DYNAMIC ECONOMETRIC MODELS
Vol. 7 – Nicolaus Copernicus University – Toruń – 2006

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Interest Rate Modeling and Tools of Financial Econometrics

1. Financial Econometrics Models

One of the most important areas of finance, where a considerable development has been recently observed, is **financial econometrics**. In our opinion, the relatively simple way of describing financial econometrics can be given as follows:

Financial econometrics models are derived for data given as financial time series and applied either in order to verify some underlying hypotheses formulated by financial theory or to identify some properties existing in financial data.

It is worth to mention that there are two general types of financial econometrics models:
- models of dynamic econometrics, adapted and applied for financial data;
- models developed solely for financial data.

It is commonly believed that the birth of financial econometrics is connected with the derivation of ARCH model (Engle (1982)). Since that time the enormous amount of models was developed and used in practice.
The most commonly used groups of financial econometrics models can be divided into:
- univariate models (ARIMA–GARCH) and multivariate models (VARIMA–MGARCH);
- structural models (VAR) and reduced type models (ARIMA);
- price models (ARIMA) and volatility models (GARCH);

- linear and non-linear models;
- deterministic volatility (GARCH) and stochastic volatility (SV) models.

Key factors being driving forces of financial econometrics are:

- technological progress, particularly in the area of computer technology, which makes practical implementation of rather sophisticated econometric tools relatively easy;
- data availability, particularly in the form of time series of stock prices, interest rates and exchange rates, which makes easy to verify the proposed models;
- development of econometric tools (for example estimation and testing methods), which allows for fine tuning the methodology;
- development of financial theory, which delivers tools (including possible hypotheses) to be verified in practice;
- practical problems to be solved by new models.

One of the main tendencies observed in the applied research is the growing integration of financial economics, financial mathematics and financial econometrics through empirical financial problems. One can say that financial econometrics verifies hypotheses of financial economics and uses some tools developed by financial mathematics.

Although proved considerable progress, financial econometrics still faces a lot of challenges. Among the most important ones are:

- modeling multivariate financial time series, where the crucial point is the estimation of volatility and dependence (correlation), both notions particularly important in risk management;
- ultra high frequency data modeling; where data is given for each particular transaction rather than for each time moment;
- modeling time series under assumptions of random error having distribution other than normal distribution (univariate or multivariate);
- implementation of models derived by financial theory based on continuous time stochastic processes with the use of data given in discrete time units.

One of the most challenging applied tasks faced by financial econometric methods is modeling interest rates. This paper gives some introductory and synthetic remarks as far as the modeling interest rates is concerned.

2. Interest Rate Modeling – Introductory Remarks

Interest rates are – besides exchange rates, stock prices and commodity prices – main time series modeled by financial econometrics. However, from the point of view of modeling, there are several differences between interest rates and the other three types of financial time series, namely:

1. There are many possible types of interest rates that can be of modeling interest, including spot rates, forward rates, yields to maturity. Usually, they are taken as risk-free rates. In addition, one can consider risky rates, they refer to debt instruments (like bonds) issued by corporations of different rating categories (AA, BBB, etc.).
2. Instead of single interest rate for each time moment, one has to consider the whole **term structure of interest rates.** Particularly for each time moment we have theoretically infinite number of interest rates, each one referring to possible period, for example one day, one month, three months, ten years, etc. Term structure of interest rates, therefore, is a function that for each considered period gives the value of respective interest rate. Due to the graphical presentation of this relation, the other used name used for the term structure of interest rates is **yield curve**. In practice the number of considered interest rates for each time moment is of course finite, however this number is large – more than dozen. In addition, these different rates are strongly related to each other, which means that interest rate modeling is actually multivariate task.
3. There is developed financial theory of interest rates, which means that econometric models of interest rates should be consistent with this theory.
4. Some theoretical rates are not observable, which means that they should be extracted from existing data.

It is worth to mention that models of interest rates have very wide application in financial practice. Among the most important applications are:

1. Monetary policy.
2. Interest rate derivative instruments pricing.
1. Interest rate risk management.
2. Investment strategies with the use of debt instruments.
3. Corporate finance.
4. Credit derivative instrument pricing.
5. Credit risk management.

Financial econometrics models developed for stocks prices and exchange rates are prices models (for example ARIMA type models) and volatility models (for example GARCH type models). Similar feature is observed for interest rates. However, here in addition one can consider risk-free and risky rates. In our opinion, one can distinguish four possible types of interest modeling. They are ordered here with respect to availability of models and applied research, namely:

1. Modeling term structure of interest rates – these are price models developed for risk-free rates.

2. Modeling term structure of interest rates volatility – these are volatility models developed for risk-free rates.
3. Modeling term structure of credit spreads – these are price models developed for credit spreads (credit spread is the difference between risky rate and risk-free rate).
4. Modeling term structure of credit spread volatility – these are volatility models developed for credit spreads (by no doubt, these models at the very early stage of development).

