility on the price of the lookback call option (see Fig. 1) and standard call option (see Fig. 6).

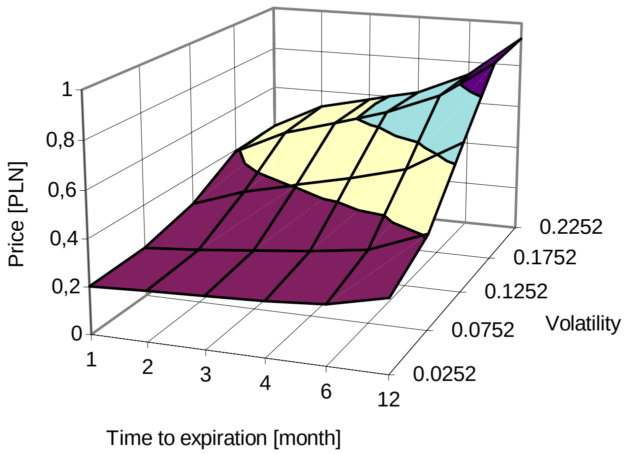

Fig. 5. The effect of volatility and time to expiry on the prices of the lookback call option
Source: Author's calculations.

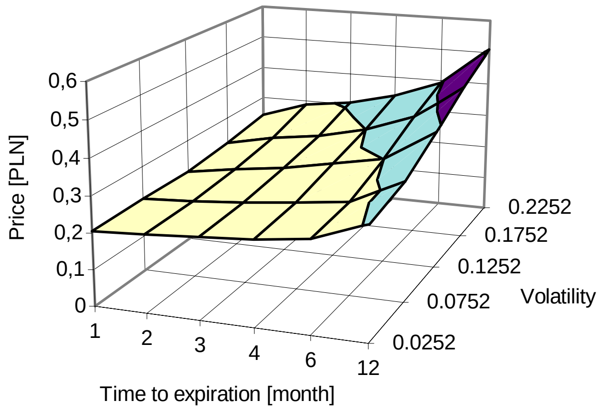

Fig. 6. The effect of volatility and time to expiry on the prices of the standard call option
Source: *Author's calculations.*

The examined example the lookback options are more expensive than the standard options for each volatility and time to expiry. The gap in the price decreases in case of the fall in volatility and time to expiry. Longer time to expiry and the growth in volatility results in the considerable price increase of the lookback call option.

4. Summary

The floating–strike lookback call option entitles the owner to purchase the underlying instrument at the lowest price reached by the underlying instrument in the option exercise time. The lookback option is more expensive than the standard option.

The value of the payoff function at expiry of the floating–strike lookback call option depends on two factors that influence the volatility of the underlying instrument:

– current price of the underlying instrument at expiry,

– lowest price of the underlying instrument in the option exercise time.

The prices of the lookback options react dramatically to the fluctuation of the volatility. The prices of the lookback options of the longer time to expiry are particularly sensitive to the fluctuations of volatility.

Consequently, the floating–strike lookback option is an exceptionally attractive financial instrument for those investors who use the option in speculative transactions on the price volatility market of the underlying instrument.

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