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Measuring Conditional Dependence of Polish Financial Returns

1. Introduction

Identification of conditional dependence structure between financial instruments undoubtedly belongs to the main challenges of modern finance and insurance. Among others, it is a crucial task in pricing basket derivatives or in portfolio risk management. The basic theoretical framework applied in this area is that of multivariate volatility model describing the volatility of several time series jointly in order to exploit possible connection between their dynamics. Very useful and popular tools for modeling the dynamic conditional covariances or correlations among various assets are parametric multivariate GARCH (MGARCH) models (see a survey by Bauwens et al., 2003). In their conventional form MGARCH models assume that the standardized innovations follow multivariate Gaussian process. In a more general setting, usually a multivariate elliptical distribution for the innovation is allowed. The multivariate normal distribution is, however, not consistent with such stylized facts about financial return distributions like asymmetry and tail-fatness, and the most common non-Gaussian elliptical distribution – multivariate Student's *t* – imposes, often unrealistically, the same degrees of freedom for all marginal distributions. Recently, Lee and Long (2005) proposed a new model named Copula-based Multivariate GARCH model (C-MGARCH) which permits modeling conditional correlations and possible more hidden dependence separately and simultaneously in the case where the standardized residuals of considered financial returns are allowed to be non-elliptically distributed and dependent. The class of C-MGARCH models includes MGARCH models as special cases but because of incorporating a copula apparatus C-MGARCH models can capture and describe the dependence structure that may be neglected by the conditional covariance.

In this paper we apply C-MGARCH methodology to model conditional dependency between pairs of selected Polish financial returns. We compare the dynamic conditional correlations estimated by means of C-DCC model belonging to the C-MGARCH class with those obtained with Engle's DCC model (Engle 2002). We also compare the 1-day ahead conditional correlation forecasts calculated with DCC and C-DCC models. In addition, by using Euclidean matrix norm, we evaluated how the implied conditional covariance forecasts fit the matrices of cross products of actually realized daily returns. Our main finding is that the conditional correlations obtained with the applied C-DCC model (for all the considered pairs they are almost everywhere positive) are, in fact totally, much lower than the ones estimated with Engle's DCC model. As regards the point forecasts of matrices of cross products of daily returns, we find that DCC models are better in that task but, altogether, we find the results not very impressive.

2. Multivariate Parametric Volatility Models

For a multivariate return series $r_{\mathrm{t}} = (r_{1,t}, \ldots, r_{k,t})'$, consider the decomposition

$$r_t = \mu_t + y_t , \tag{1}$$

where $\mu_t = E(r_t \mid \Omega_{t-1})$ and Ω_{t-1} is the information set available at time $t-1$. A general multivariate volatility model for the residual process y_t is given by the equation

$$y_t = H_t^{1/2} \varepsilon_t , \tag{2}$$

where $E(y_t y_t' \mid \Omega_{t-1}) = H_t$ and thus $E(\varepsilon_t \varepsilon_t' \mid \Omega_{t-1}) = I$. A multivariate GARCH (MGARCH) model can be obtained by describing a specific parameterization for the conditional covariance matrix H_t. There exist many such parameterizations (Bauwens et al., 2003). In this paper we restrict ourselves to a very simple parameterization DCC (dynamic conditional correlation) proposed by Engle (2002) and its copula-based extension C-DCC (Lee and Long, 2005). The idea of DCC model is to consider the evolution not of the conditional covariance matrix H_t but rather the conditional correlation matrix R_t. The DCC-GARCH model is described by the following specification

$$y_t \mid \Omega_{t-1} \sim N(0, H_t), \tag{3}$$

$$H_t = D_t R_t D_t = \left(\rho_{ij,t} \sqrt{h_{ii,t} h_{jj,t}}\right), \tag{4}$$

$$D_t = \mathrm{diag}\left(\sqrt{h_{11,t}}, \ldots, \sqrt{h_{kk,t}}\right), \tag{5}$$

$$h_{ii,t} = \omega_i + \sum_{j=1}^{q} \alpha_{ij} y_{i,t-j}^2 + \sum_{j=1}^{p} \beta_{ij} h_{ii,t-j} \,, \tag{6}$$

$$R_t = \left(\mathrm{diag}(Q_t)\right)^{-1/2} Q_t \left(\mathrm{diag}(Q_t)\right)^{-1/2} , \tag{7}$$

$$Q_t = \left(1 - \sum_{m=1}^{M} \alpha_m - \sum_{n=1}^{N} \beta_n \right) \overline{Q} + \sum_{m=1}^{M} \alpha_m u_{t\text{-}m} u'_{t\text{-}m} + \sum_{n=1}^{N} \beta_n Q_{t-n} \,, \tag{8}$$

where $u_t = D_t^{-1} y_t$, and $\overline{Q}$ is the unconditional covariance matrix of the series u_t. In the following, we apply the simplest DCC model with $M = N = p = q = 1$. The DCC model can be estimated by two-step maximum likelihood method (Engle, 2002).

