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Dynamic Hedging Portfolios – Application of Bivariate GARCH Models

1. Introduction

Futures markets were originally developed to meet the needs of farmers and merchants, who were exposed to risk related to price fluctuations. Nowadays hedging is also one of the main areas application of derivatives. Different kinds of derivatives can be used in hedging strategies. This article deals only with futures contracts. When an individual or company chooses to use futures contracts to hedge a risk, the objective is usually to take a position that neutralizes the risk as far as possible. In this paper the hedging strategy which minimizes the variance of the portfolio composed of the position taken in futures contracts and the position being hedged is analysed. Traditionally used methods of estimating hedge ratios ignore one of the main properties of financial time series, namely time-varying conditional variances and covariances of returns. Recently, in many papers the GARCH models have been used to estimate time-varying hedge ratios. This approach has been first proposed by Cecchetti, Cumby and Figlewski (1988), that applied a univariate ARCH model in estimating an optimal futures hedge with Treasury bonds, obtaining a significant reduction of the ex-post portfolio variance. Multivariate GARCH models were used to estimate hedge ratios among others by Baillie and Myers (1991), Myers (1991) for commodity futures, Kroner and Sultan (1993) for currency futures, Gagnon and Lypny (1995) for interest-rate futures, Park and Switzer (1995), Tong (1996) for stock index futures.

Existing empirical work on hedging performance has led to conflicting conclusions, however most of them indicate superior hedging performance of dynamic strategy based on the GARCH framework. In this article, firstly, performance of such strategy is analysed using futures contracts on WIG20 stock index quoted on the Warsaw Stock Exchange (WSE). Secondly, hedging performance of different specifications of GARCH models is compared. In most other studies only selected formulation is used. Thirdly, hedging effectiveness of an error correction model with a GARCH error structure is also investigated. Fourthly, methods used by practitioners of the financial market to forecast variances and covariances of returns are applied in estimating hedge ratios.

The article is laid out in four sections. Section 2 outlines the competing methods of estimating hedge ratios. In Section 3 hedging performance of presented strategies is investigated using WIG20 stock index and futures contracts on WIG 20 index quoted on the WSE. Section 4 concludes.

2. Estimation techniques for hedge ratios

The hedge ratio is the ratio of the size of position taken in futures contracts to the size of the exposure (see Hull, 1995). The return on a portfolio composed of cash and futures positions is given by:

$$x_{t+1} = s_{t+1} - b_t f_{t+1}, \tag{1}$$

where $s_{t+1} = \ln S_{t+1} - \ln S_t$, $f_{t+1} = \ln F_{t+1} - \ln F_t$, S_t is spot price, F_t – futures price, b_t – hedge ratio.

The optimal hedge ratio which minimizes the variance of the hedged portfolio return can be expressed as:

$$b_t = \frac{Cov(s_{t+1}, f_{t+1} | \psi_t)}{Var(f_{t+1} | \psi_t)}, \tag{2}$$

where ψ_t is the set of information available at time t.

It can be easily shown that formula (2) is also valid when investor's preferences can be described by a quadratic utility function:

$$E(U(x_{t+1}) | \psi_t) = E(x_{t+1} | \psi_t) - \xi Var(x_{t+1} | \psi_t), \tag{3}$$

with the assumption that log futures price is a martingale, i.e.
$E(\ln F_{t+1} | \psi_t) = \ln F_t$.

If the conditional covariance matrix is time-invariant, then the constant optimal hedge ratio may be obtained constructing a linear regression:

$$s_t = \alpha + \beta f_t + \varepsilon_t, \tag{4}$$

The optimal hedge ratio, b_t, is equal to the estimated OLS coefficient β. This approach is called OLS hedging.

It has been shown by numerous studies that the data do not support the assumption that the conditional covariance matrix of returns is constant over time (see for instance Bollerslev, Chou and Kroner, 1992, Bollerslev, Engle and Nelson, 1994, Fiszeder, 2003). Therefore, we follow recent literature by employing a multivariate GARCH model, which allows the conditional variances and covariances used as inputs to the hedge ratio to be time-varying. Given the bivariate GARCH model of spot and futures prices, the time-varying hedge ratio can be expressed as:

$$b_t = \frac{h_{sf,\, t+1}}{h_{f,\, t+1}}, \tag{5}$$

where $h_{sf,\, t+1}$ is forecast from the GARCH model of conditional covariance between returns of spot and futures prices and $h_{f,\, t+1}$ is a forecast from the GARCH model for conditional variance of return on future position at time $t+1$.

