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## Modelling the Zloty–Euro Exchange Rate

### 1. Introduction

This paper continues the author's recent studies devoted to the problem of modelling the zloty–euro exchange rate<sup>1</sup>. The exchange rate is that type of macroeconomic category whose behaviour is often influenced by unexpected and significant breaks. Our past experiences *give the impression* that technical analysis used in that case seems to be a proverbial fortune telling from *dregs*. It happens that conclusions resulting from this analysis are extremely different and may wrongly suggest a feeling of helplessness in the possibility of finding the satisfactory recognition of studying some economic rules.

It seems that econometric modelling provides us with research tools, which allow us to manage this problem more efficiently. In particular, the application of dynamic models with the inclusion of integration and cointegration analysis may be very useful in such a case.

In macroeconomic studies we usually have at our disposal nonstationary series, often integrated of order one. Standard statistical tests used in univariate or multivariate analyses are inappropriate and results obtained, e.g. relating to the influence of explanatory variables, is false – such variables are not in any causal-outcome relation with a dependent variable and produce only a spurious dependency. Cointegration analysis applied in such cases protects us from building spurious regressions<sup>2</sup>, and, in addition, creates the chance of finding a long-run relationship (LRR), i.e. relation in which included variables tend to the long-run equilibrium.

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<sup>1</sup> Similar analysis based on the weekly data is done, for example, in Krauze (2003; 2004).

<sup>2</sup> See: Newbold and Davies (1978).

The next sections present various zloty–euro model specifications carried out in order to find the long-run tendency for this rate – and preliminary forecasts of the zloty–euro exchange rate. The main conclusions from our research end the paper.

## 2. Zloty–euro exchange rate models, results of estimation and testing, preliminary forecasts

The question arises whether we can formulate a univariate process which correctly and precisely enough represents the behaviour of the zloty–euro exchange rate or, if not, whether there exists any multivariate process in which variables are cointegrated. We are also interested if these solutions can be useful in the prediction process of the zloty–euro exchange rate. In empirical analysis we use the following monthly logarithmic series rates: Polish zloty–euro ( $pe_t$ ), Swiss franc–euro ( $se_t$ ), US dollar–euro ( $ue_t$ ) and British pound–euro ( $be_t$ ) from the period January 1999 – May 2003<sup>3</sup>. Graphs of  $pe_t$ ,  $ue_t$ ,  $be_t$  and  $se_t$  with inclusion of their trend functions (PET, UET, BET<sup>4</sup> and SET) are presented in Fig. 1–4, respectively.

Let us consider the following  $x_t$  relation testing for a unit root with structural shifts in mean and in trend:

$$x_t = \mu + \beta t + \delta D(T_b)_t + \theta DU_t + \gamma DT_t^* + (1 + \rho)x_{t-1} + \sum_{k=1}^K c_k \Delta x_{t-k} + e_t, \quad (1)$$

where  $\mu$  is intercept,  $\beta$  – trend coefficient,  $\delta$  – impulse break effect,  $\theta$  – change in the intercept,  $\gamma$  – change in the trend,  $(1 + \rho)$  – autoregressive parameter,  $c_k$  –  $k$ -th parameter of the augmentation term,  $e_t$  – disturbance term, and where:

<sup>3</sup> The data for  $pe_t$ ,  $ue_t$ ,  $be_t$  and  $se_t$  are obtained from: a) *Prices in the national economy in 2001. Information and statistical papers* (2002) and *Prices in the national economy in 2002. Information and statistical papers* (2003) – for the period 1999–2001, b) [www.stat.gov.pl](http://www.stat.gov.pl) and as a result of own calculations based on the daily quotations of the Polish National Bank (NBP) ( $pe_t$ ) and the Frankfurt Stock Exchange ( $ue_t$ ,  $be_t$ ,  $se_t$ ) – for the further data. It was also considered (but unsuccessfully) inclusion of real consumer price index.

