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Realization of the Congruence Postulate as a Method of Avoiding the Effects of a Spurious Relationship

1. Introduction

Already at the turn of the 19th century the influence of the nonstationarity of economic processes on detecting the true relationships among them was noticed. Yule (1926) was the first researcher who raised the problem of the danger of a spurious relationship among processes monotonically rising or falling. In his simulation experiments studying the correlation between uncorrelated integrated processes (first and second order) he proved that the distribution of the correlation coefficient was not symmetric and not nearly normal, but it was closer to a semi-ellipse for a pair of I(1) series, and U-shaped – for a pair of I(2) series. The Yule's experiments show explicitly that the correlation between processes with integral structure, i.e. between integrated processes, should be calculated on the data from which the nonstarionarity[1] has been removed, e.g. on differenced data, i.e. first or second differences, but not on the original data, i.e. on levels. Such approach enables to avoid the danger of nonsense correlation as Yule called it.

However as the seminal study referring to the importance of distinguishing the spurious correlation between economic processes from the true one is treated in the paper by Granger and Newbold (1974). Considering the relationship between unrelated white noises Granger and Newbold shown in simulation experiments that the true null hypothesis of no relationship was rejected 76 per cent of times using the standard *t*-statistic. They came to the conclusion that the standard significance tests are biased in the direction to reject the null hypothesis of no relationship even when it is true and at the same time to indicate an

[1] In this case the nonstarionarity in variance.

apparently significant but spurious relationship, because in fact the processes are independent.

The analytical results for relationships among integrated processes were given by Phillips (1986) who developed the asymptotic theory for integrated processes and in that way explained the simulation results obtained by Granger and Newbold. Phillips (1986) demonstrated that the estimation of a model describing the relationship for unrelated white noises by ordinary least squares can lead to results that cannot be interpreted within the conventional testing procedure, because the distributions of standard statistics in the model describing relationships for integrated processes are not the same as in the model describing relationships for stationary processes.

It should be clearly mentioned that the problem of a spurious relationship does not hold only for independent random walks, but also for integrated processes of the second order (see: Haldrup (1994)), fractionally integrated processes[2] (see: Marmol (1996)), stochastic unit root processes (see: Granger, Swanson (1997)) or even stationary autoregressive processes (see: Granger, Hyung, Jeon (1998)).

From that results that the problem of a spurious relationship should be considered in the wider context of the model specification error. Then the model specification can be improved by taking into account the information about the internal structure of processes under investigation. This demonstrated the importance and at the same time necessity of studying the internal structure for all analysed processes. The approach including the information about the internal structure, i.e. the concept of congruent modelling, lead to the obtaining such a model specification which ensures required properties of a residual process, i.e. white noise properties.

The purpose of this paper is to present the method of avoiding the danger of a spurious relationship and their effects on statistical inference. This method consists in specifying the econometric model in such a way to ensure the realization of the congruence postulate in the Zieliński sense, i.e. the congruence of harmonic structures for both sides of a model.

To the approaches satisfying the congruence postulate belong: the concept of congruent modelling (see: Zieliński (1984), (2002)) and the concept of 'general to specific' modelling (see: Hendry (2000)). The differences between both concepts consist in different approaches to starting model specification.

The construction of a dynamic congruent model starts with the relationship for white noises components of processes Y_t and X_{it}, i.e. with the following relationship: $\varepsilon_{Yt} = \sum_{i=1}^{k} \rho_i \varepsilon_{Xit}, + \varepsilon_t$, where ε_{Yt}, ε_{Xit} are white noise components of processes Y_t and X_{it} respectively. For that model the congruence postulate is

[2] Fractionally integrated processes are processes with long memory – see: e.g. Granger (1980), Granger, Joyeux (1980), Diebold, Rudebusch (1989), Hosking (1981), Koop, Ley, Osiewalski, Steel (1997), Piłatowska (1999), (2000).

satisfied, because the spectral density functions for both sides of the model are parallel to the frequency axis. The model specification for real process Y_t and X_{it} is being obtained on the relationship for white noise components using the information about the internal structure of studied processes, i.e. degree of polynomial trend, type of cyclical fluctuations, order of autoregression for all given processes. The empirical verification of the realization of the congruence postulate is conducted by testing the white noise properties of a residual process[3] ε_t.

