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Empirical Verification of Money Demand Models: Non-linear Cointegration Analysis

1. Introduction

It is very likely that no subject has attracted more attention, consumed more pages in economic journals and resulted in more data mining of the same data sets as the search for a stable and interpretable money demand model[2]. The money demand equation is one of the most important relationships in the monetary theory and practice, so as the proper specification of the functional form of this relationship is fundamental for realisation of any monetary policy. The direct inflation targeting (DIT) policy undertaken by many central banks causes money demand modelling to be somewhat less important. However, changes in the monetary aggregates still belong to the set of indicators considered in executing monetary policy.

In empirical macroeconomics there exists a clear distinction between long and short term relationships. Long term money demand modelling belongs to the classical applications of standard linear cointegration analysis (see, for example, Johansen, Juselius, 1990, Ericsson, 1998). Nowadays the assumption of linearity of a long term relationship as well as symmetricity and proportionality of the adjustment to a long term value is often relaxed, so as researchers more and more often make use of non-linear cointegration analysis (see Vinod, 1999, Bae, de Jong, 2004), non-linear error correction models (see Lütkepol i in., 1999, Escribano, 2004) and non-linear co-trending analysis (see Cushman, 2002).

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[2] An overview of approaches to money demand modelling from a historical perspective can be found in Hoffman, Rasche (1996).

The aim of the paper is an empirical verification of real money demand relationships in Polish economy with the help of a parametric approach to the non-linear cointegration analysis. This research was supplemented with the examination of non-linear cointegration with the help of a method, which does not require a parametric or non-parametric estimation of a long term equation, as well as the tests for Bierens' (2000) non-linear co-trending. The rest of the paper is structured as follows: In section 2 the underlying macroeconomic relationships are derived and in section 3 some methodological issues are presented. Section 4 includes all empirical result, while section 5 shortly concludes.

2. Money Demand Modelling

The starting point for money demand modelling is the general equilibrium model, in which a representative household maximises its multiperiod utility function of consumption and leisure of the form

$$V(C,L)=\sum_{j=t}^{\infty}U(C_j,L_j)\beta^{j-t} \tag{1}$$

where C_j and L_j denote consumption of a certain good and leisure at time j, respectively, U is a concave utility function, which increases with a decreasing rate with respect to both arguments and β is a discount factor reflecting the time preference of the household. It is assumed that the time L_j, which agents have at their disposal, is divided between free time and time for purchasing and the latter is a function of real account balance M_t/P_t (M_t denotes the nominal account balance, while P_t is the price level). Furthermore, two financial assets are considered, where the first one is money M with the nominal rate of return R_t^M, and the second is the so-called alternative asset with the rate of return $R_t^W > R_t^M$. The budget constraint of the household is in the form

$$W_t + M_t = (1+R_{t-1}^W)W_{t-1} + (1+R_{t-1}^M)M_{t-1} - P_tC_t\,. \tag{2}$$

The money demand function is obtained as a side-product of maximising the utility function (1) under the constraint (2). In general, the functional form of money demand depends on the assumptions concerning the functions U and L. Assuming that L depends on M_t/P_t only and the utility function is the CES function in the form

$$U_t=\left[\omega C_t^{\delta}+(1-\omega)(M_t/P_t)^{1-\delta}\right]^{1/\delta}, \tag{3}$$

where $\delta<1$ and $\delta\neq 0$, the double-logarithmic functional form is obtained:

$$\ln\left(\tfrac{M_t}{P_t}\right)=\ln(1-\delta)-\ln\beta\tfrac{\omega}{1-\omega}+\ln\left(C_t\right)-\gamma\ln\left(R_t^W-R_t^M\right), \tag{4}$$

where $\gamma=1/(1-\delta)$ (see Stracca, 2001).

The double-logarithmic model (4) assumes that the interest rate elasticity of money does not depend on the interest rate itself (or on the term structure of interest rates). However, liquidity preferences of agents may depend on the kind of a monetary regime, i.e. the level of inflation and, as such, the level of the nominal interest rate. These phenomenon is well described by the semilogarithmic function in the form:

$$\ln\left(\tfrac{M_t}{P_t}\right) = k + \phi \ln(C_t) - \gamma\left(R_t^W - R_t^M\right) \tag{5}$$

(see Ericsson, 1998), which assumes that the interest rates elasticity is an increasing function of the interest rates spread.