<sup>4</sup> Graph of BET includes two additional breaks in the deterministic trend of  $be_t$ , occurred in September 2000 and December 2002 in order to observe easier breaks in mean of this variable in December 2000. These endeavours in equation (1c) are not necessary as a consequence of inclusion variable  $be_{t-1}$ .

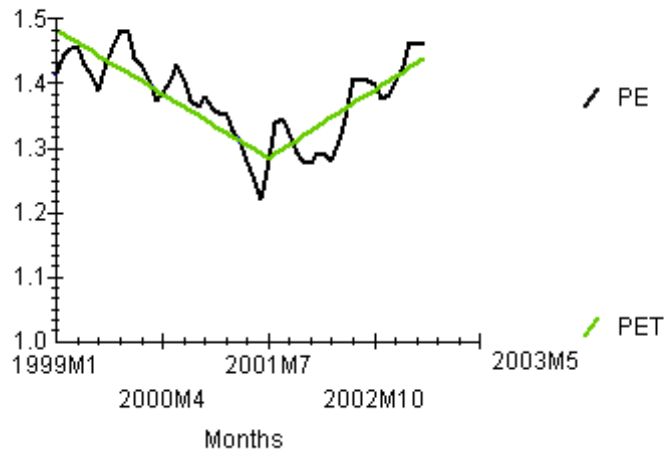


Fig. 1. Monthly series of PE and  $(pe_t)$  and PET

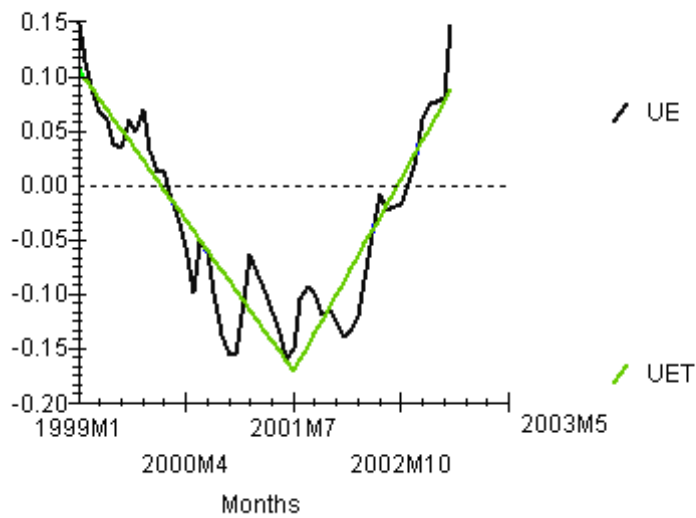


Fig. 2. Monthly series of UE  $(ue_t)$  and UET

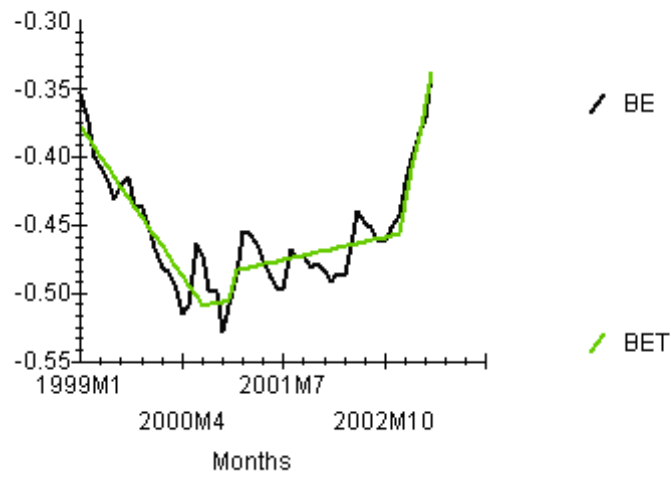


Fig. 3. Monthly series of BE ( $be_t$ ) and BET

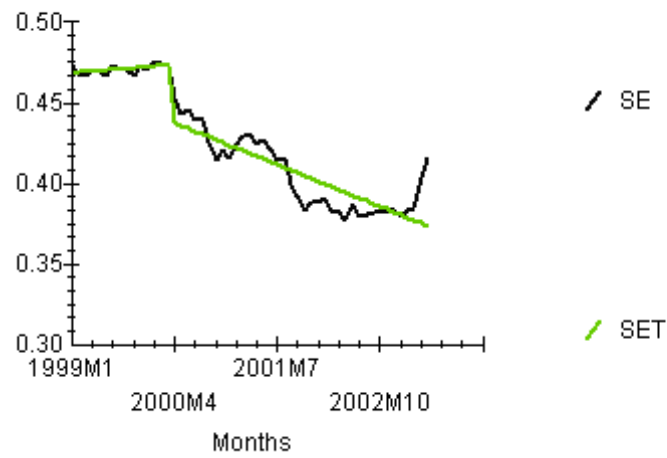


Fig. 4. Monthly series of SE ( $se_t$ ) and SET

$$D(T_b)_t = \begin{cases} 1 & \text{dla } t = T_b + 1, \\ 0 & \text{dla } t \neq T_b + 1, \end{cases} \quad DU_t = \begin{cases} 1 & \text{dla } t > T_b, \\ 0 & \text{dla } t \leq T_b, \end{cases} \quad DT_t^* = \begin{cases} t - T_b & \text{dla } t > T_b, \\ 0 & \text{dla } t \leq T_b, \end{cases}$$

are appropriate dummy variables, while  $\lambda_b$  denotes the known timing break fraction  $\lambda_b = T_b / T$ , ( $\lambda_b = 0.1, 0.2, 0.3, \dots, 0.9$ ; or an unknown timing break fraction ( $0 < \lambda_b < 1$ )).

The following ordinary least squares (OLS) equation of  $pe_t$  is selected:

$$p\hat{e}_t = 0.542 - 0.002t + 0.005DT_t^* + 0.637pe_{t-1} + 0.509\Delta pe_{t-1}, \quad (1a)$$