In the concept of 'general to specific' modelling the general unrestricted model (GUM) is being built, in which the autoregression orders for given processes are established arbitrarily as the maximum order depending on the number of observations, number of processes, next the degree of polynomial trend is generally equal to one and cyclical fluctuations are taken as constant. In that model the congruence criteria are tested, i.e. lack of autocorrelation of a residual process, lack of ARCH effect, normal distribution of residual process, residual variance homosdestasticity, parameter constancy. It should be underlined that the realization of these criteria ensures the flat spectral density function. Therefore the concept of 'general to specific' modelling satisfies also the congruence postulate in the Zieliński[4] sense.

The effectiveness of this method consisting in building the model in such a way to satisfy the congruence postulate will be presented on the results of simulation experiments.

2. A Spurious relationship and its effects on statistical inference

Studying the relationship between independent random walks y_t and x_t in the following form

$$y_t = \alpha + \beta x_t + u_t, \qquad t = 1,2,\ldots,T, \tag{1}$$

Phillips (1986) show that the distributions of standard statistics, especially t-distributions, t_α, t_β, do not have any limiting distributions, i.e. they diverge in

[3] Testing of white noise properties should consist in testing for variance homoscedasticity, lack of autocorrelation and normal distribution of residual process.

[4] In both concepts from the starting general versions of a model the insignificant variables are eliminated, but in the concept of congruent modelling the a posteriori selection method is used, and in the concept of 'general to specific' modelling – the automatic procedure of reduction the general to specific model, which is curried out by *PcGets* module of *OxMetrics* package (see: Hendry, Krolzig (1999), (2001)). It should be underlined that the ability of both selection procedures, i.e. a posteriori method and PcGets, to detect the generating model was showned in the results of Monte Carlo simulation experiments (see: respectively Kufel (2002), Krolzig, Hendry (2001)).

distribution as the sample size T increases. This means that there are not any critical values, which are asymptotically correct for those statistics. Hence for any fixed critical value of standard t-statistic, the rejection rate of the null hypothesis, H_0: $\beta = 0$, of no relationship will tend to increase with sample size. The same result was found by the simulation of Granger and Newbold (1974)[5]. Further, the regression coefficients do not converge to constants as the sample size increases, i.e. the constant $\hat{\alpha}$ in (1) has a divergent asymptotic distribution, whereas the coefficient $\hat{\beta}$ in (1) possesses a non-degenerate limiting distribution ($\hat{\beta}$ converges to random variable).

If we consider independent random walks with drift

$$y_t = \mu_y + y_{t-1} + u_t, \tag{2}$$

$$x_t = \mu_x + x_{t-1} + v_t, \tag{3}$$

the results are altered (see Entorf (1992)). The distribution of coefficient $\hat{\beta}$ in model (1) converges to μ_y/μ_x, i.e. $\hat{\beta}$ converges to constant rather than a random variable (like for two random walk without drift). Thus, the results from spurious relationship depend on whether we consider random walk processes with drifts or with no drifts.

Phillips (19986) shows that in the case of a relationship between independent random walks the R^2 statistic converges to a non-degenerate random variable in the limit as T increases, and the Durbin-Watson statistic converges to zero as the sample size tends to infinity, whereas for two genuinely related series the DW statistic converges to a non-zero value. Hence the low values of the DW statistic and high values of the R^2 statistic ($R^2 > DW$) may indicate the case of a spurious relationship as the simulation evidence advanced by Granger and Newbold demonstrated. Therefore the behaviour of DW statistic provides one way of discriminating between a spurious and genuine relationship, but a test based on this statistic may have poor power properties in small samples.

The problem of a spurious relationship for two independent random walks does not disappear, even if the potential nonstationarity by including the time variable will be taken into account and the model of the form

$$y_t = \alpha + \beta x_t + \gamma t + u_t. \tag{4}$$

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[5] It should be noted that similar results were obtained by Phillips (1986) for the model with a vector of independent integrated processes as regressor. The F statistic for testing the insignificance of parameter vector has divergent distribution as T increases. However the rate of divergence for the F statistic is equal $O(T)$ and is faster than for the t-statistic, $O(T^{1/2})$. Hence in model for many regressors the higher rejection percentage of the null of insignificance is expected when using F statistic with regard to whole parameter vector than t-statistic with regard to selected parameters.

will be estimated. Durlauf and Phillips (1988) demonstrated that as before the constant $\hat{\alpha}$ in (4) has divergent distribution (i.e. $\hat{\alpha}$ has the variance, which increases as sample size increases), the coefficient $\hat{\beta}$ in (4) possesses a non-degenerate limiting distribution (does not converge to zero) and $\hat{\gamma}$ tends in probability to zero. In (4) only the time trend coefficient has a consistent estimator. Tests for H_0: $\beta = 0$ diverge in distribution as T increases, leading to a false rejection of the null hypothesis of no relation.

