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Maria Szmuksta-Zawadzka, Jan Zawadzki
Technical University of Szczecin, Agricultural University of Szczecin

Forecasting on the Basis of "Parsimonious" Hierarchical Models

Short time series are quite often used to forecast economic phenomena. One of their advantages is that they do not require the description of structural changes in time. There is also every indication that the updated or most recent information is central to the forecasting. Classic time series models used for this purpose, i.e. models with dummy variables or trigonometric polynomial would require the estimation of m-1 parameters for the description of constant seasonality. Trend parameters should also be taken into consideration in this respect as well. The above would lead to the substantial decrease in number of degrees of freedom.

Authors of this paper suggest using "parsimonious" regular hierarchical models for the forecasting based on short time series. Let's focus on full regular hierarchical models before studying the "parsimonious" ones. Their number is a sum of permutations and permutations with repetitions of divisors p_i (i = 1, 2,…, s) of fluctuation cycle length m.
The divisors shall meet two conditions at a time:

$$2 \le p_i \le \frac{m}{2}, \tag{1}$$

$$\prod_i p_i = m. \tag{2}$$

An example of the analytic representation of two-level hierarchical model is as follows:

$$Y_t = \beta_1 t + \beta_0 + \sum_{s=1}^{p_1} d_{0s} Q_{st} + \sum_{r=1}^{p_2} d_{0rs} Q_{rst} + U_t, \tag{3}$$

where the following conditions are satisfied $\sum_{s=1}^{p_1} d_{0s} = \sum_{r=1}^{p_2} d_{0rs} = 0.$

A form of the matrix of explanatory variables (made of components corresponding to certain degrees of hierarchy) is shown as an example of the H34 model for monthly data.

The first level of hierarchy represents the number of a quarter (K1, K2, K3), the other one the number of a month of the quarter (MK1, MK2).

Matrix X43 is presented below:

$$X_{43} = \begin{array}{c} \begin{matrix} \text{K1} & \text{K2} & \text{K3} & \text{MK1} & \text{MK2} \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 & 0 \\ -1 & -1 & -1 & 0 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \end{array} \qquad (4)$$

The number of components for certain stages that is smaller by one results from the summation to zero of parameters d_{0k}, d_{0sk}. One of advantages of the hierarchical models is the smaller number of parameters required to describe the shaping of seasonal fluctuations. The maximum number of parameters describing constant seasonal fluctuations amounts to the half of cycle fluctuation length *m*. They exhibit a very important property – components for the complete data belonging to different hierarchy levels are not correlated. Thus, it is possible to measure a share of each degree in explaining the seasonal variance similarly to models with trigonometric polynomial.

The subject of this paper, as mentioned before, is the application of "parsimonious" regular hierarchic models for the description and forecasting of economic variables with seasonal fluctuations.

A hierarchical "parsimonious" model is the one in which the number of statistically significant parameters are fewer than the sum of divisors of cycle fluctuation length *m* deducted by the number of hierarchy levels.

The result of introducing statistically significant components belonging to certain tiers (hierarchy degrees) into the model only is the increase in the degrees of freedom.

The selection of statistically significant components can be done two ways. Firstly, statistically insignificant components can be eliminated from a preliminarily estimated equation containing all the components. It caries risk of eliminating the components that are close to the significance limit. Therefore the step-by-step elimination of components starting from the ones of the lowest absolute values $|t_j|$ can be proposed.

Secondly, common methods of selecting explanatory variables can be used, whereas "explanatory" variables in hierarchical models are columns corresponding to certain components. The forward stepwise variable selection method covered by the STATGRAPHICS is worth recommending. This particular method makes it possible to track the 'profitability' of introducing successive component.

The development of "parsimonious" model has to consider the fact that the decrease in the number of estimated parameters would result in the loss of some *quantum* of information. The size of the 'loss' can be measured by differences in the explanation of seasonal variance by classic and hierarchical models.

Models producing the lowest 'loss' should only be selected for forecasting purposes.

The above considerations would be illustrated by two examples of variables that differ not only in the length of fluctuation cycles, but in harmonic structure as well.