(4.35)    (-3.55)    (3.82)    (-4.40)    (4.37)

$$R^2 = 0.916, \quad DW = 1.87, \quad h = 0.55 [0.58], \quad T_b = 31,$$

where  $t$ -values are in parentheses (at  $pe_{t-1}$   $t$ -value for  $\hat{\rho}$  is given – this convention is used below in all the unit root equations),  $R^2$  is the determination coefficient,  $DW$  is the Durbin-Watson statistic,  $h$  is (normally distributed) the Durbin statistic, while in square parentheses the  $p$ -value is given (i.e. the significance level relating to  $h$ ). An unknown timing (endogenous) break in the trend in July 2001 was estimated and growing tendency in the zloty–euro exchange rate appeared at a level of about 0.3 (=0.5–0.2)% per month. Zivot-Andrews' test critical value (ZA) for the case of an unknown timing break in trend, at 5% significance level<sup>5</sup> equals to  $-4.42$  and it is slightly lower than the empirical  $t$ -value at  $pe_{t-1}$ , i.e.,  $-4.40$ . Consequently, there is no reason to reject the null hypothesis of the unit root in  $pe_{t-1}$  (the alternative hypothesis that this variable is stationary with a shift in the trend could be accepted at a 10% significance level because in that case the critical value equals  $-4.11$ ). The first differences of  $pe_t$  are stationary and, thus, this variable is integrated of order 1.