However, the effects of a spurious relationship may be removed if in building the model the information about internal structure of given processes will be used, i.e. the model will be specified according to the concept of congruent modelling. This is evidenced by the simulation results presented below.

3. Possibilities of avoiding the effects of a spurious relationship – simulation analysis

In the presented Monte Carlo simulation experiment the results for two models are compared, i.e. model (A) of the form (see also model (1))

$$y_t = \alpha_0 + \alpha_1 x_t + u_t \tag{A}$$

and model (B) taking into account the autoregressive structure of processes y_t and x_t

$$y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 x_{t-1} + \alpha_3 y_{t-1} + u_t \tag{B}$$

which are independent autoregressive processes of the form

$$y_t = \beta_y y_{t-1} + \varepsilon_{yt}, \tag{C}$$

$$x_t = \beta_x x_{t-1} + \varepsilon_{xt}, \tag{D}$$

where $\beta_x = \beta_y = 0, 0.5, 0.7, 0.9, 1.0,$ ε_{yt} and ε_{xt} stand for independent white noises of given processes, sample size $n = 120$, number of replications $m = 1000$.

For $\beta_x = \beta_y = 1$ we have to do with independent random walks, i.e. the case studied by Granger and Newbold (1974). Table 1 displays the rejection percentage of the true null hypothesis of no relation between y_t and x_t, H_0: $\alpha_1 = 0$, for the t-statistic and the null hypothesis of the lack of autocorrelation, H_0: $\rho_1 = 0$, for the DW statistic, where ρ_1 stands for first order autocorrelation coefficient.

Table 1. Rejection percentage of the null hypothesis H_0: $\alpha_1 = 0$ for the t-statistic and the null H_0: $\rho_1 = 0$ for the DW statistic in the case of two independent processes x_t and y_t

	$\beta_x = \beta_y = 0$	$\beta_x = \beta_y = 0.5$	$\beta_x = \beta_y = 0.7$	$\beta_x = \beta_y = 0.9$	$\beta_x = \beta_y = 1.0$
	Model (A): $y_t = \alpha_0 + \alpha_1 x_t + u_t$				
Per cent $\lvert t \rvert > 1.98$	6	24.7	22.8	51.8	77.1
Per cent $DW < d_l$	5.9	100	100	100	100
	Model (B): $y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 x_{t-1} + \alpha_3 y_{t-1} + u_t$				
Per cent $\lvert t \rvert > 1.98$	6	5.3	5.2	5.1	5.2
Per cent $DW < d_l$	0	4	0.8	3.4	4.8

Source: Author's calculations.