The first of the considered variables is the production of electric power by months in years 2000-2004, the year 2004 is a period of empirical verification of forecasts.

Table 1 lists shares of certain sine and cosine components (and of harmonics, too) in the explanation of the seasonal variance for forecasted variable.

Table 1. Harmonic structure of electric energy production (%)

j	Fluctuations period	Sine comp.	%	Cosine comp.	%	Total (harmonics)
1	12 months	S1	5.12	C1	89.01	94.13
2	6 months	S2	0.15	C2	0.31	0.45
3	4 months	S3	0.29	C3	0.37	0.66
4	3 months	S4	2.15	C4	0.24	2.39
5	2.4 months	S5	1.67	C5	0.48	2.14
6	2 months	S6	0.00	C6	0.23	0.23
Total			9.37		90.63	100.00

Source: authors' own calculations.

The variance would be a point of reference for measurements of shares in the variation of certain levels and whole hierarchic models as well.
The information given in the table shows that the first harmonic (12-month period) has the largest share that amounts to 94.13%. The fourth and fifth harmonics have the share over 2%, (fluctuation periods of 3 months and 2.4 months respectively).
Table 2 gives shares of certain degrees (hierarchy levels) and entire models in explaining the seasonal variance of the forecast variable.

Table 2. Shares of certain levels of hierarchical models in explaining the seasonal variance of electric power production (%)

	Level I	Level II	Level III	Total
H26	0.05	3.04		3.09
H34	65.00	0.77		65.77
H43	84.35	2.47		86.82
H62	82.03	0.29		82.32
H232	0.05	2.58	0.06	2.69
H223	0.05	0.48	0.29	0.82
H322	65.00	0.48	0.29	65.77

Source: authors' own calculations.

The above data shows that the share of certain levels in explaining the seasonal variance varies significantly. It ranges from 0.05% for the first level of model H223 and second level of models H26 and H232 through 84.35% for the first level of model H43.
Consequently, the explanation degrees of the total seasonal variation are differentiated. Two groups of hierarchical models can be identified in this respect. The first group comprises models H223, H232 and H26 with shares below 3.10%. The other four models have shares amounting to 65.77% at least. The largest share of explaining the seasonal variance is exhibited by model H43 (86.82%).
The synthetic presentation of results of selecting significant components and measures of *ex post* forecast accuracy on the basis of "parsimonious" and full models will follow.

Columns three through five, table 3, show the information on the number of statistically significant components for certain levels (hierarchy degrees) in "parsimonious" models.

The components have been selected by means of forward stepwise selection procedure. Levels, for which all the components have been found statistically significant are printed in boldface. Such identification has been revealed for the first level in models O34, O43 and O322.

Table 3. Numbers of estimated parameters in "parsimonious" hierarchical models of variable EN and ex post forecast relative errors (%)

	Model	„Parsimonious" models			Total	Full hier. Model	Forecast errors	
		Level I	Level II	Level III			Parsim. model	Full model
1	O26	0	0	-	0	6	8.38	8.39
2	O34	**2**	0	-	2	5	5.07	3.86
3	O43	**3**	1	-	4	5	3.91	3.91
4	O62	3	0	-	3	6	4.61	3.87
5	O223	0	0	0	0	4	8.38	8.37
6	O232	0	0	0	0	4	8.38	8.37
7	O322	**2**	0	0	2	4	5.07	5.09
8	Q				11	11	3.09	3.40
9	SC				4	11	3.40	3.40

Source: authors' own calculations.

Trend parameters have only been found statistically significant for three models that belong to the first group (shares in explaining the seasonal variation below 3.10%), that is why zero-elements are found inline corresponding to certain hierarchy degrees. One parameter for the second level of model O43 out of the ones that appear by components belonging to the second and third levels has been found significant.

Column six includes numbers of estimated parameters in "parsimonious" models that are sums of elements contained in three preceding columns. They range from null through four.

The two last lines give numbers of statistically significant parameters in classic models. All parameters that are LSM estimates of seasonally components have been statistically significant in a model with dummy variables (Q). Four harmonic components, namely S1, S4, S5 and C1 have proved significant in a model with the trigonometric polynomial (OSC).