From the other side money demand of a particular household may have a discontinuity for a certain value of the nominal interest rate, for which an income from a deposit does not compensate transaction costs. Due to this liquidity preferences will stay at a high constant value when the interest rates spread is below a certain threshold. However, the threshold may vary for different agents and in aggregate values we will observe a certain general increase of the interest rate elasticity instead. To describe this effect Hoffman, Rasche (1996), p. 103, Ashworth, Evans (1998) and Stracca (2001) suggest a log-inverse specification in the form:

$$\ln\left(\tfrac{M_t}{P_t}\right) = k + \phi \ln(C_t) + \tfrac{\gamma}{R_t^W - R_t^M}\,, \tag{6}$$

The three suggested functional forms of the money demand equation may be written down with the help of the Box-Cox transformation as

$$\ln\left(\tfrac{M_t}{P_t}\right) = k + \phi \ln(C_t) - \gamma\, \tfrac{\left(R_t^W - R_t^M\right)^\lambda}{\lambda}\,. \tag{7}$$

where $\lambda = 0, 1$ and -1 correspond to (4), (5) and (6), respectively. Then the interest rate elasticity is equal $-\gamma\left(R_t^W - R_t^M\right)^\lambda$. The question of which of the three forms is the proper one is mainly of empirical nature.

3. Methodological Issues

In the paper by non-linear cointegration we understand the existence of a co-mixing relationship. The notion "co-mixing" was introduced into econometrics by Dufrénot and Mignon (2002) as a generalisation of the Granger and Hallman's concept of non-linear cointegration (see Granger, Hallman, 1991) to the case of non-linear processes. To define the notion we first need to recall the so-called mixing condition. A stochastic process is strongly mixing (or α-mixing) if the dependence between past and future events becomes negligible when the time span between two events increases. More formally, if X_t is a sequence of random variables and $F_s^t = \sigma(X_s, \ldots, X_t)$ is the σ-algebra generated by the sequence, then X_t is α-mixing when $\alpha_m \to 0$ with $m \to \infty$, where α_m are coefficients in the form

$$\alpha_m = \sup_t \sup_{\{F \in F_{-\infty}^t, G \in F_{t+m}^{\infty}\}} \left| \mathrm{P}(G \cap F) - \mathrm{P}(G)\,\mathrm{P}(F) \right| . \tag{8}$$

A generalised I(d) process is defined as such a non-mixing random sequence, which becomes strongly mixing after applying the difference operator d times. Now consider two non-mixing processes X_t and Y_t and define a non-linear function f which is measurable with respect to an appropriate σ-field. These processes are said to be co-mixing if

(1) there exists a sequence $f(X_t, Y_t, \theta)$ that is α-mixing for $\theta = \theta^*$ and non-mixing for $\theta \neq \theta^*$;

(2) or if there exists a sequence $f(X_t, Y_t, \theta)$ that is non-mixing but with dependence structure weaker than the dependence inherent to both X_t and Y_t (see Dufrénot and Mignon, p. 217).

To investigate the co-mixing property we can use methods requiring parametric or nonparametric estimation of the function f or, alternativelly, methods which do not require such an estimation. In the first case residuals from the potential long term relationship are tested for mixing condition with the help of, for example, the KPSS test, the rank test for unit roots, the modified R/S analysis or significance tests of mutual information coefficients, higher order moments and other measures of non-linear dependence (like maximal correlation coefficients or different entropy measures. Besides, tests for short memory in information suggested by Aparicio and Escribano (1998) in the context of the so-called cointegration in information might be used. In the second group of tests for co-mixing we can single out the rank test for non-linear cointegration suggested by Breitung (2001), tests based on first differences of ranges and tests for cointegration in information.

As we place relatively more emphasis on the parametric approach to non-linear cointegration analysis, we should carefully choose an estimation method. One possibility is the fully modified ordinary least squares method of Phillips and Hansen (1990). First of all, the method is more robust to non-linearity of adjustment processes than the OLS method (some simulation results for case of bilinear processes are provided by Charemza, Makarova, 1999). This property is very important in our context, as we admit stable non-linear adjustment processes while testing for strong mixing. Secondly, the FMOLS method corrects for the bias resulting from autocorrelation of residuals. Finally, it admits endogenous regressors, while in money demand modelling the so-called scale variable (DGP, for example) often turns out not to be weakly exogenous as regards parameters of a long term relationship (comp. Fagan, Henry, 1998).