3. Copula-based Extensions of MGARCH Models

Let η_1, η_2 be random variables with distribution functions *F* and *G*. If *F* and *G* are continuous then the copula of (η_1, η_2) is the function *C* with domain $[0,1] \times [0,1]$ which is the (restricted to the square $[0,1] \times [0,1]$) joint distribution function of the variables $U = F(\eta_1)$ and $V = G(\eta_2)$. If *H* is the joint distribution function of (η_1, η_2) then, in the above situation, by Sklar's theorem (1959), there exists the unique copula *C* such that

$$H(x, y) = C(F(x), G(y)) \,. \tag{9}$$

Thus copula allows to decompose the joint distribution into two parts: marginal distributions and dependence structure. We are not going to report details concerning general properties and applications of copulas, referring to (Nelsen, 1999). The main improvement in MGARCH model structure proposed by Lee and Long (2005) consists in rejecting the very restrictive condition (3) and postulating instead that

$$\varepsilon_t = \Sigma_t^{-1/2} \eta_t \,, \quad \eta_t = (\eta_{1,t}, \eta_{2,t})' \,, \tag{10}$$

$$\eta_t \,|\, \Omega_{t-1} \sim C_t(F_t(\cdot), G_t(\cdot); \theta_t) \,, \tag{11}$$

where C_t is possible time-varying copula of $(\eta_{1,t}, \eta_{2,t})$ dependent on parameter vector θ_t and the marginal distribution functions F_t and G_t of $(\eta_{1,t}, \eta_{2,t})$ are also allowed to vary in time. The main advantage of the reported approach is that the ε_t are still uncorrelated but can be dependent and the dependence structure is controlled by hidden variables $(\eta_{1,t}, \eta_{2,t})$ and their copula C_t. The offdiagonal element $\sigma_{12,t}$ of the covariance matrix Σ_t of η_t is determined by the copula and the marginal distribution functions F_t and G_t. By Heoffding's Lemma (Lehmann, 1966) it can be computed by the formula

$$\sigma_{12,t} = E(\eta_{1,t}\eta_{2,t} \,|\, \Omega_{t-1}) = \iint_{R^2} \big(C_t(F_t(x), G_t(y)) - F_t(x)G_t(y)\big)\, dxdy\,. \qquad (12)$$

The log-likelihood function for η_t has the form

$$L(\Theta, \eta_t) = \ln(f_t(\eta_{1,t})) + \ln(g_t(\eta_{2,t})) + \ln(c_t(F_t(\eta_{1,t}), G_t(\eta_{2,t}))), \qquad (13)$$

where f_t, g_t, and c_t are the corresponding density functions. One can easily derive the corresponding formula for r_t, taking into account that $r_t = \mu_t + y_t$ and $y_t = H_t^{1/2}\Sigma_t^{-1/2}\eta_t$. The C-DCC model reduces to the conventional DCC model when $C_t = C$ is the product copula and the marginal distributions are standard normal.

4. The Data and Model Specification

The data we analyze consist of daily returns on two exchange rates EUR/PLN and USD/PLN, and three sub-indices of the stock index WIG published by the Warsaw Stock Exchange. The sub-indices under scrutiny are WIG-construction (WIG-con), WIG-IT and WIG-food. All observations are from the period November 17, 2000 – March 23, 2005. They are divided into two groups. The first 990 observations were used for in-sample estimation. On the base of the remaining 102 returns from November 2, 2004 we have done one-day-ahead forecasting. Our analysis concerns the returns defined as

$$r_t = 100(\ln P_t - \ln P_{t-1}),$$

where P_t is the closing quotation on day *t*. Basic descriptive statistics for the return series are presented in Table 1.