The next column includes numbers of estimated parameters describing the seasonality in full models (hierarchical and classic ones). They are usually higher in the hierarchical models compared to the "parsimonious" ones.

The two last columns give relative errors of *ex post* forecasts obtained on the basis of "parsimonious" and full models. In case of all hierarchical models belonging to the first group the value of forecast errors has exceeded 8% and it has been almost equal for the full and parsimonious models.

Such a result is not surprising because all parameters describing the seasonality in the considered models have been statistically insignificant.

Model O43 has been found the best among the ones belonging to the second group because only one component belonging to the second level (MK1) has been insignificant. In case of model O62 the forecast error has been about 0.7% higher. Estimating three parameters in this model instead of six parameters in the full models has not affected the *ex post* forecast accuracy. The next accurate models have been models O34 and O322 in which only parameters belonging to the first level have been significant. In this case, estimating the smaller number of parameters has not affected the forecast accuracy. Relative forecast errors calculated on this basis have not exceeded 5%.

The comparative analysis of the explanation degree of the seasonal variation and forecast accuracy reveals that the higher the explanation degree the better is the forecast accuracy. Relative forecast errors for classic models have been about 3.4%, in case of parsimonious model with trigonometric polynomial (OSC) only the relative forecast errors have been lower and amounted to 3.09%. The comparison of the forecast accuracy obtained n the basis of classic and hierarchical models suggests that model O43 can only be used for forecasting the production of electric power.

Modeling and forecasting results for the best hierarchical models (O43 and H43) and classic models (Q and OSC) are depicted in Figs. 1 and 2.

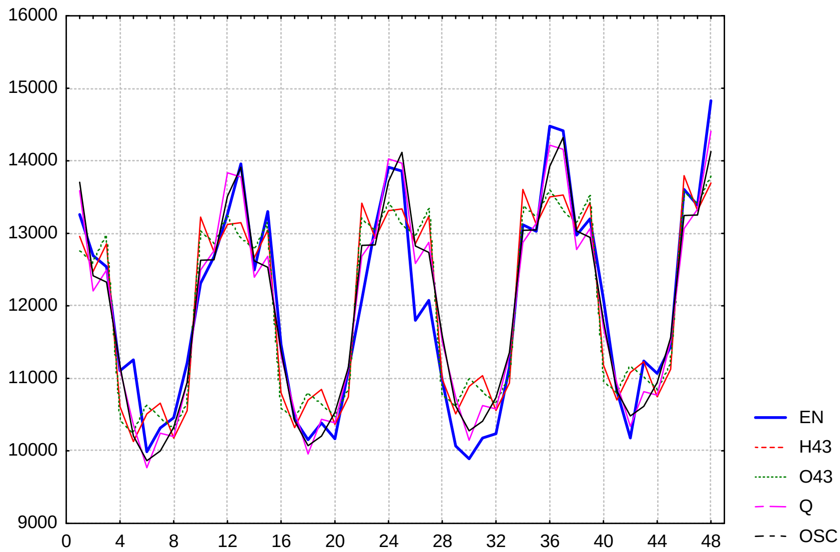

Fig.1 Actual and fitted values of electric power production
Source: authors' elaboration.