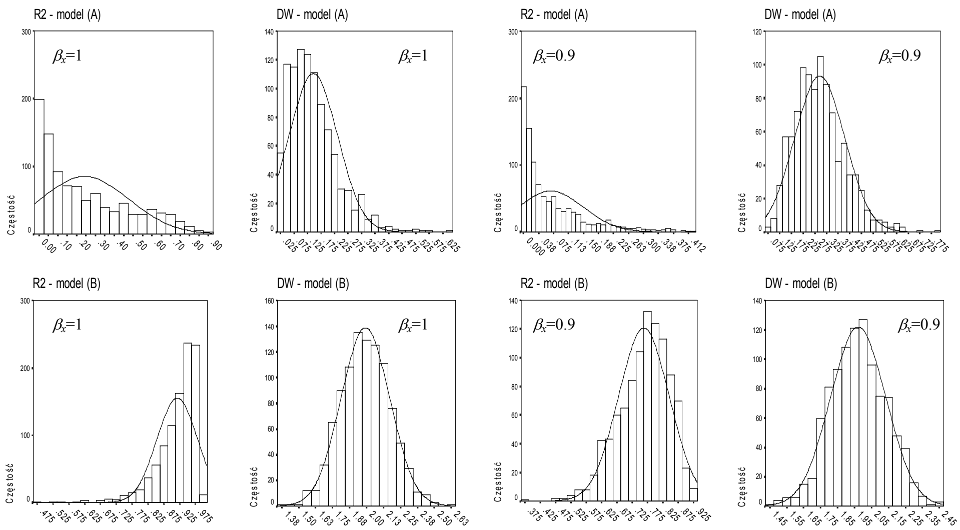

Fig. 1. Distribution of determination coefficient R2 and DW statistic in model (A) and (B) for $\beta_x = \beta_y = 1.0$ and $\beta_x = \beta_y = 0.9$ in the case of two independent processes x_t and y_t

Results in Table 1 indicate that the true null hypothesis H_0: $\alpha_1 = 0$ is rejected 77.1 per cent of times for the case of two independent random walks (for $\beta_x = \beta_y = 1.0$), similar as in the experiment of Granger and Newbold (1974), and for the case of two independent autoregressive processes, e.g. for $\beta_x = \beta_y = 0.9$ – 51.8 per cent of times, similar as in experiment of Granger, Hyung and Jeon (1998)[6] – see: results for model (A) in Table 1. Figure 1 shows the distribution

[6] Insignificant differences observed between simulation results in table 1 and those obtained by Granger and Newbold (1974), as well by Granger, Hyung and Jeon (1998) are caused by the different sample sizes fixed in experiments, i.e. the assumed number

of R^2 (at Fig. 1 – R2) for model (A) does not tend to zero, and the DW statistic converges to zero. These results provide evidence of a spurious relationship.

The results of testing H_0: $\alpha_1 = 0$ for model (B), which takes into account autoregressive structure of given processes, completely differs from those for model (A). Using the t-statistic the null of no relation is rejected only 5 per cent of times independently of parameter values β_x and β_y, hence the correct inference of no relation is observed[7]. The DW statistic for model (B) has a standard distribution, because it converges to its limiting value equalling 2. This means the lack of first order autocorrelation for a residual process. The distribution of the R^2 statistic does not converge to zero[8], because the autoregresive relation between y_t and y_{t-1} exits, however this distribution converges to a certain non-zero value. This is an evidence that the effects of underdifferencing do not appear[9].

Summing up, the approach using information about the internal structure of given processes according to the concept of congruent modelling leads to the obtaining of the model specification which enables the detection of a genuine relationship and ensures the required properties of a residual process, i.e. white noise properties.

Moreover, it should be clearly underlined that effects of omission of the internal structure elements in the case of dependent processes, i.e. effects of building non-congruent models in the Zieliński sense, on the distribution of t-statistic and DW statistic, are very similar to those obtained in the case of independent processes. Table 2 presents the simulation results for model (A) (omission of autoregressive structure) and model (B) (inclusion of autoregressive structure) assuming that white noises ε_{yt} and ε_{xt} in model (C) and (D) are dependent, i.e. $\varepsilon_{yt} = \rho\varepsilon_{xt} + \varepsilon_t$, where $\rho = 0.5$ and 0.9.

In the case of model (A) – see Table 2 – which does not include the autoregressive structure of processes y_t and x_t, the rejection percentage of the true null hypothesis of relation, H_0: $\alpha_1 = \rho$, is considerably higher than the nominal significance level of 5%; e.g. for $\beta_x = \beta_y = 0.9$ the null is rejected 50 per cent of times. Additionally, in all cases (except $\beta_x = \beta_y = 0$, i.e. when y_t and x_t are white noises) the first order autocorrelation occurs. The distribution of the DW statistic converges to zero, and the determination coefficient R^2 has divergent distribution (see: Fig. 2 – model (A)) – similar as for two independent processes (see: Fig. 1 – model (A)).

of observations was as follows: in Granger, Newbold – $n = 50$, in Granger, Hyung and Jeon – $n = 100$, and in experiment presented in table 1 – $n = 120$.

[7] Similar results were obtained when testing the hypothesis H_0: $\alpha_2 = 0$.

[8] The distribution of R^2 converges to zero for $\beta_x = \beta_y = 0$, when x_t and y_t are white noises.

[9] For effects of underdifferencing and overdifferencing see results of simulation study in Piłatowska (2003).