Specification of multivariate GARCH model which is one of the most frequently used for several time series is the BEKK representation:

$$H_t = CC' + \sum_{i=1}^{q} D_i\, \varepsilon_{t-i}\varepsilon'_{t-i}\, D_i' + \sum_{j=1}^{p} E_j H_{t-j} E_j', \tag{6}$$

where H_t is the $N \times N$ symmetric conditional covariance matrix, ε_t is the $N \times 1$ vector of errors, C is an upper triangular parameter matrix, D_i and E_j are the $N \times N$ parameter matrices. The advantage of this formulation is a positive definiteness of H_t and time-varying conditional correlations between the returns.

Different formulations of mean equations are analysed. The simplest specification assumes that spot and futures returns are constant:

$$s_t = \alpha_{s0} + \varepsilon_{s,\, t}, \tag{7}$$

$$f_t = \alpha_{f0} + \varepsilon_{f,\, t}, \tag{8}$$

$$\varepsilon_t \big| \psi_t \sim N(0, H_t), \tag{9}$$

where $\varepsilon_t = (\varepsilon_{s,\, t}, \varepsilon_{f,\, t})'$ and H_t is given by (6).

In order to capture short term relations between spot and future returns the VAR–BEKK model is considered:

$$s_t = \alpha_{s0} + \sum_{i=1}^{k} \alpha_{si} s_{t-i} + \sum_{i=1}^{k} \beta_{si} f_{t-i} + \varepsilon_{s,\, t}, \tag{10}$$

$$f_t = \alpha_{f0} + \sum_{i=1}^{k} \alpha_{fi} s_{t-i} + \sum_{i=1}^{k} \beta_{fi} f_{t-i} + \varepsilon_{f,\,t}\,, \tag{11}$$

$$\varepsilon_t \big| \psi_t \sim N(0, H_t)\,, \tag{12}$$

where H_t is given by (6).

In the absence of transaction costs, market microstructure effects, or other impediments to their free operation, the efficient markets hypothesis and the absence of arbitrage opportunities imply that the spot and corresponding futures markets react contemporaneously and identically to new information. There has been some debate in the literature as to whether this implies that the two markets must be cointegrated. The conditional mean equations of the model employed in this article represent a bivariate Vector Error Correction Mechanism, which may be written as:

$$s_t = \alpha_{s0} + \sum_{i=1}^{k} \alpha_{si} s_{t-i} + \sum_{i=1}^{k} \beta_{si} f_{t-i} + \gamma_s (S_{t-1} - \gamma F_{t-1}) + \varepsilon_{s,\,t}\,, \tag{13}$$

$$f_t = \alpha_{f0} + \sum_{i=1}^{k} \alpha_{fi} s_{t-i} + \sum_{i=1}^{k} \beta_{fi} f_{t-i} + \gamma_f (S_{t-1} - \gamma F_{t-1}) + \varepsilon_{f,\,t}\,, \tag{14}$$

$$\varepsilon_t \big| \psi_t \sim N(0, H_t)\,, \tag{15}$$

where H_t is given by (6). That is, we adopt the cointegration assumption.

Although the unconditional distribution for ε_t in the GARCH model with conditional normal errors as given by (9), (12) and (15) has fatter tails than the normal distribution, for many financial time series it does not adequately account for leptokurtosis. That is why bivariate Student-*t* distribution with unknown degrees of freedom is additionally applied instead of (9). Other specifications of the distribution of ε_t are possible but the GARCH model with conditional Student-*t* distribution of ε_t is adequate to account for the fat-tailed properties of the data and is relatively easy to estimate.

