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Space-Time Modelling of the Unemployment Rate in Polish Poviats

A b s t r a c t. The purpose of the article is to model the unemployment rate in Poland in its spatial and time dimensions. The spatial lag models with the neighbourhood matrix based on the common border of poviats were used in the study. The analysis of the changes over time of the parameters in the space models gave a foundation to apply the space-time model with the parameters linear dependent on time. Such an approach enabled us to work out models with good statistical properties and with a possibility of providing a correct economic interpretation of the parameters.

K e y w o r d s: spatial econometrics, spatial model, space-time model.

Introduction

The authors show interest in modelling economic phenomena that are characterized by spatial dependencies. The spatial approach requires the use of specific methods and models taken from spatial econometrics. The term of spatial econometrics was introduced by J. H. P. Paelinck in 1974. Since that year the problems of spatial econometrics have been discussed in abundant works, such as, for instance, Clif and Ord (1981), Anselin (1988), Zeliaś (1991), Anselin, Florax and Rey (2004), LeSage and Pace (2004), Haining (2005), Arbia (2006), Szulc (2007), Bivand, Pebesma and Gómez-Rubio (2008), LeSage and Pace (2009), Suchecki (2010). In the present paper the authors will conduct a spatial analysis of the unemployment in Poland with a view to researching its properties and existing dependencies within the selected determinants. A correct description of this phenomenon, regarded as one of the major social and economic problems, constitutes a significant task of spatial econometrics.

The article is an attempt to present two research approaches used in modelling of economic processes, i.e. the spatial and space-time modelling the processes. The models falling into the former category were applied to provide a spatial description of the character of the unemployment rate, though only in its static aspect. The models consider the spatial dependencies showing the influence of the unemployment observed in one poviat on the unemployment level in other poviats. In addition, the global spatial tendency of the phenomenon was characterized by means of the spatial trend. The dependencies between the unemployment rate and the spatial processes defined as ‘determinants’ were adequate interpreted.

The purely spatial analysis of the unemployment rate carried out in the years 2004–2009 constituted a starting point for conducting a dynamic spatial analysis. It was observed that the models in specific years possessed some common features such as the same spatial trend and a similar parameter of the spatial autocorrelation. These settlements formed a basis for the space-time approach also referred to as ‘dynamic’.

The next step was to estimate the space-time model for the unemployment rate in Polish poviats in the years 2004–2009, with time-varying parameters of the space-time trend and the structural parameters. The analysis of the parameters showed linear dependence with regard to time. Consequently, the final version of the hypothetical space-time model with parameters that change linearly relative to time was selected. This choice allowed the number of the parameters of the model to be significantly limited. The space-time approach is infrequently used in spatial econometrics, though it allows the enriching of spatial analyses with a time dimension. The methodology relating to space-time modelling and few of applications can be found e.g. in the works of Anselin (1988), Szulc (2007), LeSage and Pace (2009).

1. Methodology

The models used for the purposes of the paper are to reflect the spatial and space-time dependencies relative to the unemployment rate. The models that were applied included static ones as well as two types of dynamic spatial models. Throughout the present paper n denotes the number of the ordered units of space. The letters X and Y denote the processes observed in space only (the static version) or in space and time (the dynamic version) and they possess either a single or double index dependent on their use.

One of the methods of taking into consideration the spatial relations observed in economic processes is to introduce into the formal description the so-called the spatial weights matrix (denoted as $\mathbf{W} = (w_{ij})$, $i, j = 1,...,n$). This matrix determines the intensity of connections between separated areas of space. Usually the form of the matrix can reflect only the geographical structure of the researched area; however, it may consider some other characteristics (properties) of the space, such as economic ones (see Anselin, 1988). For the dynamic models our assumption is that the spatial matrix is constant over time.

With the specified **W** matrix, we can determine a lag operator for the spatial process *Y*, which determines the influence on the observation of the process in the given location $i = 1,...,n$ of the observations in other locations, that is:

- $(\mathbf{W}Y)_i = \sum_{j=1,...,n} w_{i,j} Y_j$ – in the spatial version,
- $(\mathbf{W}Y_t)_i = \sum_{j=1,...,n} w_{i,j} Y_{jt}$, where $Y_t = (Y_{it})_{i=1,...,n}$ – in the space-time version.

Moreover, it is assumed that for a given area it is possible to determine the global force of the relations occurring in the process between various locations. This is the so-called spatial autocorrelation of the process, incorporated into the model by adding the spatial autoregression component with the parameter *ρ*.

When defining models the so-called spatial or space-time white noise *ε*, i.e. collections of the uncorrelated random variables with the following properties:

$$\varepsilon_i \sim N(0, \sigma^2), \quad i = 1,...,n, \tag{1}$$

$$\varepsilon_{it} \sim N(0, \sigma^2), \quad i = 1,...,n \text{ and } t = 1,...,T, \tag{2}$$

are used.

