DYNAMIC ECONOMETRIC MODELS Vol. 6 – Nicolaus Copernicus University – Toruń – 2004

Grażyna Trzpiot, Alicja Ganczarek University of Economics in Katowice

Risk on the Polish Energy Market

1. Introduction

During the last few years the Polish energy market has developed. The Polish Power Exchange came into existence. The Day Ahead Market (DAM) was the first market, which was established on the Polish Power Exchange. This whole-day market consists of the twenty-four separate, independent markets where participants may freely buy and sell electricity. The breakthrough in the development of the Polish Power Exchange was made on 1st July 2000, when the first transaction was completed on the DAM. The strength of the Exchange is that all participants of the market can buy and sell electric energy, regardless of whether they are producers or receivers of electricity.

Many markets attempt to compensate for the lack of equilibrium between supply and demand by storing goods. We do not store electricity. Electricity is delivered only at the moment when there is demand for it. Since 1st September, 2001 the Balance Market (BM) has been in operation. This is a technical market, which maintains the balance on the Polish energy market. On 1st July, 2002 the BM introduced additional prices: Price Accounting Deviations of sale – PADs – and Price Accounting Deviations of purchase – PADp. These prices should help to forecast future demand for the electric energy on the whole-day and futures market.

Risk on the market is as high as change in the price. If we compare day's change in the price for petroleum at 1-3% and for gas at 2-4% with change in the price for electric energy at 10-50%, we can see that both producers and consumers of energy are forced to protect themselves against losses. In Poland the forward energy market is developed outside the exchange. Since 1^{st} October 2002, on the Polish Power Exchange we have had a futures market with the futures contracts on the delivery of monthly, weekly and in peak-hours 7–10 p.m. electric energy.

2. Measures of risk

When we take financial decisions, at the same time we take risk. The notion of risk is a property of the future. We have many sources of risk: the changes in a price, uncertainty about fulfilling the terms and conditions of contracts, lack of possibility of closing a position on a financial market, the changes in law and the risk of a strategy.

If we want to estimate the future risk we have to measure it. There are a lot of different measures of risk. We can divide them into three groups: the measure of volatility, the measure of sensitivity and measures of downside risk. In this paper we present quantile downside risk measures, such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) and compare the results with the measure of risk-standard deviation.

2.1. Value-at-Risk

The downside risk measure measures unwilling deviations from the expected rate of return. VaR is such a loss in value, which will not be exceeded in a given period of time with the given probability $\alpha \in (0,1)$

$$P(W \le W_0 - VaR) = \alpha \tag{1}$$

where W_0 is a present value, W is a random variable, such as investment at the end of duration (see: Blanco (1998), Jajuga, Jajuga (1998), Ogryczak, Ruszczyński (2002) and Weron, Weron (2000)).

VaR tries to answer the question: How much money can we lose over a period of time T with probability α ? It is the figure that represents an estimate of how much we may lose as a result of market movements in a particular horizon and for a given confidence level (see: Blanco (1998)).

The definition of this potential loss depends on two main parameters:

- the horizon over which the potential loss is measured; it is not the same to measure the excepted loss over a one day period as over a one week period;
- the degree of confidence; it is a measure of the degree of certainty of the VaR estimate (see: Blanco (1998)).

The horizon can be a function of either the position or the investor. In the former case, the longer horizon for estimating risk can be the result of the time it takes for the position to be liquidated or neutralized. Risk is measured over the period until investment objectives are reviewed and reassessed. We must remember that a long period is merely the sequence of several short periods of risk. When choosing a horizon, consider (see: Blanco (1998)):

- unwind period how long, on average, does it take to reverse a market position or individual trade?
- attention period how often, on average, do we re-examine our portfolio and its mark-to-market or hedging trades?

 accounting period – how long does it take until the next financial investment must be done?

The probability of incurring losses longer than our VaR will be $(1-\alpha)100\%$, being $\alpha 100\%$ the confidence level. For example, for a 95% confidence level ($\alpha = 0,05$), the probability that the losses will exceed VaR is 5%. The common choices for a confidence level are (see: Blanco (1998)):

- 95%; with a 95% level and a one-day horizon, losses in the excess of VaR will occur about once in every twenty days,
- 99%; with a 99% level and a one-day horizon, losses in the excess of VaR will occur about once in every one hundred days.

There are three main methodologies to calculate VaR: variance-covariance, Monte-Carlo simulation, historical simulation (see: Blanco (1998)).