The hedging performance of different specifications of multivariate GARCH models are also examined. One simple assumption that could be made to reduce the number of parameters is to specify that the (r,s) th element in H_t only depends on the corresponding (r,s) th element in $\varepsilon_{t-i}\varepsilon'_{t-i}$ and H_{t-j}. This assumption amounts to taking D_i and E_j to be diagonal matrices.

In the constant conditional correlations model of Bollerslev (1990), which is outside the BEKK class, the time-varying conditional covariances are parameterised to be proportional to the product of the corresponding conditional standard deviations:

$$H_t = D_t \Gamma D_t\,, \tag{16}$$

where D_t is the $N \times N$ diagonal matrix with the conditional standard deviations and Γ is the $N \times N$ matrix of time-invariant conditional correlations. If the conditional variances are all positive and the conditional correlation matrix Γ is positive definite, then the conditional covariance matrix H_t is guaranteed to be positive definite for all t.

Methods used by practitioners of financial market can also be applied to forecast variances and covariances of returns in (2) (see Zangari, 1996, Litterman and Winkelmann, 1998). Forecasts of variances and covariances of returns based on the moving average model are given by:

$$\sigma^2_{f,t+1} = \frac{1}{k-1} \sum_{i=t-k+1}^{t} (r_{f,i} - \bar{r}_f)^2 , \tag{17}$$

$$\sigma_{sf,t+1} = \frac{1}{k-1} \sum_{i=t-k+1}^{t} (r_{s,i} - \bar{r}_s)(r_{f,i} - \bar{r}_f) , \tag{18}$$

where $\sigma^2_{f,t+1}$ and $\sigma_{sf,t+1}$ are forecasts of variances and covariances of returns at date $t+1$ respectively, $r_{s,i}$ and $r_{f,i}$ are returns on spot and future positions at date i, $\bar{r} = \frac{1}{t}\sum_{i=1}^{t} r_i$.

Forecasts of variances and covariances of returns based on the exponential smoothing model can be formed as follows:

$$\sigma^2_{f,t+1} = (1-\alpha)\, r^2_{f,t} + \alpha\, \sigma^2_{f,t} , \tag{19}$$

$$\sigma_{sf,t+1} = (1-\alpha)\, r_{s,t} r_{f,t} + \alpha\, \sigma_{sf,t} , \tag{20}$$

where $0 < \alpha < 1$. The choices of the moving average estimation period (k) in (17)–(18) and value of smoothing parameter (α) in (19)–(20) are arbitrary and should be determined empirically.

3. Hedging performance for the WIG20 stock index

Hedging effectiveness of presented strategies is investigated using the WIG 20 stock index and futures contracts on the WIG 20 index quoted on the WSE. WIG20 index is a portfolio index of the 20 largest and most actively traded stocks. Futures contracts on WIG 20 are the most heavily traded futures contracts quoted on the WSE. The period investigated is January 04, 1999 to December 31, 2002 (996 daily returns). Hedging performance of selected strategies is evaluated for data from the year 2002 (249 daily returns). The most actively traded contract is always analysed. In order to avoid problems of scarce liquidity and other distortions, the rollover to the next contract is carried out one week before the last trading day. For each trading day in the year 2002 the portfolio composed of the WIG 20 stock index and futures contracts on the WIG 20 index are constructed.

Hedging strategies, which are considered in this article, differ only in the method of estimating hedge ratios in (2). For each strategy the mean and the standard deviation of the hedged portfolio returns in the year 2002 were calculated (ex post). All results are presented in Table 1. For comparison the same results are presented for the unhedged portfolio (hedge ratio equal to zero) and naive strategy (hedge ratio equal to one). As was to be expected hedging a portfolio leads to significant risk reduction measured by the standard deviation of portfolio returns. Conventional OLS hedging approach (equation (4)) provides further reduction of risk (see Table 1).

Other hedging strategies use the GARCH models to estimate time-varying hedge ratios (formula (5)). Firstly BEKK ($p = q = 1$) representation which guarantees positive definite of a conditional covariance matrix and is a relatively simple specification of a multivariate GARCH model is used. The influence of different formulations of mean equations on hedging performance is analysed. The following specifications are considered: constant spot and future returns (equations (7–9)), VAR model (equations (10–12)), vector error correction model (equations (13–15)). Capturing short term (VAR model) and long term (VECM model) relations between the spot and future contracts results in risk reduction of hedged portfolio, however decrease is negligible.