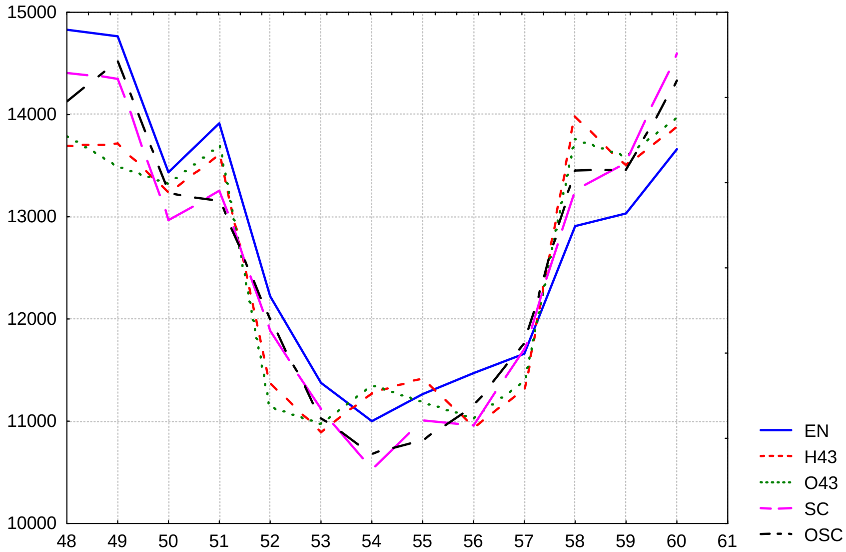

Fig.2 Ex *post* forecasts of electric power production (EN)
Source: authors' elaboration.

Another variable subjected to predictive modeling and forecasting was a decade balance of *a'vista* deposits (DEP). The variable has exhibited 36-decade fluctuations in an annual cycle. The estimation period has covered 108 decades, and the further 9 decades have formed a period of empirical verification of forecasts. Due to the large size of a table including shares of all 35 harmonics, a list of the harmonics of seasonal variation explanation share above 1% is presented. The relevant numerical data is shown in the table 4.

Table 4. Shares of selected harmonic components and harmonics in explaining the seasonal variation of decade balance of *a'vista* deposits (%)

j	Fluctuations period	Sine comp.	%	Cosine comp.	%	Total (harmonic)
1	36 dec.	S1	26.25	C1	4.74	30.99
2	18 dec.	S2	0.78	C2	2.96	3.74
11	3.27 dec.	S11	0.68	C11	1.15	1.83
12	3 dec.	S12	52.07	C12	5.82	57.89
		rest	2.84	rest	2.71	5.55
Total			82.62		17.38	100.00

Source: authors' own calculations.

The information given in the above table shows that the variable concerned has different characteristics of the harmonic structure from the production of electric power. The twelfth harmonic (3-decade cycle) has proved to be the largest as far as the share in explaining the seasonal variance is concerned –

57.89%. The shares of sine and cosine components are 52.07% and 5.82% respectivey. The next largest one is the first harmonic (36-decade cycle) with the share of 30.99%.The second harmonic (18-decade cycle) has the share of approx. 3%. The eleventh harmonic has been the last one with the share exceeding 1% (a cycle slightly above a month 36/11).

The total share of the four harmonics concerned in explaining the seasonal variance is 93.67%, whereas the share of the remaining fourteen is slightly above 6%.

There are 25 hierarchical models for decade data, incl. of 7 – two levels models, 12 – three levels models and 6 –four levels models. Shares of certain degrees (levels) in explaining the seasonal variation for all 25 equations are given in table 5.

The last column lists percentage factors of explaining the seasonal variance by hierarchical models that are sums of numbers found in four preceding columns.

The analysis of the numerical data included shows that two sets of hierarchical models that differ in a degree of explaining the seasonal variance can be identified. The first group comprises 15 equations of last divisors as follows: 3, 6, 9, 12 and 18. They are characterized by percentage factors of explaining the seasonal variation above 80%. Model H66 exhibits the largest measure of 98.77%, and the smallest one – model H2233 (80.82%).

The second group includes 10 models of divisor 2 or 4 with percentage factors ranging from 26.30% (model H3322) through 49.34% (model H182). Columns two through five, table 5, lists percentage factors for certain levels in explaining the total seasonal variations (2-, 3- and 4-level).

The single level shares range from 0.18% for the lowest levels in models with the last digit of two through 64.59% in model H312. The latter is just by 1.21 per cent smaller than the second level in model H218 and by 2.2 per cent for the same level in model H312. The shares range from 57.89% through 59.2% for the lowest levels in the further 11 models with the last divisor of 3, 6 or 9.

This information may be the basis for a conclusion that the degree of explanation of the seasonal variance in the estimation of predictive property of hierarchical models seems to be a better criterion than the estimation of a standard deviation of the random factor.