The most commonly used of the three VaR methods is **variancecovariance**. It is based on the analysis of the volatilities and correlation between the different risks. The main issues that have to be solved in order to calculate analytic the VaR are the following: the systematic measurement of actual markets for the production of data applicable to the vertex set chosen and the reduction of firm exposures to a form which can be analysed using vertex dataset. In order to be compatible with the available data, every instrument in a portfolio needs to be reduced to a collection of cash flows in order to derive a synthetic portfolio from the assets we hold. The synthetic portfolio is made up of positions in the risk factors or vertices for which we have volatilities and correlations. The main problem of this method is to have a set of risk factors small enough to be manageable, but comprehensive enough to capture the risk exposures of the firm. Once we construct the cash flow map, we only need to perform basic matrix manipulation to calculate the VaR of our portfolio.

Monte Carlo simulation is based on the generation of random scenarios of prices for which the portfolio is revaluated. Looking at the hypothetical profits and losses under each scenario, it is possible to construct a histogram of expected losses from which VaR is calculated. In this method we need a correlation and volatility matrix to generate the random scenarios. To perform Monte Carlo simulation it is necessary to have pricing models for all the instruments in our portfolio, and it is a procedure that is computationally intensive. The main advantage is that this is a forward-looking assessment of risk, and it deals with options and non-linear positions as we conduct a full valuation of the portfolio for each price scenario.

Historical simulation consists in revaluing the portfolio of several hundred historical scenarios and is built on a hypothetical distribution of profit and losses based on how the portfolio would have behaved in the past. This simulation has the advantage that it does not use estimates like in variances and covariances, and we do not make any assumptions about the distribution of the portfolio returns. However, we are assuming that the past risk reflects the future risk, which in energy markets is a very extreme assumption. Historical simulation is definitely not the method to use to capture risk on energy markets. To

calculate VaR through historical simulation we need a database with historical prices for all the risk factors that we want to include in the simulation, and pricing models to revaluate the portfolio of each price scenario. We can think of a historical simulation as a special case of the Monte Carlo simulation in which all the scenarios are defined ex-ante according to the past behaviour of market prices. In Table 1 we compare the three methods:

How we use the variance-covariance method to calculate VaR:

Noticed by $Q_{\alpha}(W)$ a α -quantile we can write:

$$Q_{\alpha}(W) = W_0 - VaR$$

Noticed by $Q_{\alpha}(R)$ a α -quantile of rate of return we can write:

$$Q_{\alpha}(R) = \frac{W_{\alpha} - W_{0}}{W_{0}} \quad \text{or} \quad Q_{\alpha}(R) = \ln(\frac{W_{\alpha}}{W_{0}}).$$
(3)

(2)

We have now

$$VaR = -Q_{\alpha}(R)W_{0} \text{ or } VaR = (1 - e^{Q_{\alpha}(R)})W_{0}, \qquad (4)$$

where R means rate of return¹ (see: Weron, Weron (2000)).

VaR for single contract on electric energy:

Value of contract in moment t we can write as (see: Weron, Weron (2000)): $X = \alpha(U - K)$

$$X_{t} = q(U_{t} - K),$$
(5)
where U_t is running price of energy, K price of realization of contract,

$$q = N \cdot h \cdot W,$$
(6)

N is the number of days of delivery of energy, h is the number of hours of delivery every day, W is the amount of energy delivered every hour, q > 0 for long position, q < 0 for short position.

When we analysis changing of price of contract during (t-1,t) we have:

$$\Delta X = q\Delta U, \qquad D^2(\Delta X) = q^2 D^2(\Delta U). \tag{7}$$

We can estimate variance of contract on base historical data by

$$D^{2}(\Delta X) = q^{2}U^{2}D^{2}(\frac{\Delta U}{U}), \qquad (D^{2}(\frac{\Delta U}{U}) \approx \frac{D^{2}(\Delta U)}{U^{2}}).$$
(8)

Standard deviation for values of contract we can write as:

$$\sigma_{\rm K} = q \cup \sigma,$$
 (9)
where σ is a variability of price of energy.

$$VaR = Q_{\alpha}(R)qU, VaR = (1 - e^{Q_{\alpha}(R)})qU^{2}.$$
 (10)

¹ If we assume normal distribution of rate of return, we can write for example $Q_{0,01}(R) = 2,33 Q_{0,05}(R) = 1,64$, if we assume normal distribution of logarithmic rate of return, we have $(1 - e^{Q_{0,01}(R)}) = 2.33$, $(1 - e^{Q_{0,05}(R)}) = 1.64$.