In empirical macroeconomics the choice of a functional form of model for nonstationary variables is mostly based on linear cointegration analysis, where as the proper model the one for which cointegration takes place is chosen. From the other hand, diagnostic tests and non-nested hypotheses tests for error correction models can be used. These are the error correction models, which are subject to goodness of fit tests, comparison of predictive abilities and properties of residuals. However, a long term relationship is controlled for stability of parameters, what might be viewed as a test for proper functional form and is also interpreted as a test for cointegration (see Hansen, 1992). Both equations – the short term and the long term ones – are checked as regards the reasonability of statistical and economic interpretation and the validity of imposed restrictions. We should, however, emphasise that non-nested hypotheses tests for cointegrating relationships have not been worked out yet. Such tests would make it possible directly to compare different long term specifications, including the nonlinear ones. For this reason Ashwoth and Evans (1998) suggest using such tests (for example, the standard *J* test) to stationary error correction models. An alternative to this approach might be non-linear cointegration analysis, which admits non-linearity of an adjustment process in a very general form. The central idea of this article is to treat tests for co-mixing as a method of choosing the proper functional form for long term relationships. The tests will point at specifications, which give adjustment series with the shortest memory in mean. It is also worth noticing that the relaxation of the assumption of linearity of the long term relationship and the replacement of standard cointegration tests with tests for co-mixing will enable finding long term relationships where linear cointegration tests fail. Thanks to this a second verification of many macroeconomic and financial hypotheses will take place.

4. Empirical Results

In the empirical investigation the following time series for the data span 01.1993-02.2004 (134 observations) were used: seasonally adjusted real modified broad money M3+[3], seasonally adjusted industrial production in 1995 prices, average value of interest on current accounts in Polish Zloty as measure of R_t^M and average value of interest on 3-month deposits in Polish Zloty in main commercial banks in Poland as measure of R_t^W. In what follows we use the following abbreviations: *lnM3r* – logarithm of M3+, *lnip* – logarithm of industrial production, *l_s* – interest rates spread, *lnl_s* – logarithm of *l_s*, *invl_s* – inverse of *l_s*.

In the preliminary analysis we made use of the KPSS test to check nonstationarity of the variables. It turned out that all series can be treated as I(1) processes. However, in case of *lnip* and *invl_s* the hypothesis of trend-stationarity

[3] The modified M3+ aggregate was suggested by Kot (2004) as a quantity adjusted to the so-called Belka's tax from November 2001 and the methodological change in the statistical data published by the National Bank of Poland from March 2002. I would like to thank Adam Kot from the Department of Macroeconomic and Structural Analyses at the NBP for providing me with this time series.

was not rejected at the 5% significance level, while in case of *l_s* at the same significance level the hypotheses of stationarity and trend-stationarity were rejected for first differences. Thus the nonlinear transformations of the series *l_s* clearly affect its statistical properties (comp. Ashworth, Evans, 1998). Next the Phillips-Hansen's FMOLS estimator was used to obtain estimates of parameters in three models generated by equation (7). Additionally, next three equations with a linear trend in the long run relationship were estimated. The deterministic components might be viewed as an effect of the monetization process in Polish economy (comp. Kot, 2004). Estimation outputs and results of stability tests for the long term relationships (see Hansen, 1992) are given in Table 1. The first three models have parameters estimates with signs and sizes consistent with the underlying macroeconomic theory, whereat the semilogarithmic model has also stable parameters. In case of the models with trends signs of parameters are no longer interpretable, but the structural parameters seem to be stable over time.

Non-linear cointegration was firstly examined with the help of the KPSS test, in which the small-sample critical values of Sephton (1996) for residuals from a long term relationship were used – see Table 2. Furthermore, the modified R/S analysis was performed (see Lo, 1991) together with tests for significance of mutual information coefficients and the Breitung's cointegration test, which is invariant to monotonic transformations of variables (see Breitung, 2001). Results are presented in Tables 3 – 5. Additionally, the non-linear co-trending test of Bierens (2000) was executed[4] (see results in Table 6). As we can conclude from the tables, the KPSS test indicates the presence of cointegration only in case of models 1 and 6 (at the 5% significance level).

[4] Assuming that processes under study are stationary around non-linear trends, non-linear co-trending means that there exist one or more linear combinations of variables, which are stationary around a linear trend. Bierens (2000) points out that it is practically impossible to distinguish between non-linear cointegration and non-linear co-trending in finite samples without a strong a priori belief concerning properties of the initial series.