It may be therefore expected that the accuracy of the *ex post* forecast accuracy obtained on the basis of equations classified to the first group is higher than the accuracy of forecasts obtained on the basis of predictors classified to the second group.

Table 5. Shares of certain levels of hierarchical models in explaining the seasonal variance of variable DEP

Model	Level I	Level II	Level III	Level IV	Total
H218	22.30	63.24	–	–	85.54
H312	26.61	64.95	–	–	94.56
H49	27.09	59.27	–	–	96.36
H66	36.02	62.75	–	–	98.75
H94	42.65	0.48	–	–	43.13
H123	34.74	57.89	–	–	52.63
H182	49.16	0.18	–	–	49.34
H922	35.73	0.00	0.18	–	35.91
H229	22.30	0.20	59.27	–	81.77
H292	22.30	16.67	0.18	–	39.15
H632	33.37	0.00	97.89	–	46.56
H623	33.37	0.00	57.89	–	91.26
H334	24.35	1.78	0.40	–	26.53
H343	24.35	0.82	57.89	–	83.06
H433	27.09	0.42	57.89	–	85.40
H326	24.35	0.55	58.13	–	83.02
H362	24.35	13.93	0.18	–	38.45
H236	22.30	3.52	58.13	–	83.95
H623	22.30	57.84	3.73	–	83.92
H2233	22.30	0.20	0.42	57.89	80.82
H2332	22.30	3.52	13.01	0.18	39.02
H2323	22.30	3.52	0.00	57.89	83.71
H3322	24.35	1.78	0.00	0.18	26.30
H3232	24.35	0.55	13.01	0.18	38.09
H3223	24.35	0.55	0.00	57.89	82.78

Source: authors own calculations.

The selection of significant component in "parsimonious" models has been carried out, as previously, by the forward stepwise selection procedure. Table 6 lists the information on the number of estimated parameters for certain levels of "parsimonious" models (columns second through fifth). Levels, for which all the components have been found statistically significant are printed in boldface.

Table 6. Numbers of estimated parameters in "parsimonious" hierarchical models of variable DEP and relative errors of ex post forecast (%)

	Model	"Parsimonious" models				Total	Full hier. model	Forecast errors	
		Level I	Level II	Level III	Level IV			Pars. model	Full model
1	O218	**1**	11	–	–	12	18	3.09	2.86
2	O312	**2**	9	–	–	11	13	3.98	3.86
3	O49	**3**	8	–	–	11	11	3.61	3.61
4	O66	4	5	–	–	9	10	3.51	3.32
5	O94	4	0	–	–	4	11	4.42	4.64
6	O123	7	**2**	**–**	–	9	13	3.55	3.36
7	O182	7	0	–	–	7	18	4.66	4.55
8	O229	**1**	0	4	–	5	9	3.44	3.16
9	O236	**1**	0	5	–	6	8	3.64	3.15
10	O263	**1**	0	**2**	–	3	8	3.67	3.07
11	O292	**1**	3	0	–	4	10	4.44	4.38
12	O326	**2**	0	2	–	4	8	3.82	3.82
13	O362	**2**	1	0	–	3	8	5.02	4.75
14	O334	**2**	0	0	–	2	7	4.88	4.87
15	O343	**2**	0	**2**	**–**	4	7	3.86	3.86
16	O433	**3**	0	2	–	5	7	3.85	3.58
17	O623	4	0	**2**	–	6	8	3.54	3.35
18	O632	4	1	0	–	5	8	4.59	4.56
19	O922	4	0	0	–	4	10	4.42	4.53
20	O2233	**1**	0	0	**2**	3	6	3.68	3.28
21	O2323	**1**	0	0	**2**	3	6	3.68	3.06
22	O2332	**1**	1	1	0	3	6	4.65	4.41
23	O3223	**2**	0	0	**2**	4	6	3.86	3.85
24	O3232	**2**	0	1	0	3	6	4.73	4.71
25	O3322	**2**	0	0	0	2	6	4.88	4.81
	Q					8	35	3.62	3.53
	SC					5	35	3.59	3.53

Source: authors ‘own calculations.