 $^{^2}$ With the assumption of normal distribution we have: VaR_{99\%} = 2.33 qU\sigma, VaR_{95\%} = 1.64 qU\sigma.

Table 1. Methodologies to calculate VaR

	Variance-Covariance	Monte Carlo Simula- tion	Historical Simu- lation	
Easiness of Interpretation	Intuitive, although intermedi- ate steps difficult to explain	Intuitive, but computa- tional aspects more difficult to explain in a non-technical fashion	Very intuitive and easy to explain and interpret	
Accuracy of VaR estimates	Depends on validity of as- sumptions (low optionality, stable variances- covariance, normality of return)	Depends on assumptions about variance and covariance, number of simulations and distribu- tion of prices	Is the historical period choice representative of all possible future market scenarios?	
Distributional assumptions about portfolio returns	Portfolio returns are inde- pendents and distributed normally	None, only distributional assumptions about risk factor returns to simulate random paths.	None, but implicit assumption that past return behav- iour is representa- tive of future returns.	
Volatilities and Correlation matrices	Required, correlation matrix must be positive-definite.	Required, correlation matrix must be positive- definite.	Not required.	
Amount of historical data needed for estimation of volatilities/correlation or for performing historical simu- lation	Exponentially weighted moving average methods require only a few months of historical data.	Exponentially weighted moving average methods require only a few months of historical data.	Depends on mar- ket, structural changes, and seasonality effects.	
How does it deal with optionality	Delta method. It can be a poor approximation for portfolios with strong option- ality, specially with exotic options. Delta-gamma ap- proach improves treatment but still not perfect.	Full valuation approach, we can look at changes in volatilities as well as prices of the underlying from day to day.	Full valuation approach.	
Data requirements	Can use risk metrics dataset or create own from historical price series.	Can use risk metrics dataset or create own from historical price series.	Absolute depend- ence on historical data, risk factors not represented in the dataset is ignored.	
Analysis of VaR for Risk management	Incremental and component VaR analysis possible, possi- ble to go from risk measure- ment to risk management.	Study of worst-case hypothetical scenarios, does not allow incre- mental VaR analysis.	Absolute depend- ence on past events, does not allow incremental VaR analysis.	
Computational Inten- sity/hardware requirements	Simple matrix multiplication once cash flow map is ob- tained, relatively fast for most portfolios.	Computationally inten- sive, all the portfolio instruments must be revalued for each price scenario.	Fairly easy to implement, but all instruments pric- ing functions are required.	
Length of horizon	Static approach, assumes portfolio is valued on the effective date of calculation, most effective for very short time horizons.	Introduces the effects of time on portfolio returns mark-to-horizon.	Can be adjusted, but there is a problem a data availability.	

Source: C. Blanco (1998).

Let U means running value of energy and R is a rate of return then we have (see: Weron, Weron (2000)):

...(**D**)

VaR for prices of electric energy:

$$VaR_{99\%} = -Q_{0,01}(R)U \text{ or } VaR_{99\%} = (1 - e^{Q_{0,01}(R)})U,$$

$$VaR_{95\%} = -Q_{0,05}(R)U \text{ or } VaR_{95\%} = (1 - e^{Q_{0,05}(R)})U.$$

VaR for contract on electric energy:

$$VaR_{99\%} = -Q_{0,01}(R) \cdot q \cdot U \text{ or } VaR_{99\%} = (1 - e^{Q_{0,01}(R)}) \cdot q \cdot U,$$

$$VaR_{95\%} = -Q_{0,05}(R) \cdot q \cdot U \text{ or } VaR_{95\%} = (1 - e^{Q_{0,05}(R)}) \cdot q \cdot U.$$

VaR for portfolio of electric energy:

With the assumption of normal distribution we can write:

$$VaR_{99\%} = -2.33 \sqrt{\sum_{i=1}^{n} q_i^2 U_i^2 \sigma_i^2 + 2\sum_{i=1}^{n} \sum_{j>i}^{n} q_i q_j \rho_{ij} U_i \sigma_i U_j \sigma_j},$$

$$VaR_{95\%} = -1.64 \sqrt{\sum_{i=1}^{n} q_i^2 U_i^2 \sigma_i^2 + 2\sum_{i=1}^{n} \sum_{j>i}^{n} q_i q_j \rho_{ij} U_i \sigma_i U_j \sigma_j},$$

where U_i – is a price of ith contract of energy, σ_i – is a variability (standard deviation) of price of ith contract of energy, ρ_{ij} – is a correlation coefficient measurement of ith and jth contract of energy, n – is a number of components of portfolio (see: Weron, Weron (2000)).