Table 1. Descriptive statistics for the return series (November 17, 2000 – March 23, 2005)

	EUR/PLN	USD/PLN	WIG-con	WIG-IT	WIG-food
Mean	0.0053	-0.0332	0.0274	-0.0335	0.0839
Std. Dev.	0.6680	0.6846	1.1766	2.0556	1.0216
Min.	-3.11	-2.5038	-4.5241	-11.0701	-4.3276
Max.	5.5271	4.2208	5.3392	7.8320	6.3689
Skewness	0.8684	0.5734	0.3324	0.0936	0.3832
Kurtosis	8.6606	5.5623	4.8328	4.8392	5.7619

We estimated the DCC models given by the equations (1) – (8). For our C-DCC models, we assumed the standardized Student's t distribution with ν_1 and ν_2 degrees of freedom for the marginals $\eta_{1,t} = \eta_1$ and $\eta_{2,t} = \eta_2$. As the copula of (η_1, η_2) we have chosen Frank copula

$$C^{Frank}(u,v;\theta) = -\frac{1}{\theta}\ln\left[1 - \frac{(1-\exp(-\theta u))(1-\exp(-\theta v))}{1-\exp(-\theta)}\right]. \tag{14}$$

5. Empirical Results

The estimated parameters of the DCC and C-DCC models are presented in tables 2 and 3. Prior to the parameter estimation we filtered the pairs of raw return series by means of VAR(1) model and so we could assume $\mu_t = 0$. We do not report here the VAR(1) parameters, as well as those of the univariate GARCH models. In the case of C-DCC model, due to the joint estimation, we had to replace the equation of form (8) by the following one

$$Q_t = CC' + \alpha\, u_{t-1}u'_{t-1} + \beta\, Q_{t-1}. \tag{15}$$

Table 2. Parameters of the estimated DCC models

DCC parameters		α	β
EUR/PLN	USD/PLN	0.0225 (0.0095)	0.9430 (0.0249)
WIG-con	WIG-IT	0.0149 (0.0086)	0.9751 (0.0167)
WIG-con	WIG-food	0.0272 (0.0154)	0.9425 (0.0371)
WIG-IT	WIG-food	0.0077 (0.0048)	0.9920 (0.0028)

Table 3. Parameters of the estimated C-DCC models

C-DCC parameters	EUR/PLN USD/PLN	WIG-con WIG-IT	WIG-con WIG-food	WIG-IT WIG-food
ν_1	7.2426 (0.6594)	6.1399 (0.4288)	5.5649 (0.4498)	0.4242 (1.0474)
ν_2	8.6038 (0.9974)	7.2975 (0.6722)	5.5725 (0.3983)	5.4255 (0.6931)
α	0.0390 (0.0246)	0.0017 (0.0032)	0.0390 (0.0269)	0.0092 (0.0083)
β	0.9288 (0.0216)	0.9686 (0.0201)	0.8087 (0.0912)	0.9900 (0.2437)
θ	-1.7967 (0.3994)	-2.3559 (0.4228)	-0.8176 (0.4242)	-1.5359 (0.4161)
C	0.6247 0 0.0473 0.2499	0.1563 0 0.0112 0.0649	1.4979 0 0.0585 0.2152	0.1122 0 0.0635 0.2962
σ_{12}	-0.2608	-0.3276	-0.1184	-0.2194

It is not obvious to predict from the values of the parameters how the corresponding series of dynamic conditional correlations can differ from each other. As we can see in the following figures 1-4, the estimated conditional correlations from C-DCC models are much lower than those from DCC models for almost all the time. The same is also true for the forecasts presented in figures 5-8. We also tried to check how the conditional covariance matrices implied by the obtained conditional correlation forecast fit the matrices of cross products of the actually realized daily returns but we found that result not very impressive and do not report it here.