The above result motivates us to carry out a multivariate analysis. We should determine the orders of integration of variables, which can potentially be included in the cointegrating relationship (in which the linear combination of these variables is stationary). Consequently, the following testing equations of  $ue_t$ ,  $be_t$  and  $se_t$  are selected:

$$u\hat{e}_t = 0.018 - 0.002t + 0.059D(T_{00.06})_t + 0.048D(T_{00.12})_t + 0.052D(T_b)_t + 0.006DT_t^* + 0.788ue_{t-1} + 0.351\Delta ue_{t-k}, \quad (1b)$$

(1.23)    (-2.13)    (3.02)    (2.45)    (2.76)    (2.96)    (-2.29)    (2.86)

$$R^2 = 0.957, \quad DW = 2.04, \quad h = -0.19 [0.85], \quad T_b = 31,$$

<sup>5</sup> See: Zivot and Andrews (1992). The critical values of the ZA test used below come from this cited paper.

$$\begin{aligned}
b\hat{e}_t = & -0.022 + 0.001t + 0.045D(T_{00.06})_t - 0.028D(T_{00.08})_t - 0.035D(T_{00.10})_t \\
& \quad \quad \quad (-1.25) \quad (5.27) \quad (4.03) \quad (-2.55) \quad (-3.18) \\
& + 0.034D(T_{01.08})_t + 0.045D(T_b)_t - 0.021DU_t + 0.993be_{t-1}, \\
& \quad \quad \quad (3.09) \quad (4.02) \quad (-3.22) \quad (-0.18) \\
R^2 = & 0.942, \quad DW = 1.62, \quad h = 1.42 [0.15], \quad T_b = 24,
\end{aligned} \tag{1c}$$

$$\begin{aligned}
s\hat{e}_t = & 0.007 + 0.0002t - 0.013D(T_b)_t - 0.008DU_t + 0.981se_{t-1}, \\
& \quad \quad \quad (1.25) \quad (1.25) \quad (-1.83) \quad (-2.38) \quad (-2.38) \\
R^2 = & 0.972, \quad DW = 2.01, \quad h = -0.04 [0.97], \quad T_b = 15,
\end{aligned} \tag{1d}$$

where subscript  $b$  at additional impulse variables is substituted by the date of break (two last figures of year and month). Estimated in (1b)–(1d) trend coefficients are  $-0.2\%$ ,  $0.1\%$  and  $0.2\%$ . An unknown timing shift in the trend of  $ue_t$  ( $+0.6\%$ ), shifts in the mean of  $be_t$  ( $-2.1\%$ ) and  $se_t$  ( $-0.8\%$ ) appeared in July 2001, December and March 2000, respectively. A few impulse variables in (1b) and (1c) allow to estimate one-time effects of strong disturbances (of maximum order  $+5.2\%$  and  $+4.5\%$ , respectively). They have remarkable influence already at a 1% significance level. It may suggest the existence of exogenous breaks. At last we treat them in unit root testing as endogenous breaks strengthening our valuation criterion. Empirical  $t$ -values at variables  $ue_t$  and  $be_t$ , i.e.  $-2.29$  and  $-0.18$  are distinctly higher than the corresponding critical values of the ZA test (at 5% significance level, i.e.  $-4.42$  and  $-4.80$ ), and thus, all these variables are nonstationary. Their first differences are already stationary, and therefore, they are integrated of order 1. Furthermore, we re-estimate (using OLS) equations (1b)–(1d) including variable  $pe_{t-1}$ . This new regressor in each case appears not to be causal in Granger sense<sup>6</sup> at a 5% significance level.

The above results give a chance to find a cointegrating relationship which includes the considered above exogenous variables. Such a relation for variable  $y_t$  can be written as:

$$y_t = \mu_0 + \beta_0 t + \delta_0 D(T_c)0_t + \theta_0 DU0_t + \gamma_0 DT0_t^* + x_t d + \xi_t, \tag{2}$$

where  $\mu_0$ ,  $\beta_0$ ,  $\delta_0$ ,  $\theta_0$ ,  $\gamma_0$ ,  $d$ ,  $\xi_t$  are: intercept, trend parameter, impulse break effect, shift in mean, shift in trend, vector of parameters of strongly exogenous variables ( $x_t$ ) and disturbance term, while dummy variables are given as:

<sup>6</sup> It means, that determining some forecasts, based on these relations, we shouldn't use information relating to the past values of an endogenous variable; see: Granger (1969).

$$D(T_c)0_t = \begin{cases} 1 & \text{dla } t = T_c + 1, \\ 0 & \text{dla } t \neq T_c + 1, \end{cases} \quad DU0_t = \begin{cases} 1 & \text{dla } t > T_c, \\ 0 & \text{dla } t \leq T_c, \end{cases} \quad DT0_t^* = \begin{cases} t - T_c & \text{dla } t > T_c, \\ 0 & \text{dla } t \leq T_c, \end{cases}$$

where  $\lambda_c (= T_c / T)$  is known ( $\lambda_c = 0,1, 0,2, 0,3, \dots, 0,9$ ) or an unknown ( $0 < \lambda_c < 1$ ) break fraction.

Using appropriate statistical criteria the following OLS distributed lag model (DL) is selected:

$$\begin{aligned} p\hat{e}_t = & 1.066 - 0.001t + 0.056 D(T_{99.10})0_t + 0.080 D(T_{00.10})0_t + 0.045 D(T_{00.11})0_t \\ & - 0.053 D(T_{01.06})0_t + 0.041 D(T_{01.09})0_t + 0.043 D(T_{02.07})0_t \\ & + 0.052 D(T_{02.09})0_t + 0.013 DT0_t^* + 0.928 u\hat{e}_t - 0.805 b\hat{e}_{t-1}, \\ R^2 = & 0.918, \quad DW = 1.20, \quad T_b = 48, \end{aligned} \quad (2a)$$

where fitted values of  $ue_t$  and  $be_t$  are obtained in estimation of (1b) and (1c), treated as marginal equations. We find that the estimate of endogenous shift of +1.3% in trend of  $pe_t$  occurred in December 2002 (at a decreasing trend of order 0.1% monthly, to this break point). Now, this shift is stronger and occurs later than in equation (1a). The re-estimated (OLS) version of equation (2a) including residual regressors from (1b)–(1d) is as follows:

$$\begin{aligned} p\hat{e}_t = & 1.024 - 0.001t + 0.051 D(T_{99.10})0_t + 0.075 D(T_{00.10})0_t \\ & + 0.041 D(T_{00.11})0_t - 0.056 D(T_{01.06})0_t + 0.044 D(T_{01.09})0_t \\ & + 0.062 D(T_{02.07})0_t + 0.048 D(T_{02.09})0_t + 0.016 DT0_t^* \\ & + 0.949 u\hat{e}_t - 0.904 b\hat{e}_{t-1} - 1.015(ue_t - u\hat{e}_t) + 0.126(be_{t-1} - b\hat{e}_{t-1}), \\ R^2 = & 0.929, \quad DW = 1.26, \quad T_b = 48, \end{aligned} \quad (2a')$$

where regressor  $(ue_t - u\hat{e}_t)$  has significant influence on  $pe_t$  ( $t = -5.16$ ), therefore,  $ue_t$  is not weakly exogenous, and thus not strongly exogenous. It inclines us to undertake further attempts to find a cointegrating relationship.

Then, we will check whether it might be achieved by using the following (OLS) autoregressive distributed lag model (ADL) of  $pe_t$ :

$$\begin{aligned}
p\hat{e}_t = & 0.279 - 0.0003t + 0.035D(T_{99,08})0_t - 0.029D(T_{00,03})0_t \\
& \quad (2.73) \quad (-1.50) \quad (2.36) \quad (-2.03) \\
& + 0.046D(T_{00,10})0_t + 0.060D(T_{01,07})0_t + 0.039D(T_{02,07})0_t \quad (2b) \\
& \quad (3.12) \quad (3.93) \quad (2.67) \\
& + 0.005DT0_t^* + 1.046pe_{t-1} + 0.414pe_{t-2} + 0.446ue_t - 0.541b\hat{e}_{t-1}, \\
& \quad (1.79) \quad (9.68) \quad (-4.95) \quad (5.87) \quad (-4.76) \\
R^2 = & 0.966, \quad DW = 1.24, \quad T_b = 48,
\end{aligned}$$