2.2. Conditional Value-at-Risk

Next downside measure is CVaR. CVaR can be called the Expected Shortfall – ES (see: Jajuga, Jajuga (1998), Ogryczak, Ruszczyński (2002)):

$$\mathrm{ES}_{\alpha}(\mathbf{X}) = \mathrm{E}\{\mathbf{X} \mid \mathbf{X} \le \mathrm{Q}_{\alpha}(\mathbf{X})\}. \tag{11}$$

The VaR quantity represents the maximum possible loss, which is not exceeded with the probability α . The CVaR quantity is the conditional expected loss given the loss strictly exceeds its VaR:

$$ES_{\alpha}(R) = E\{R \mid R \le VaR_{\alpha}(R)\}.$$
(12)

CVaR defined as the mean of the quantile of worst realizations. The definitions ensure that the VaR is never more than the CVaR, so portfolios with low CVaR must have low VaR as well. CVaR is a function of α for fixed x.

For discrete distribution $\{(R_i, p_i) | i = 1, ..., n \sum_{i=1}^{n} p_i = 1\}$ we can write:

ES_{\alpha} (R) =
$$\frac{1}{\alpha} \sum_{i=1}^{k} R_i p_i$$
, $\sum_{i=1}^{k} p_i = \alpha$. (13)

For continuous distribution with the cumulative distribution function F_x we defined this measure as:

$$\mathrm{ES}_{\alpha} = \frac{1}{\alpha} \int_{0}^{\alpha} F_{\mathrm{X}}^{(-1)}(t) \mathrm{d}t \quad 0 < \alpha \le 1,$$

$$(14)$$

where $F_X^{(-1)}(p) = \inf\{\eta; F_X(\eta) \ge p\}$.

CVaR is an alternative measure of risk, but has better properties than VaR. Recently Pflug (2000) proved that CVaR is a coherent risk measure having the following properties: transition-equivariant, positively homogeneous, convex, monotonic, stochastic dominance of order 1, and monotonic dominance of order 2. Minimizing the CVaR of portfolio is closely related to minimizing VaR, as already observed from the definition of these measures (see: Rockafellar, Uryasev (2000)).

Let U means running value of energy and R is a rate of return then we have:

CVaR for prices of electric energy:

aR for prices of electric energy: $CVaR_{99\%} = ES_{0.01}(R)U$ or $CVaR_{95\%} = ES_{0.05}(R)U$.

Let $q = N \cdot h \cdot W$ like for VaR, then we have:

CVaR for contract on electric energy:

 $CVaR_{99\%} = ES_{0.01}(R)qU$ or $CVaR_{95\%} = ES_{0.05}(R)qU$. If we write **R** like a rate of return of portfolio, we can write:

CVaR for portfolio of electric energy:

$$CVaR_{99\%} = ES_{0.01} (\mathbf{R}) \sum_{i=1}^{n} q_i U_i,$$

$$CVaR_{95\%} = ES_{0.05} (\mathbf{R}) \sum_{i=1}^{n} q_i U_i.$$

Risk on the Polish Energy Market

For estimation of risk on the Polish Energy Market we took into consideration the price of contracts on electric energy, the price of electric energy on DAM and BM quoted from 01. 10. 2002 to 20. 12. 2002. In Table 2 we presented average measures for each value. Prices and rates of return for each value relate to 1 MWH electric energy. Already in the initial analysis in Table 2 we can see, that on the BM and the DAM change in the price is higher than on the futures market. The prices of contracts are more stable, standard deviation amounts to 7% of the level of the average price for the most diverse contract FFW48-02. On the BM and the DAM variation coefficients of prices range from 11% to 35%. This analysis shows, that we should look at changes in prices in distribution tails. We can say, that on the average in the investigated period the price of energy on the whole-day market had an increasing tendency and we cannot say the same about prices on the futures market.