The GARCH model with conditionally normal errors results in a leptokurtic unconditional distribution. However, the degree of leptokurtosis induced by the time-varying conditional variance often does not capture all of the leptokurtosis present in high frequency financial data. That is why in formula (9) bivariate Student-*t* distribution is applied. However the introduction of *t*-Student distribution does not improve hedging effectiveness.

The BEKK model is a reasonable compromise between generality and parsimony of the GARCH model. This is achieved by complicated nonlinear restrictions imposed on the general VECH–GARCH(1, 1) model (see Osiewalski and Pipień (2002) for a detailed presentation and discussion). The influence of further simplifications of the multivariate GARCH model on hedging performance is also analysed. The BEKK model in which D_i and E_j in equation (6) are diagonal matrices and constant conditional correlations model (formula (16)) are applied. Simplification of the conditional covariance matrix leads to the decrease of mean standard deviation estimates of hedged portfolios. The highest reduction of risk is in the case forecasts from the constant conditional correlations are used to estimate a hedge ratio. This model is computationally simple and is relatively easy to ensure the positive definiteness of the conditional covariance matrix during the optimisation. However none of the hedging strategies based on the GARCH framework are more effective (in terms of risk reduction) than OLS hedging.

Table 1. Effectiveness of different hedging strategies

Estimation techniques for hedge ratios	Mean standard deviation	Risk reduction (in %)	Mean return $(\times 10^{-4})$
Unhedged (b = 0)	0.015328	0	-1.4054
Naive (b = 1)	0.006420	58.12	0.5172
OLS	0.006242	59.28	-0.1127
BEKK–N	0.006339	58.64	-0.5907
VAR–BEKK–N	0.006310	58.83	0.3803
VECM–BEKK–N	0.006300	58.90	0.1736
BEKK–t	0.006349	58.58	-0.5268
BEKK–N – diagonal matrices	0.006289	58.97	1.5051
Constant cond. correlations–N	0.006246	59.25	1.1532
Moving variance and covariances	**0.005758**	**62.43**	1.4885
Exponential smoothing	0.005858	61.78	**2.6580**

The mean return and mean standard deviation of the hedged portfolios are presented. Risk reduction is measured by mean standard deviation in comparison with unhedged portfolio.

Methods used by practitioners of the financial market can be applied to forecast variances and covariances of returns namely: the moving average model (equations (17)–(18)) and exponential smoothing model (equations (19)–(20)). The moving average estimation period and the value of smoothing parameter are chosen to minimize the mean standard deviation of the hedged portfolios in a pre-sample. The application of the moving average model and the exponential smoothing model results in the increase of hedging performance. The lowest estimate of the mean standard deviation of the hedged portfolio is in case forecasts of variances and covariances are from the moving average model. However it must be emphasized that such good performance of the moving average and the exponential smoothing models results from applied method of selection of the moving average estimation period and the value of smoothing parameter.

In Table 1 the mean return in the year 2002 is also presented. The realized return can be an additional criterion considered in the hedging strategy selection. Because the strategy which minimizes the variance of the hedged portfolio is considered in this paper, that is why the mean return is of secondary importance.

Conclusions

Different methods estimating hedge ratios are presented in the paper. Hedging performance of these strategies is investigated using the WIG20 stock index and futures contracts on WIG 20 index quoted on the WSE. In our empirical example, none of the strategies based on the GARCH framework are more effective than OLS hedging. Simplification of the model used to estimate hedge ratios does not significantly decrease effectiveness of applied strategies (it refers to both mean and variance equations). The lowest estimate of the mean standard deviation of the hedged portfolio is in case the forecasts of variances and covariances are from the moving average model. However, it must be remembered that the moving estimation period in the moving average model and the value of the smoothing parameter in the exponential smoothing model are chosen for each forecasting period separately to produce the best fit in the pre-sample. If the estimation period and value of parameter are chosen arbitrarily, it does not provide such good results. All results presented in this paper should be treated as introductory and further analysis for other portfolios are necessary.

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