3. Interest Rate Models

Any type of econometric model should be well-suited to the facts observed in real world. The same statement refers to interest rate models. The basic facts observed in the financial markets are:

- interest rates are mean-reverting, which means that there is a level in interest rates to which these rates approach in long term;
- the changes of interest rates corresponding to different periods (for example one month rate and one year rate) are not perfectly correlated;
- volatility of short term rates is higher than volatility of long term rates;
- most changes of interest rates can be explained by three factors:

a) parallel movement: all rates changes by the same amount – up or down;
b) slope changes: short term rates change by more – up or down – or less than medium term rates and the same is true for medium term rates comparing to long term rates; this means change of the slope of yield curve;
c) curvature changes – medium term rates change more – up or down – or less than short term and long term rates.

There are many interest models proposed in the literature and used in empirical research. They can be classified into two main groups:

- yield curve approximation methods;
- interest rate dynamics models.

Yield Curve Approximation Methods

The main feature of these methods is to use available data on interest rates to fit function to these data. The analysis of the available methods leads us to the following systematization:

- direct method, called bootstrapping method, where unobserved interest rates are derived from observed rates;
- spline method, where yield curve is composed of particular segments;

– yield curve econometric models – here the model is proposed, in which particular parameters can be given some interpretation related to the characteristics of yield curve.

Among the models proposed within the third group, we should mention Svensson model (Svensson (1994)). This model was used by some central banks to model term structure of interest rates. The extended version of Svensson model is given as:

$$r_m = \beta_0 + \beta_1 \exp\left(\frac{1-\exp(-m/\delta_1)}{m/\delta_1}\right) + \beta_2 \exp\left(\frac{1-\exp(-m/\delta_1)}{m/\delta_1} - \exp(m/\delta_1\right)$$
$$+ \beta_3 \exp\left(\frac{1-\exp(-m/\delta_2)}{m/\delta_2} - \exp(m/\delta_2)\right) + u.$$

Here:

r_m – interest rate corresponding to *m* years;

β_0 – parameter corresponding to long term interest rate;

β_1 – parameter corresponding to the parallel movement of interest rates;

β_2, β_3 – parameters corresponding to possible humps in the yield curve.

The detailed description of yield curve approximation methods is given by Martinelli, Priaulet and Priaulet (2003).

Interest Rate Dynamics Models

Contrary to yield curve approximation methods, these models try to explain the behavior of interest rates, rather than just to find the approximation for the data. By no doubt, there is variety of these models and the development of these models is still in progress.

We give here two important systematizations of interest rate dynamics models.

1. Classification with respect to the formal structure of the model:

A. Classical financial econometrics models

These are financial econometrics models applied to interest rates. Since interest rates corresponding to different periods are related, it is multivariate modeling.

B. Binomial tree models

Here the dynamics of interest rates is described by discrete time stochastic process, where in each period interest rate can move in two directions.

C. Stochastic differential equations models

Here the dynamics of interest rates is described by continuous time stochastic process, which can be represented as stochastic differential equation. One of the most often used structures for interest rates is the Ornstein-Uhlenbeck process, given as:

$$dr_t = \kappa(\theta - r_t)dt + \sigma dZ_t,$$

where:

κ – parameter, interpreted as the speed of mean reversion,

θ – parameter, interpreted as the long term interest rate,

σ – volatility parameter.

2. Classification with respect to the derivation of interest rates:

A. Endogenously derived models (factor models)

These are models where the interest rate dynamics is explained by few (one to three, as a rule) underlying factors, being "driving forces" of the dynamics. Many of these models can be described in the framework of stochastic differential equations.

B. Arbitrage models

These are models developed by financial theory, based on the same idea as classical option pricing models in Black-Scholes-Merton framework. This is non-arbitrage idea, in which price of financial instrument (here debt instrument based on interest rates) is determined in such a way that arbitrage strategy is not possible – arbitrage strategy is the strategy of no initial investment, no risk and positive inflow. Among the most well known and advocated arbitrage models of interest rate dynamics is the model proposed by Heath, Jarrow and Morton (1992).

Interest rate dynamics models are the most advanced models of interest rates. One of the models, which on one hand is rather sophisticated, on the other hand it leads to nice interpretation, is the model proposed by Chen (1996). This is three-factor model, where the factors are: short term rate, mean level short term rate and short term rate volatility. This model is given as:

$$dr_t = \kappa(\theta_t - r_t)dt + \sqrt{v_t}\sqrt{r_t}\,dZ_{1t},$$

$$dv_t = \gamma(\vartheta - v_t)dt + \xi\sqrt{v_t}\,dZ_{2t},$$

$$d\theta_t = \varphi(\lambda - \theta_t)dt + \eta\sqrt{\theta_t}\,dZ_{3t}.$$

As one can see, all three factors are modeled by Ornstein-Uhlenbeck process, so they are considered to be mean-reverting and in addition there is volatility part described by Wiener process.

One of the most important problems in the application of these models is the estimation of its parameters.

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