There are all models for the first level for which first divisors are greater than four. All components have been statistically significant for the second level in models O66 and O123. This has been found in models of the last divisor of three for the last two levels. Column six includes the information on the total number of estimated parameters in the "parsimonious" models. Column seventh lists the numbers of parameters in the full hierarchical models The comparison of the numbers found in these columns reveals that the number of estimated parameters in the "parsimonious" models has been greater at least by one than in the full models. The greatest difference for predictors belonging to the first group has been found in model O218 (six parameters), and in model O94 (seven parameters) within the second group.

In the classic model with dummy variables there have been 9 statistically significant parameters out of 35 parameters d_{0k} (significance level $\alpha = 0.12$). The model with a trigonometric polynomial has exhibited five statistically significant components (S_1, C_1, C_2, C_{12}, S_{12}).

The last two columns include estimates of relative errors of the *ex post* forecast for "parsimonious" and full models respectively.

The information given in the columns reveals that the explanation degree of the seasonal variation has affected the accuracy of the *ex post* forecasts. They have ranged from 2.86% through 3.86% for predictors classified to the first group (share in explaining the seasonal variance above 80%). In case of the second group they have ranged from 4.38 through 4.81 per cent.

Such relations occur in case of the "parsimonious" models as well. The estimation of the smaller number of parameters has slightly influenced the increase in estimates of relative errors of the forecasts – mainly by approx. 0.2 per cent.

The last two lines give the information about the number of statistically significant parameters of time series and the forecast errors. There have been 8 statistically significant parameters out of 35 in models with dummy variables (QI).

The model with a trigonometric polynomial has had 5 statistically significant parameters at the following harmonic components: S1, S12, C1, C2 and C12. (OSC) The relative errors obtained on the basis of the equations concerned are greater than the errors of forecasts obtained on the basis of the "parsimonious" hierarchical models of the first group. The errors are greater by less than 0.1 per cent for the further five hierarchical models. Forecasting results for the four best hierarchical predictors and classic ones with statistically significant parameters are shown in Fig. 3.

The graphical presentation has been omitted since the curve for 108 observations would be absolutely illegible.

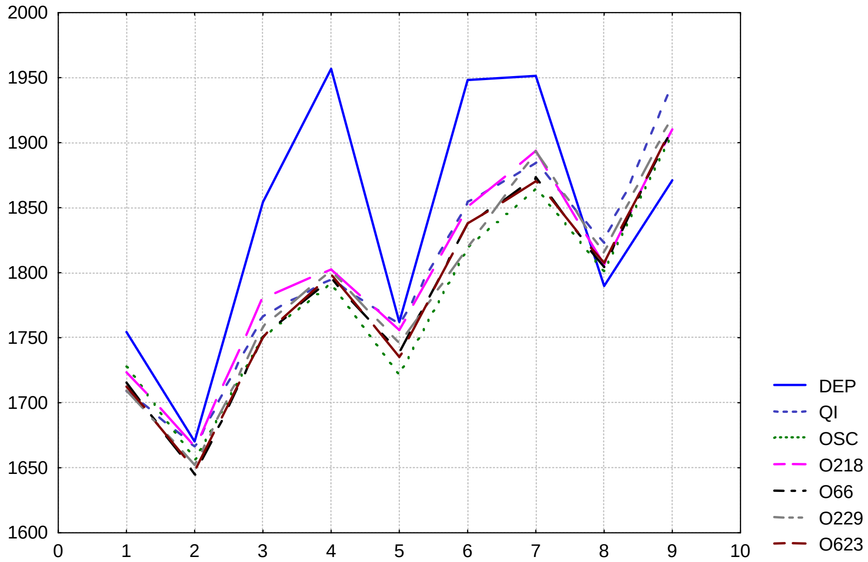

Fig. 3 Forecasts and deposit balance status
Source: authors elaboration.

This study reveals that "parsimonious" hierarchical models can be successfully used for the forecasting of economic variables that exhibit the seasonal fluctuations of 12-month and 36-decade cycles

References

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