Table 1. Fully modified ordinary least squares estimates and results of Hansen's stability tests

Equation	L_C statistic (p-value)	Mean$F$ statistic (p-value)	Sup$F$ statistic (p-value)
$\ln M3r = \underset{(\pm 1.44730)}{-0.61524} + \underset{(\pm 0.28744)}{1.69401} \ln ip - \underset{(\pm 0.07170)}{0.23369} \ln l\_s + u1$	1.69027 (0.01)	14.54576 (0.01)	82.37271 (0.01)
$\ln M3r = \underset{(\pm 1.02675)}{-0.54707} + \underset{(\pm 0.20775)}{1.65377} \ln ip - \underset{(\pm 0.00742)}{0.04195} l\_s + u2$	0.33611 (0.13)	4.74383 (0.13)	40.15577 (0.01)
$\ln M3r = \underset{(\pm 1.64357)}{-0.82577} + \underset{(\pm 0.34578)}{1.63386} \ln ip + \underset{(\pm 0.43506)}{0.54049} invl\_s + u3$	4.74225 (0.01)	96.50684 (0.01)	985.1895 (0.01)
$\ln M3r = \underset{(\pm 1.39580)}{8.14065} + \underset{(\pm 0.00151)}{0.00915} t - \underset{(\pm 0.314642)}{0.350239} \ln ip + \underset{(\pm 0.06352)}{0.11060} \ln l\_s + u4$	0.14303 (>0.2)	2.33146 (>0.2)	8.33663 (>0.2)
$\ln M3r = \underset{(\pm 1.70016)}{9.17038} + \underset{(\pm 0.00185)}{0.01006} t - \underset{(\pm 0.38022)}{0.56697} \ln ip + \underset{(\pm 0.01095)}{0.02367} l\_s + u5$	0.12100 (>0.2)	4.13433 (>0.2)	18.21213 (0.04)
$\ln M3r = \underset{(\pm 1.33860)}{7.28036} + \underset{(\pm 0.00126)}{0.00808} t - \underset{(\pm 0.28541)}{0.10356} \ln ip - \underset{(\pm 0.27230)}{0.38191} invl\_s + u6$	0.16337 (>0.2)	3.01464 (>0.2)	11.32775 (>0.2)

Computed by the author.

Table 2. Results of the KPSS test for residual series

Series	u1	u2	u3	u4	u5	u6
KPSS statistic	0,08719	0,222678	0,378465	0,219823	0,484276	0,153363
Critical values	Small sample critical values (T = 125): 0.170 (α = 10%), 0.229 (α = 5%), 0.402 (α = 1%); Asymptotic critical values: 0.164 (α = 10%), 0.219 (α = 5%), 0.389 (α = 1%) – see Sephton (1996).					

Computed by the author.

Results of the R/S analysis let us reject the hypothesis of short memory in mean in case of models 2, 4 and 6. Looking at the outcomes of significance tests for MI coefficients we can conclude that residuals from model 2 have the shortest memory in information, while the longest memory in information is present in residuals from model 1.

Table 3. Results of modified R/S analysis

Series	u1	u2	u3	u4	u5	u6
V statistic	1,0695	0,7598	1,3357	0,9473	1,1475	0,9566
q	12	14	17	20	16	26
Critical values	Small sample quantiles of V statistic (see Dufrenot, Mignon, 2002, p. 162, Tables 3.A.2 and 3.A.3) for q = 10: 0.81 (1%), 0.9 (5%), 1.54 (95%), 1.65 (99%), for q = 25: 0.95 (1%), 1.05 (5%), 1.68 (95%), 1.91 (99%). Asymptotic quantiles (see Lo, 1991, Table 2): 0.721 (0.5%), 0.809 (2.5%), 0.861 (5%), 1.747 (95%), 1.862 (97,5%), 2.098 (99,5%).					

Computed by the author. *q* was chosen according to the Andrews formula (see Lo, 1991).

