DYNAMIC ECONOMETRIC MODELS
Vol. 6 – Nicolaus Copernicus University – Toruń – 2004

© Copyright by The Nicolaus Copernicus University Scientific Publishing House

Ewa Dziawgo
Nicolaus Copernicus University in Toruń

Approximation of Basket Call Option Price

1. Introduction

Exotic options are characterized by the income structure different from the structure of standard options. The exotic options are traded on the over-the counter derivatives market. Basket options are exotic, correlative options. Their pricing encompasses the correlations among the base assets behaviour.
The aim of the paper is to present the approximation methods of the basket call option price. This article presents pricing of the basket call option on USD, EUR and GBP. In the article a tree-month basket call options was priced. Based on the selected basket call options the analysis of the basket options price and the average price of the standard options was carried out.

2. Basket options and their pricing

The basket options are issued on more than one base instrument. Their price is directly related to the correlation coefficient of the base instruments. The base instrument of the basket options is the basket composed of a few instruments: shares, currencies or indexes. In 1993, in the field of basket options pricing, Gentle suggested the approximation of arithmetic average by the properly weighted geometric mean (Weron, Weron (1998)).

Let S_t^i (for $i = 1,2,...,k$) be the price processes of k underlying financial instruments (which are shares). The process S_t^i is described by the geometric Brownian motion:

$$dS_t^i = S_t^i\left(rdt + \sigma_i dB_t^i\right), \tag{1}$$

where:

B_t – one dimensional Brownian motion with respect to the martingale measure Q,
r – risk-free rate of interest,
$\sigma_i \in R$, is a volatility of share price[1].
The equation (1) shows, that the share price amounts to:

$$S_t^i = S_0 \exp\left[\sigma_i B_t^i + (r - 0{,}5\sigma_i^2)t\right]. \quad (2)$$

The payoff function of the basket call option end value, whose base instrument is composed of k shares, amounts to[2]:

$$f_T = \left(\sum_{i=1}^{k} w_i S_T^i - K\right)^+ , \quad (3)$$

where:
K – option exercise price,
$S_T{}^i$ – price of asset „i" at time T,
T – time to the option expiry date,
k – number of assets on the underlying option,
w_i – weight of asset „i", where: $w_i \geq 0$ and $\sum_{i=1}^{k} w_i = 1$.
The modified weights for the fixed $t \in (0;T)$ amount to:

$$\tilde{w}_i = \frac{w_i S_t^i}{\sum_{j=1}^{k} w_j S_t^j} = \frac{w_i F_{S^i}(t,T)}{\sum_{j=1}^{k} w_j F_{S^j}(t,T)}, \qquad \text{for } i = 1,2,...,k$$

where:
$F_{S^i}(t,T)$ – forward price in time t of the fixed i – on the asset with the expiry date T,
$F_{S^i}(t,T) = e^{r(T-t)} S_t^i$.
The payoff function of the basket option for the modified weights amounts to:

$$f_T = \left(\sum_{j=1}^{k} w_j F_{S^j}(t,T)\right)\left(\sum_{i=1}^{k} \tilde{w}_i \tilde{S}_T^i - \tilde{K}\right)^+ , \quad (4)$$

where:

© Copyright by The Nicolaus Copernicus University Scientific Publishing House

[1] The volatility of the shares is calculated as a standard deviation of the return rate on shares.

[2] $f_T = \left(\sum_{i=1}^{k} w_i S_T^i - K\right)^+ = \max\left[\sum_{i=1}^{k} w_i S_T^i - K; 0\right]$

$$\widetilde{S}_T^i = \frac{S_T^i}{F_{S^i}(t,T)} \qquad \text{for } i = 1,2, \ldots, k,$$

$$\widetilde{K} = \frac{K}{\sum_{j=1}^{k} w_j F_{S^j}(t,T)} = \frac{e^{-r(T-t)}K}{\sum_{j=1}^{k} w_j S_t^{\,j}}.$$