Table 2. Rejection percentage of the null hypothesis H_0: $\alpha_1 = \rho$ for the t-statistic and the null H_0: $\rho_1 = 0$ for the DW statistic in the case of two dependent processes x_t and y_t

	ρ	$\beta_x = \beta_y = 0$	$\beta_x = \beta_y = 0.5$	$\beta_x = \beta_y = 0.7$	$\beta_x = \beta_y = 0.9$	$\beta_x = \beta_y = 1.0$
		Model (A): $y_t = \alpha_0 + \alpha_1 x_t + u_t$				
Per cent $\mid t \mid > 1.98$	0.5	4.6	20.4	24.1	50.3	78.7
$t = (a_1 - \rho)/S(a_1)$	0.9	5.8	17.7	23.9	50.5	81.0
Per cent $DW < d_l$	0.5	4.4	100	100	100	100
	0.9	5.7	100	100	100	100
		Model (B): $y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 x_{t-1} + \alpha_3 y_{t-1} + u_t$				
Per cent $\mid t \mid > 1.98$	0.5	4.6	5.9	5.6	4.6	5.0
$t = (a_1 - \rho)/S(a_1)$	0.9	5.8	4.2	3.5	6.1	6.2
Per cent $DW < d_l$	0.5	0.0	2.8	1.0	1.1	2.6
	0.9	0.0	4.8	2.0	1.5	3.1

Source: Author's calculations.

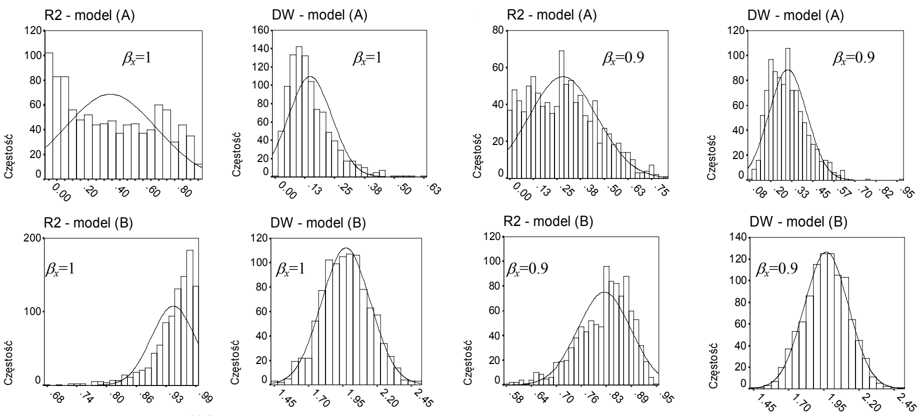

Fig. 2. Distribution of determination coefficient R2 and DW statistic in model (A) and (B) for $\beta_x = \beta_y = 1.0$ and $\beta_x = \beta_y = 0.9$ in the case of two dependent processes y_t and x_t

Including the autoregressive structure of studied processes improves considerably the distribution of the t-statistic and DW statistic (see model (B) in Table 2). The t-statistic has standard distribution; the true null hypothesis of the relation is rejected only about 5–6 per cent of times. The first order autocorrelation does not occur, because the rejection percentage of the null does not exceed the 5 per cent nominal significance level. The distribution of DW statistic converges to standard value equalling 2, and the R^2 statistic has convergent distribution (see Fig. 2 – model (B)).

4. Conclusions

The presented results indicate that the problem of a spurious relationship for independent processes (see Table 1 and Fig. 1) and effects of a spurious relationship for dependent processes (see Table 2 and Fig. 2) disappear if in building a model the information about the internal structure of studied processes will be included so as the residual process has the white noise properties. Such a strategy is realized in the concept of congruent modelling.

Moreover, the extended model specification has a great importance for the correct interpretation of model parameters. It is known (see Zieliński (1986)), that parameters in an econometric model measure the total influence of a given explanatory process on the endogenous process, which consists of a direct and indirect influence. The direct influence lies in that the explanatory process effects the endogenous process with that its part which is not correlated with omitted explanatory processes. Whereas the indirect influence is the effect of a given explanatory process with that its part which is correlated with omitted processes. Hence including to a model the omitted processes decreases the indirect influence, and at the same time leads to obtain the parameters of a better cognitive value. This is especially required from the causal point of view.

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