Table 4. Mutual information coefficients for residual series

Series/MI for lags τ	u1	u2	u3	u4	u5	u6
1	**1.1043e+00**	**9.1175e-01**	**1.0685e+00**	**8.4832e-01**	**9.4230e-01**	**1.0974e+00**
2	**1.0105e+00**	**7.8200e-01**	**9.9144e-01**	**8.3809e-01**	**8.8673e-01**	**8.6910e-01**
3	**9.8193e-01**	**8.0223e-01**	**1.0293e+00**	**7.4910e-01**	**8.5670e-01**	**8.0049e-01**
4	**8.2012e-01**	6.2670e-01	**8.8581e-01**	**6.8407e-01**	**7.7411e-01**	**7.7264e-01**
5	**8.2721e-01**	5.9408e-01	**8.5963e-01**	**6.9836e-01**	**7.3474e-01**	**7.4377e-01**
6	**8.1120e-01**	5.8905e-01	**7.4999e-01**	**6.7412e-01**	**7.8739e-01**	**6.9303e-01**
7	**8.1151e-01**	5.3442e-01	**8.7265e-01**	6.5983e-01	**7.6589e-01**	**6.8443e-01**
8	**7.8598e-01**	5.0615e-01	**8.0626e-01**	5.7403e-01	**7.4720e-01**	5.9801e-01
9	**8.0888e-01**	5.1473e-01	**7.8930e-01**	5.3848e-01	**6.7017e-01**	5.7706e-01
10	**7.6350e-01**	4.7280e-01	**7.8355e-01**	5.2891e-01	6.1450e-01	5.6796e-01
20	**7.1692e-01**	5.7941e-01	6.5327e-01	5.0540e-01	6.5962e-01	6.2209e-01
30	**7.2798e-01**	4.3516e-01	6.4212e-01	3.9248e-01	5.2284e-01	6.3743e-01
40	6.6038e-01	5.2889e-01	**7.4695e-01**	5.3435e-01	5.9551e-01	6.3584e-01

Computed by the author. Critical value in testing for significance obtained with the Monte Carlo method based on 50 simulations of white noise process from N(0,1) distribution: 0.67 (for α = 10%). Significant values are in bold.

Table 5. Results of the Breitung's nonlinear cointegration test

$\Xi^*[2]$ statistic	0.0173
Critical values	0.0221 (a = 10%), 0.0188 (α = 5%), 0.0142 (α = 1%) - see Bruzda (2003).

Computed by the author. The null hypothesis of the lack of cointegration is rejected when the test statistic is less than the appropriate critical value. In order to maintain the assumption of equal monotonicity the series l_s i lnl_s are taken with the opposite sign. In this way one value of the test statistic or the case of the regression without trend was obtained.

The Breitung's test generally provides evidence in favour of the presence of (possibly non-linear) cointegration without the trend component, while the nonlinear co-trending tests indicates the presence of two vectors giving combinations of variables, which are stationary around a linear trend.

Table 6. Results of the non-linear co-trending tests

Hypothesised no. of co-trending relationships	λ_r^* statistic	10% critical region	5% critical region
1	0.05890	>0.35183	>0.46577
2	0.25542	>0.53501	>0.67420
3	1.54156*	>0.70366	>0.86038

Computed by the author. * denotes the rejection of the null hypothesis.

In the last step of our empirical investigation vector error correction models based on fully modified residuals were estimated and the *J* tests for variance encompassing were performed (see results in Table 7). Each of the 6 VEC models indicated the lack of weak exogenity of *lnip*, while the *J* tests do not single out models, which encompass other.

Table 7. Results of the *J* test

H0: M1 or M2 encompass M3		H0: M1 or M3 encompass M2		H0: M2 or M3 encompass M1	
t = -0.97317	t = -0.92226	t = -0.12312	t = -0.11201	t = 0.00268	t = 0.00733
H0: M4 or M5 encompass M6		H0: M4 or M6 encompass M5		H0: M5 or M6 encompass M4	
t = 1.19404	t = 1.22377	t = 1.15355	t = 1.23068	t = 0.76008	t = 0.78664
H0: M4 encompasses M1		H0: M5 encompasses M2		H0: M6 encompasses M3	
t = 0.00352		t = -0.12903		t = -0.91910	

Computed by the author.

5. Conclusions

Any comparison of different non-linear specifications for nonstationary variables is problematic because of the lack of an appropriate theory of statistical inference in the case of non-nested hypotheses testing for cointegrating relationships. In such circumstances some authors suggest using non-nested hypotheses tests for stationary error correction models. An alternative approach can be based on non-linear cointegration analysis, which might be helpful in searching for stable long term relationships, where linear cointegration tests fail, as well as in choosing the proper functional form of this relationship by pointing at adjustment processes with relatively short memory in mean.

Among the six compared functional forms of the money demand equation for Polish economy, according to the suggested methodology, best results were obtained for models 1 and 2, whereat the Hansen's tests and tests for significance of mutual information coefficients distinguished the semilogarithmic model, while the KPSS tests and the modified R/S analysis singled out the double-logarithmic functional form. The popular tests for variance encompassing applied to error correction models were not able to distinguish between the alternative specifications.

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