The arbitrage price of the basket call option C_t^B in time t amounts to:

$$\begin{aligned} C_t^B &= e^{-r(T-t)}\left(\sum_{j=1}^{k} w_j F_{S^j}(t,T)\right) E^Q\left(\left(\sum_{i=1}^{k} \widetilde{w}_i \widetilde{S}_T^i - \widetilde{K}\right)^+ \middle| F_t\right) = \\ &= \left(\sum_{j=1}^{k} w_j S_t^{\,j}\right) E^Q\left(\left(\sum_{i=1}^{k} \widetilde{w}_i \widetilde{S}_T^i - \widetilde{K}\right)^+ \middle| F_t\right) \end{aligned} \tag{5}$$

The weighted arithmetic mean $\sum_{i=1}^{k} \widetilde{w}_i \widetilde{S}_T^i$ is replaced with the properly weighted geometric mean. Then

$$\breve{C}_0^B = \left(\sum_{j=1}^{k} w_j S_t^{\,j}\right) E^Q\left(\prod_{i=1}^{k} \left(\widetilde{S}_T^i\right)^{\widetilde{w}_i} - \hat{K}\right)^+ \tag{6}$$

is the approximation of the basket call option price[3],
where:

$$\hat{K} = \widetilde{K} + E^Q\left(\prod_{i=1}^{k} \left(\widetilde{S}_T^i\right)^{\widetilde{w}_i} - \sum_{i=1}^{k} \widetilde{w}_i \widetilde{S}_T^i\right)$$

Because:

$$\begin{aligned} \prod_{i=1}^{k} \left(\widetilde{S}_T^i\right)^{\widetilde{w}_i} &= \prod_{i=1}^{k} \left(\frac{S_0^i \exp(\sigma_i B_T^i + (r - 0{,}5\sigma_i^2)T)}{S_0^i e^{rT}}\right)^{\widetilde{w}_i} = \\ &= \prod_{i=1}^{k} \left(\exp(\sigma_i B_T^i - 0{,}5\sigma_i^2 T)\right)^{\widetilde{w}_i} = \exp\left(\sum_{i=1}^{k} \sigma_i B_T^i \widetilde{w}_i - 0{,}5T\sum_{i=1}^{k} \sigma_i^2 \widetilde{w}_i\right) = \\ &= \exp(\eta_T - 0{,}5T\sum_{i=1}^{k} \sigma_i^2 \widetilde{w}_i) \end{aligned} \tag{7}$$

where:

$$\eta_T = \sum_{i=1}^{k} \sigma_i B_T^i \widetilde{w}_i\,, \; Var(\eta_T) = \sum_{i,j=1}^{k} \rho_{ij} \sigma_i \sigma_j \widetilde{w}_i \widetilde{w}_j T = v^2 T\,,$$

[3] At time $t = 0$.

© Copyright by The Nicolaus Copernicus University Scientific Publishing House

$v^2 = \sum_{i,j=1}^{k} \rho_{ij}\sigma_i\sigma_j \tilde{w}_i\tilde{w}_j$, $\rho_{i,j}$ – correlation coefficient between base instruments.

Then the expected values of the weighted geometric mean and weighted arithmetic mean amount to:

$$\begin{aligned} &E^Q\left(\exp(\eta_T - 0{,}5T\sum_{i=1}^{k}\sigma_i^2\tilde{w}_i)\right) = \exp\left(0{,}5T\left(v^2 - \sum_{i=1}^{k}\sigma_i^2\tilde{w}_i\right)\right)\cdot \\ &\cdot E^Q\left(\exp(\eta_T - 0{,}5Var(\eta_T))\right) = \exp\left(0{,}5T\left(v^2 - \sum_{i=1}^{k}\sigma_i^2\tilde{w}_i\right)\right) = c \end{aligned} \tag{8}$$

and

$$\begin{aligned} &E^Q\left(\sum_{i=1}^{k}\tilde{w}_i\tilde{S}_T^i\right) = \sum_{i=1}^{k}\tilde{w}_i S_0^{-i} E^Q\left(e^{-rT}S_T^i\right) = \\ &= \sum_{i=1}^{k}\tilde{w}_i S_0^{-i} S_0^i e^{-rT+rT} = \sum_{i=1}^{k}\tilde{w}_i = 1 \end{aligned} \tag{9}$$

$$E^Q\left(\prod_{i=1}^{k}\left(\tilde{S}_T^i\right)^{\tilde{w}_i} - \sum_{i=1}^{k}\tilde{w}_i\tilde{S}_T^i\right) = c - 1 \tag{10}$$