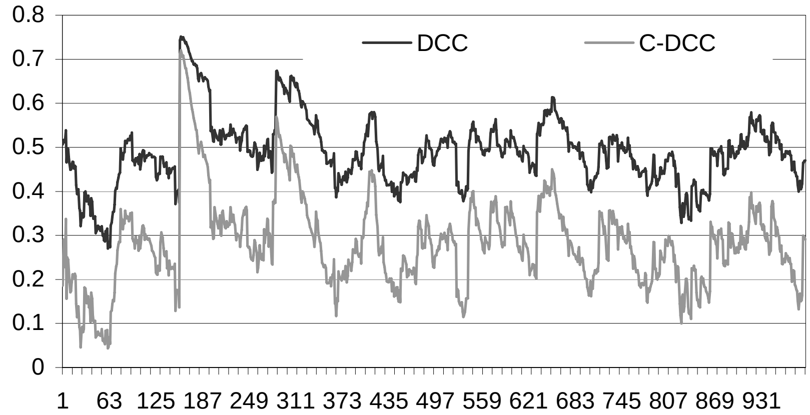

Fig. 1. Comparison of the conditional correlations for EUR/PLN and USD/PLN

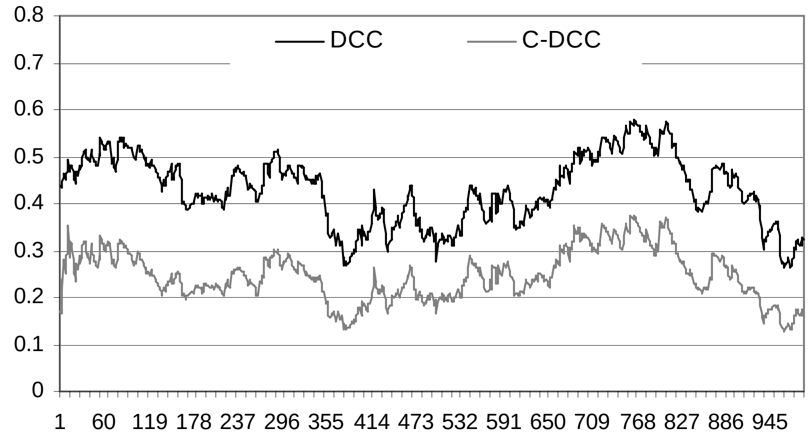

Fig. 2. Comparison of the conditional correlations for WIG-con and WIG-IT

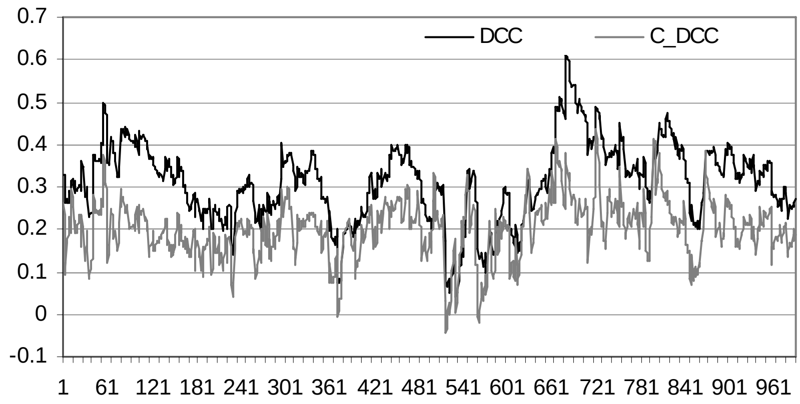

Fig. 3. Comparison of the conditional correlations for WIG-con and WIG-food

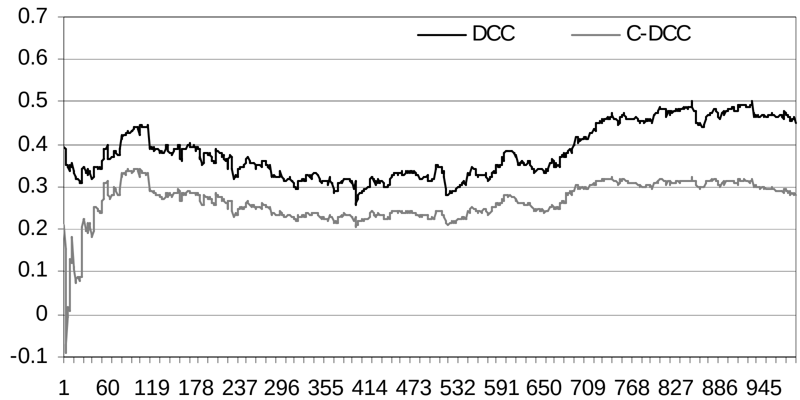

Fig. 4. Comparison of the conditional correlations for WIG-IT and WIG-food

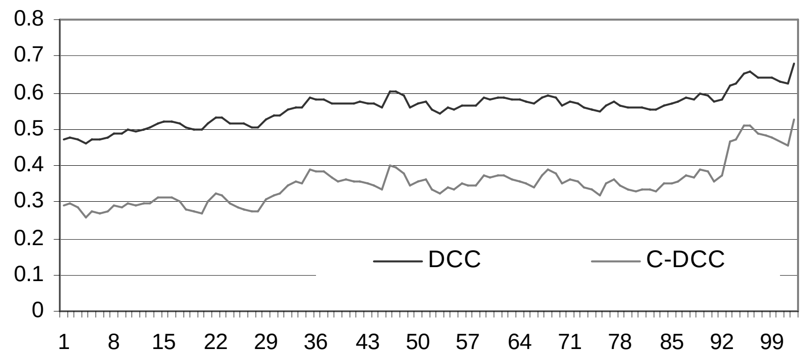

Fig. 5. Forecasts of the conditional correlations for EUR/PLN and USD/PLN

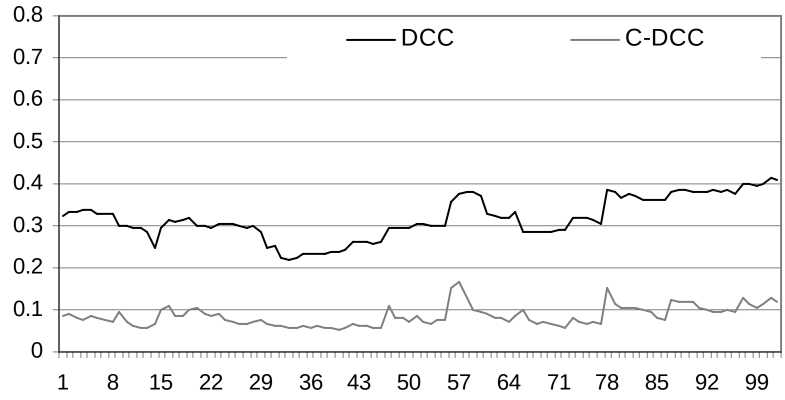

Fig. 6. Forecasts of the conditional correlations for WIG-con and WIG-IT

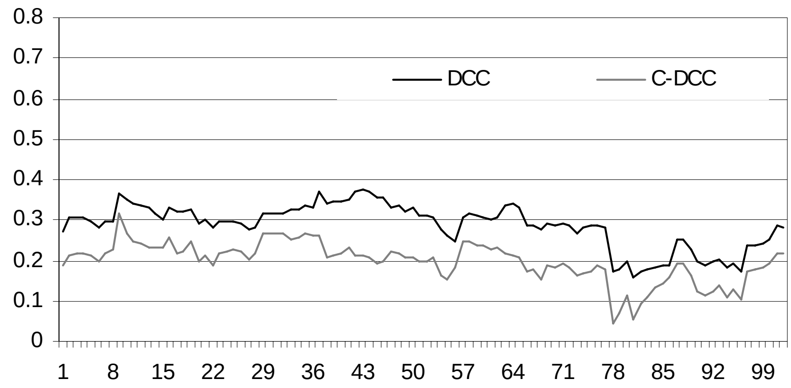

Fig. 7. Forecasts of the conditional correlations for WIG-con and WIG-food

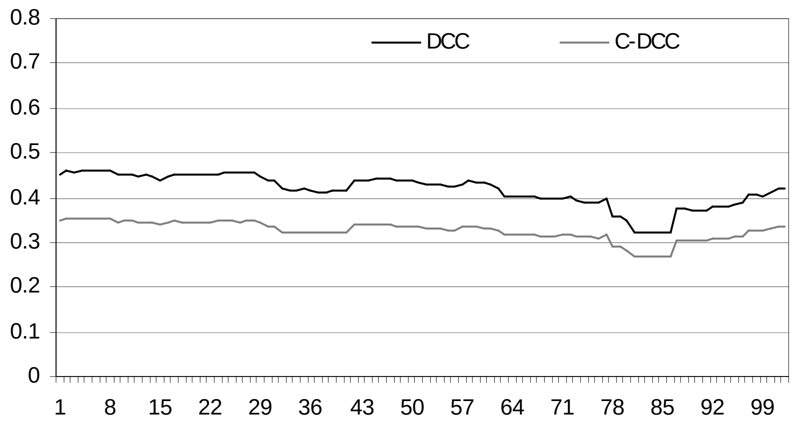

Fig. 8. Forecasts of the conditional correlations for WIG-IT and WIG-food

6. Conclusions

In this paper we model conditional dependence for pairs of selected Polish financial returns using the C-DCC model. The model belongs to the class C-MGARCH that includes the conventional multivariate GARCH models and by incorporating copula methodology allows to model the conditional correlations and the remaining dependence separately and simultaneously without limitation to elliptically distributed errors. We find that the copula-based models applied by us produce much lower estimates of the conditional correlation than the standard DCC models do. In our opinion, a possible explanation of this phenomenon, apart from the two-stage estimation imperfections, is that the copula improved model structure managed to identify and separate the dependence that was mistakenly qualified as linear conditional correlation by the simpler model. Our results regarding the application of C-DCC and DCC models to forecasting the matrices of return cross products are not very impressive though the simpler model has gone better. We suppose that the results in this subject could look more interesting if we would compare the forecasts with the so-called daily realized covariance (Andersen et al. 2005) matrices calculated on the basis of intraday quotations.

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