Contracts on electric	Parameters of price			Parameters of rates of return		Parameters of loga- rithmic rates of return	
energy	mean	S	V	mean	s Z	mean	s
FFM01-03	125.43	1.63	0.01	-0.0001	0.0055	-0.0001	0.0055
FFM02-03	126.23	1.53	0.01	-0.0254	0.1603	0.0010	0.0051
FFW01-03	122.89	3.82	0.03	-0.0024	0.0241	-0.0027	0.0247
FFW02-03	126.69	5.89	0.05	-0.0045	0.0309	-0.0050	0.0313
FFW03-03	124.95	2.15	0.02	-0.0026	0.0093	-0.0026	0.0095
FFW04-03	124.98	2.11	0.02	-0.0025	0.0091	-0.0026	0.0093
FFM13-02	124.11	4.80	0.04	0.0015	0.0514	0.0002	0.0511
FFW45-02	127.79	2.73	0.02	0.0005	0.0238	0.0002	0.0238
FFW46-02	125.59	5.34	0.04	0.0000	0.0405	-0.0009	0.0413
FFW47-02	128.22	5.48	0.04	-0.0033	0.0313	-0.0039	0.0335
FFW48-02	125.77	9.03	0.07	-0.0062	0.0389	-0.0070	0.0400
FFW49-02	124.94	3.60	0.03	0.0045	0.0368	0.0038	0.0365
FFW50-02	122.73	4.91	0.04	-0.0019	0.0447	-0.0030	0.0449
FFW51-02	122.93	4.44	0.04	0.0019	0.0403	0.0011	0.0402
FFW52-02	116.35	3.24	0.03	-0.0012	0.0331	-0.0017	0.0331
PW 43-02	144.46	3.08	0.02	0.0026	0.0229	0.0023	0.0231
PW 44-02	149.38	6.51	0.04	0.0057	0.0293	0.0052	0.0289
PAD	107.09	37.86	0.35	0.0087	0.1418	-0.0004	0.1344
PADs	236.67	61.08	0.26	0.0086	0.1379	-0.0002	0.1322
PADp	82.12	9.70	0.12	0.0018	0.0625	-0.0001	0.0616
DAM	108.44	24.36	0.22	0.1889	2.0467	0.0001	0.4274

Table 2. Average measures of price and rates of return of contracts

In Table 3 we presented the results of value VaR and CVaR, which we calculated with the use of the variance-covariance method. We should remember about the differences between the interpretation and magnitude of these measures. When we look at Value at Risk we can say, that with the probability of 0.99 on contract FFM01-03 we will not lose more than 3.18 zł/MWh. On contracts FFM02-03 with the probability off 0.95 we will lose nothing. We can incur the highest loss on contract FFW47-02, but with the probability off 0.99 this loss will not exceed 20.25 zł/MWh. In the same period of time on the whole-day market our losses with the probability off 0.99 will not exceed the value from 14.38 to 77.56 zł/MWh (between 7.78 and 40.55 zł/MWh with the probability off 0.95). CVaR_{99%} informs us about average of the 1% of the highest loss. For example: CVaR_{99%} = 3.18 for FFM01-03 means, that the average of 1% of the worst loss on this contract equals 1.9 zł/MWh. On the DAM the value VaR_{99%} = 28.50 informs, that on this market with the probability off 0.99 we do not lose more then 28.50zł/MWh and with the probability off 0.01 we may lose more. With the same degree of confidence on this market $CVaR_{99\%} = 34.73$, informs that 1% of the worst loss we may on average lose 34.73zł/MWh.