From the formulae (7), (8) and (10) it can be obtained:

$$E^Q\left(\prod_{i=1}^{k}\left(\tilde{S}_T^i\right)^{\tilde{w}_i} - \hat{K}\right)^+ = E^Q\left(c\exp\left(\eta_T - 0{,}5Var(\eta_T)\right) - (\tilde{K} + c - 1)\right)^+ \tag{11}$$

Substituting formula (11) in formula (6) the approximation of the basket call option price is obtained[4]:

$$\breve{C}_t^B = \left(\sum_{j=1}^{k} w_j S_t^{\,j}\right)\left(cN(d_1) - \left(\tilde{K} + c - 1\right)N(d_2)\right) \tag{12}$$

where:

© Copyright by The Nicolaus Copernicus University Scientific Publishing House

[4] It can be concluded from the following (using the following notation: $\xi = \eta_T$,$a = c$ and $b = \tilde{K} + c - 1$):
Let ξ be a variable in space (Ω, F, Q) which has a normal distribution with zero mean and variance σ >0. Then for the positive real numbers *a* and *b*, the following is true:
$E^Q\left(a\exp(\xi - 0{,}5\sigma^2) - b\right)^+ = aN(h) - bN(h - \sigma),$
where: $h = \sigma^{-1}\ln\left(\frac{a}{b}\right) + 0{,}5\sigma$; (Musiela, Rutkowski (1998)).

$$c = \exp\left[\left(\sum_{i,j=1}^{k} \rho_{ij} \tilde{w}_i \tilde{w}_j \sigma_i \sigma_j - \sum_{j=1}^{k} \tilde{w}_j \sigma_j^2\right)\frac{(T-t)}{2}\right],$$

$$d_1 = \frac{\ln c - \ln\left(\tilde{K} + c - 1\right) + \frac{1}{2} v^2 (T-t)}{v\sqrt{T-t}},$$

$$d_2 = \frac{\ln c - \ln\left(\tilde{K} + c - 1\right) - \frac{1}{2} v^2 (T-t)}{v\sqrt{T-t}},$$

$N(d)$ – cumulative probability function of the standardised normal distribution.

3. Examples

The empirical illustration is concerned with the pricing of the three-month European basket options with USD, EUR and GBP currencies as base instruments. The analysis is carried out for the call options and relates to the period between: 04.01.2003 and 04.04.2003. In the calculations the identical weights for the base instruments were accepted, namely in the case of basket options on two currencies w_i = 0.5 (for i =1,2). Moreover, for the basket option on the three base instruments $w_i = \frac{1}{3}$ (for i =1,2,3).

Fig. 1 shows the change in the basket call option price based on USD and EUR. In the mentioned period the empirical correlation coefficient between the currencies amounts to 0.54. Fig. 1 also illustrates the variation in the average price of standard options on USD and EUR.

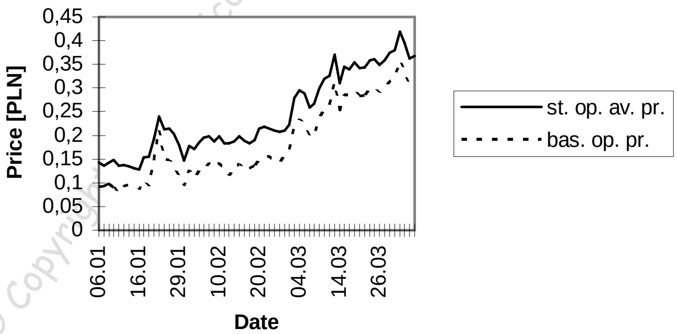

Fig. 1. Variation in the basket option price (on USD and EUR currencies) and standard option average price (on EUR and USD currencies)
Source: Author's calculations.

© Copyright by Nicolaus Copernicus University Scientific Publishing House

Fig. 2 illustrates the change in the basket option price based on USD and GBP. The empirical correlation coefficient between the currencies amounts to 0.088. Fig. 2, furthermore, presents the change in the average price of standard options on USD and GBP.