where the date of endogenous shifts in trend is the same as in (2a). Now, estimates of the trend coefficient (0,03%) and shift in the trend (0.5%,) are corrected due to the inclusion of  $pe_{t-1}$  and  $pe_{t-2}$ . Equation (2b) is re-estimated after adding residuals of  $ue_t$  and  $be_t$  from (1b) and (1c) as regressors. Estimation results are as follows:

$$\begin{aligned}
p\hat{e}_t = & 0,313 - 0,0005t + 0,038D(T_{99,08})0_t - 0,031D(T_{00,03})0_t + 0,049D(T_{00,10})0_t \\
& \quad (3,25) \quad (-2,26) \quad (2,59) \quad (-2,35) \quad (3,58) \\
& + 0,052D(T_{01,07})0_t + 0,046D(T_{02,07})0_t + 0,007DT0_t^* + 1,036pe_{t-1} - 0,469 \\
& \quad (3,64) \quad (3,21) \quad (2,26) \quad (10,30) \quad (-5,97) \\
& \cdot pe_{t-2} + 0,525ue_t - 0,676be_{t-1} - 0,266(ue_t - u\hat{e}_t) + 0,023(be_{t-1} - b\hat{e}_{t-1}), \\
& \quad (6,98) \quad (-5,91) \quad (-1,96) \quad (0,10) \\
R^2 = & 0,973, \quad DW = 2,04, \quad T_b = 48, \quad (2b')
\end{aligned}$$

Both new regressors are insignificant at a 5% level. Therefore, variables  $ue_t$  and  $be_t$  are weakly exogenous. It suggests considering a single equation model, i.e. with the exclusion of marginal equations. Consequently, the following OLS equation  $pe_t$  is selected:

$$\begin{aligned}
p\hat{e}_t = & 0,457 - 0,001t + 0,035D(T_{99,08})0_t + 0,040D(T_{00,10})0_t \\
& \quad (3,55) \quad (-2,44) \quad (2,41) \quad (2,78) \\
& + 0,052D(T_{01,07})0_t + 0,041D(T_{02,07})0_t + 0,009DT0_t^* + 1,103pe_{t-1} \quad (2c) \\
& \quad (3,57) \quad (2,94) \quad (2,54) \quad (11,55) \\
& + 0,468pe_{t-2} + 0,485ue_t - 0,633be_{t-1} - 0,456se_t, \\
& \quad (-5,80) \quad (6,73) \quad (-5,44) \quad (-2,00) \\
R^2 = & 0,970, \quad DW = 2,03, \quad T_b = 48,
\end{aligned}$$

where the date of the endogenous shift in the trend is the same as in (2a) and (2b). The estimates of this shift (+0.9%) and trend coefficient (monthly decres 0.1%) are different. ADL models in (2b) and (2c) have, e.g., distinctly better fitting than the DL model in (2a). It should be noted that due to the fact that variables  $ue_t$ ,  $be_t$  and  $se_t$  are weakly exogenous, and  $pe_{t-1}$  is not their cause, these three variables are strongly exogenous. As a result, the suggested OLS long-run relationships (LRR) based on the ADL models is the following:



$$\begin{aligned}
p\hat{e}_t = & 0.758 - 0.001t + 0.096D(T_{99.08})0_t - 0.080D(T_{00.03})0_t \\
& + 0.126D(T_{00.10})0_t + 0.162D(T_{01.07})0_t + 0.106D(T_{02.07})0_t \\
& + 0.015DT0_t^* + 1.213u\hat{e}_t - 1.471b\hat{e}_t,
\end{aligned} \tag{2*}$$

$$\begin{aligned}
p\hat{e}_t = & 1.250 - 0.004t + 0.095D(T_{99.08})0_t + 0.110D(T_{00.10})0_t \\
& + 0.142D(T_{01.07})0_t + 0.112D(T_{02.07})0_t + 0.025DT0_t^* \\
& + 1.327ue_t - 1.248be_t - 1.731se_t.
\end{aligned} \tag{2**}$$