Contracts on elec-	Rates of return				Logarithmic rates of return			
tric en- ergy	Va	aR	CV	′aR	VaR		CVaR	
	VaR _{99%}	VaR _{95%}	CVaR99%	CVaR _{95%}	VaR _{99%}	VaR _{95%}	CVaR _{99%}	CVaR _{95%}
FFM01-03	3.18	1.00	3.18	1.90	3.18	1.00	3.18	1.91
FFM02-03	1.01	0.00	1.01	0.03	1.01	0.00	1.01	0.03
FFW01-03	9.69	9.69	9.69	9.69	9.69	9.69	9.69	9.69
FFW02-03	10.51	10.51	10.51	10.51	10.51	10.51	10.51	10.51
FFW03-03	4.51	4.51	4.51	4.51	4.51	4.51	4.51	4.51
FFW04-03	4.42	4.42	4.42	4.420	4.42	4.42	4.42	4.42
FFM13-02	14.01	10.60	14.01	12.31	14.01	10.60	14.01	12.32
FFW45-02	6.63	5.72	6.63	6.18	6.63	5.72	6.63	6.18
FFW46-02	14.82	11.22	14.82	13.02	14.82	11.22	14.82	13.04
FFW47-02	20.25	2.89	20.25	11.57	20.25	2.89	20.25	11.89
FFW48-02	16.95	12.24	16.95	14.59	16.95	12.24	16.95	14.62
FFW49-02	7.17	7.17	7.17	7.17	7.17	7.17	7.17	7.17
FFW50-02	13.46	7.50	13.46	10.48	13.46	7.50	13.46	10.52
FFW51-02	12.58	6.26	12.58	9.42	13.48	6.04	13.48	9.82
FFW52-02	11.91	5.93	11.91	8.92	11.91	5.93	11.91	8.96
PW43-02	8.46	8.46	8.46	8.46	-8.46	8.46	8.46	8.46
PW44-02	10.46	10.46	10.46	10.46	10.46	10.46	10.46	10.46
PAD	38.82	19.91	46.26	30.68	38.82	19.91	46.52	31.33
PADs	77.56	40.55	95.05	63.96	77.56	40.55	95.90	65.17
PADp	14.38	7.78	16.59	11.63	14.38	7.78	16.69	11.72
DAM	28.50	16.98	34.73	24.70	28.50	16.98	34.88	24.94

Table 3. Quantile downside risk measures of contracts

In the next step we build the portfolios. The results of hedge positions on the energy market by optimal choice portfolio are in Table 4. The hedge position is possible owing to the correlation between portfolio's components.

Table 4. Value of VaR _{99%} for portfolios of contract noticed on Polish Power Exchange

No.	4	VaR99%		
1.0	X	FFW45-02	FFM01-03	2005.26
20	FFW50-02	FFM01-03	FFW45-02	2044.88
S		FPW43-02	FFW48-02	1953.39
4		FFM01-03	FFW50-02	1801.87
5		FFM01-03	FFW52-02	2170.64
6		FFM01-03	FPW43-02	2108.67

4. Conclusions

Taking into consideration quantile downside risk measures for participants of the market interested in short positions, it is more profitable to invest in the futures market. Values of prices and rates of return for the lower distribution tail are higher on the futures market and so are the values of conditional downside risk measures. This market can be characterized as more stable at the moment.

The quantile values for the upper distribution tail were not presented in this paper. For VaR this will be analogous, we can interpret them not as a fall but as the height of price and rate of return values. Also average conditional values will be similar for the upper tail of prices and they will be closer to maximum values. Accordingly we can conclude, that for participants of the market interested in long positions it is safer to invest on the futures market.

Summing up it should be stated, that quantile measures are superior to average measures. Participants of the market, not only of the market of energy, want to draw benefits from their investments and they are prepared, in return, to take some limited risk resulting from their opportunities and expectations. In this case, quantile measures of risk give them a more precise answer than average measures. They mark extreme, not only average, positions of values and they can also act as a signal for buying or selling.

References

- Blanco, C. (1998), Value at risk for energy: Is VaR useful to manage energy price risk?, *Financial Engineering Associates.*
- Jajuga, K., Jajuga, T. (1998), Inwestycje. Instrumenty finansowe. Ryzyko finansowe. Inżynieria finansowa, PWN, Warszawa.
- Ogryczak, W., Ruszczyński, A. (2002), Dual Stochastic Dominance and Quantile Risk Measures, *International Transactions in Operational Research*, 9, 661–680.
- Pflug, G. Ch. (2000), Some remarks on the value-at-risk and the conditional value-at risk, in: *Probabilistic Constrained Optimization: Methodology and Applications* (ed. Uryasev S.), Kulwer.
- Rockafellar, R. T., Uryasev, S. (2000), Optimization of Conditional Value-at-Risk. *Journal of Risk*, 2, 21–41.
- Tarczyński, W. (1997), Rynki kapitałowe (Capital Markets), Placet, Warszawa.
- Tarczyński, W. (1999), Inżynieria Finansowa (Financial Engineering), Placet, Warszawa.
- Weron, A., Weron, R. (2000), *Gieldae energii. Strategie zarządzania ryzykiem (Energy Exchange. Strategies of Risk Management)*, CIRE, Wrocław.