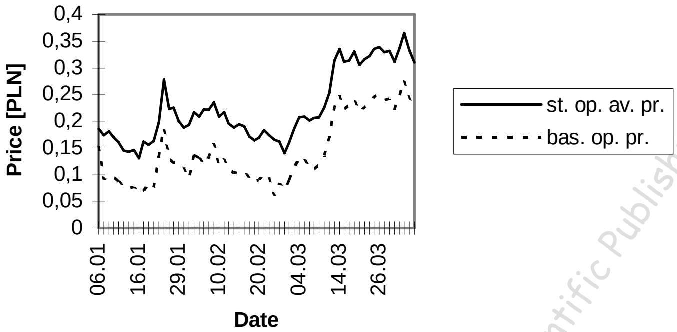

Fig. 2. Variation in the basket option price (on USD and GBP currencies) and standard option average price (on GBP and USD currencies)
Source: Author's calculations.

The variation in the basket option price based on EUR and GBP is shown in Fig. 3. That graph also depicts the change in the average price of standard options on EUR and GBP. The empirical correlation coefficient between the currencies amounts to -0.03.

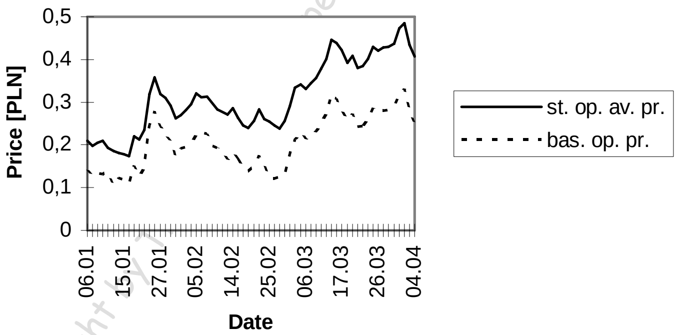

Fig. 3. Variation in the basket option price (on GBP and EUR currencies) and standard option average price (on EUR and GBP currencies)
Source: Author's calculations.

Fig. 4 presents the change in the basket option price based on three currencies: USD, EUR and GBP. Fig. 4 also exhibits the change in the average price of standard options on USD, EUR and GBP.

© Copyright by Nicolaus Copernicus University Scientific Publishing House

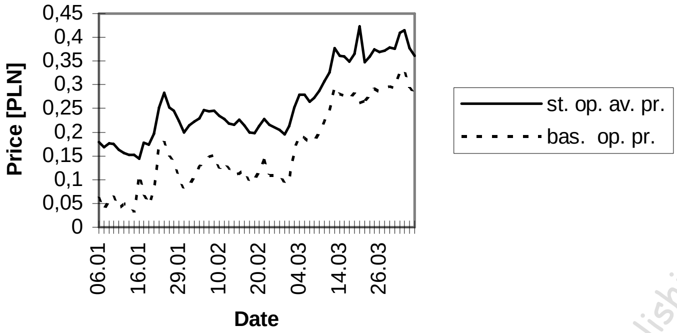

Fig. 4. Variation in the basket option price (on GBP, USD and EUR currencies) and standard option average price (on GBP, USD and EUR currencies)
Source: Author's calculations.

4. Conclusions

The analysis of the variation in option prices depicted in graphs clearly indicates that the prices of basket options are lower than the prices of standard options. This is true both for the positive and negative correlation coefficient. Taking into consideration the cost incurred by the investor in order to hedge against the risk of change in the base instruments prices, it should be stated that basket options are more attractive than standard options.

References

Hull, C. J. (1997), *Options, Futures and Other Derivatives,* Prentice-Hall International, Inc.

Musiela, M., Rutkowski, M. (1998), *Martingale Methods in Financial Modelling*, Springer-Verlag, Berlin.

Neftci, S. N. (1996), *An Introduction to the Mathematics of Financial Derivatives*, Academic Press, San Diego.

Weron, A., Weron, R. (1998), *Inżynieria finansowa* (*Financial engineering*), Wydawnictwo Naukowo-Techniczne, Warszawa.

© Copyright by The Nicolaus Copernicus University Scientific Publishing House