We consider using single equation tests for cointegration. They include residual-based tests and error correction model (*ECM*) tests. The tests from the first group were discussed by Engle and Granger (1987). The latter ones are described in Kremers, Ericsson and Dolado (1992). Below we apply extensions of both groups of testing procedures proposed by Krauze (2002). The residual-based version of cointegration test postulates using the OLS residuals ( $\hat{\xi}_t = pe_t - p\hat{e}_t$ , where  $p\hat{e}_t$  are fitted values from LRR) in the equation of the Dickey-Fuller type (*CDF*), i.e.:

$$\Delta\hat{\xi}_t = b\hat{\xi}_{t-1} + \zeta_t, \tag{3a}$$

where  $b$  ( $-2 \leq b \leq 0$ ) is the autoregressive parameter, and  $\zeta_t$  is the disturbance term. Next, the *ECM* type of the test uses the following testing equation:

$$\Delta pe_t = a\Delta p\hat{e}_t + b\hat{\xi}_{t-1} + \varepsilon_t, \tag{3b}$$

where  $b$  is the error correction coefficient,  $a$  is the short-run parameter ( $0 \leq a \leq 1$ ),  $\varepsilon_t$  is the disturbance term, while  $\zeta_t$  and  $\varepsilon_t$  are independent from  $\hat{\xi}_t$ . It is assumed that variables in a cointegrating relationship are integrated of order 1. We test  $b = 0$  (variables are not cointegrated) against the alternative  $-2 < b < 0$  (variables are cointegrated). Estimated (OLS) equations (3a) and (3b), corresponding to (2\*) and (2\*\*), are given as follows:

$$\Delta\hat{\xi}_t = \underset{(-5.79)}{-0.797}\hat{\xi}_{t-1} + \hat{\zeta}_t, \quad DW = 1.95, \tag{3a*}$$

$$\Delta pe_t = \underset{(7.71)}{0.338}\Delta p\hat{e}_t - \underset{(-7.79)}{0.481}\hat{\xi}_{t-1} + \hat{\varepsilon}_t, \quad DW = 1.20, \tag{3b*}$$

$$\Delta\hat{\xi}_t = \underset{(-5.90)}{-0.820}\hat{\xi}_{t-1} + \hat{\zeta}_t, \quad DW = 1.95, \tag{3a**}$$



logarithm 4.4500 and 4.3723 (zł/euro), respectively. The real value of this ratio in June was 4.4373, therefore, its overestimation and underestimation reached 0.29% and 1.46%, respectively. Thus, the first forecast, based on equation (2b), is distinctly more precise. However, we should remember that one-period forecasting horizon does not allow for any generalization.

### 3. Conclusions

Shifts in trend of the monthly series of Polish zloty–euro, US dollar–euro, British pound–euro and Swiss franc–euro rates in July and again July 2001, December and March 2002, respectively, were detected. All these series were nonstationary and integrated of order one. Basing on the ADL model we determined the long-run relationship. We proved that prices of the US dollar, British pound and Swiss franc in euro were weakly and strongly exogenous, cointegrated with the Polish zloty–euro rate. Two long-run relationship specifications were proposed, i.e. conditional model (including marginal process) and single-equation model. Basing on these models ex-post forecasts of zloty–euro rate were calculated. Conditional model was more useful. However, a sufficiently longer period of empirical verification of forecasts is necessary for formulating some generalization.

Further studies aiming at more precise recognition of regularities characterizing behaviour of exogenous variables are advisable. It has key meaning for achieving success in the prediction process of the zloty–euro exchange rate. Some experiences from analysis of ex post predictive information would be also very useful.

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