In the static part of the research the spatial lag model was applied and it had the following form (see Anselin,1988):

$$Y_i = \rho \cdot (\mathbf{W}Y)_i + \alpha X_i + \varepsilon_i, \tag{3}$$

where:

$i = 1,...,n$ stands for the number of the location,

$\mathbf{W} = (w_{ij})$, $i, j = 1,...,n$ is the weight matrix,

ε_i is the spatial white noise.

For the dynamic part of the research two types of models were applied. The first model is autoregressive with time-varying structural parameters and with constant spatial autocorrelation and it is defined as follows:

$$Y_{it} = \rho \cdot (\mathbf{W}Y_t)_i + \alpha_t X_{it} + \varepsilon_{it}, \tag{4}$$

where:

$i = 1,...,n$ is the number of the location,

$t = 1,...,T$ is the time index,

$\mathbf{W} = (w_{ij})$, $i, j = 1,...,n$ is the weight matrix,

ε_{it} is the white space-time noise.

Due to its slight research value the above model constitutes rather a starting point for further research and not a final form of the model that would be ready for adaptation.

The other model under consideration is a peculiar case of the model (4), in which changes of coefficients for explanatory variables are linear relative to time, that is:

$$Y_{it} = \rho \cdot (\mathbf{W}Y_t)_i + (\alpha_0 + \alpha_1 t) X_{it} + \varepsilon_{it}, \quad (5)$$

where:

$i = 1,...,n$ is the number of the location,

$t = 1,...,T$ is the time index,

$\mathbf{W} = (w_{ij})$, $i, j = 1,...,n$ is the weight matrix,

ε_{it} is the white space-time noise.

In the empirical part of the research all the spatial and space-time models were extended by the introduction of the second explanatory variable and by the spatial time-varying trend.[1]

2. Empirical Results

The empirical research was conducted based on the statistical data obtained from the official Central Statistical Office website and they concerned the registered unemployment rate in Polish poviats ($n = 379$) at the end of the years 2004–2009 ($T = 6$). The research conducted earlier by the authors (see Müller-Frączek and Pietrzak (2011a, 2011b)) showed that the unemployment rate is characterized by strong spatial interdependencies, which justified the use of the models presented in this section for the purpose of the description of the phenomenon. Also, it was observed that the unemployment rate is subject to a global spatial trend; therefore, the models used in the empirical analyses were extended by the addition of the spatial trend.

For the purpose of this research the normalized weights matrix (**W**) was used and this matrix reflects the geographical neighbourhood of poviats in the sense of the common border line. Moreover, centres of gravity were determined for the poviats whose geographic coordinates were taken for the estimation of spatial trends.

The analysis of statistical data showed that among potential unemployment determinants only two were statistically significant. Table 1 presents economic processes and their designations used in the research.

Table 1. Economic spatial processes used in modelling the unemployment rate

Process	Designation
Unemployment rate	Y
Investment outlays in thousands of PLN per capita	X_1
One hundred of entities of the national economy per 10000 inhabitants	X_2

[1] The calculations were performed in R-CRAN.

2.1. Spatial Models

In the static approach the spatial model of the unemployment rate was estimated separately for each considered year. In all the cases of the estimation of the static models contained in this section, extended by the spatial trend, only the coefficients of a first degree trend turned out to be statistically significant. Therefore, the static models used for successive years took the following hypothetical form expressed by the following equation:

$$Y = \rho \cdot \mathbf{W}Y + a + bx + cy + \alpha X_1 + \beta X_2 + \varepsilon, \qquad (6)$$

where:

x and y are geographic coordinates for the centres of gravity of poviats,

$\mathbf{W} = (w_{ij})$, $i, j = 1,...,n$ is the weight matrix,

ε stands for the white noise.

Table 2 shows the estimated parameters of the models determined by the equation (6) for individual years, where the designations used were consistent with the equation.

Table 2. The results of the estimation of the models determined by the equation (6)

Parameter		Static models					
		2004	2005	2006	2007	2008	2009
a	Estimates	16,51	15,69	13,90	11,42	9,86	12,30
	p-value	≈0,00	≈0,00	≈0,00	≈0,00	≈0,00	≈0,00
b	Estimates	0,61	0,58	0,48	0,42	0,41	0,52
	p-value	≈0,00	≈0,00	≈0,00	≈0,00	≈0,00	≈0,00
c	Estimates	-0,84	-0,73	-0,62	-0,43	-0,30	-0,45
	p-value	≈0,00	≈0,00	≈0,00	≈0,00	0,02	≈0,00
α	Estimates	-1,20	-1,27	-1,07	-0,68	-0,49	-0,42
	p-value	≈0,00	≈0,00	≈0,00	≈0,00	≈0,00	≈0,00
β	Estimates	-0,73	-0,67	-0,59	-0,55	-0,53	-0,66
	p-value	≈0,00	≈0,00	≈0,00	≈0,00	≈0,00	≈0,00
ρ	Estimates	0,66	0,65	0,65	0,64	0,64	0,60
	p-value	≈0,00	≈0,00	≈0,00	≈0,000	≈0,00	≈0,00

The statistical quality of the models was examined by calculating the respective characteristics for the residuals. These were the following: the value of the determination coefficient, the value of the Global Moran's statistics, and significance of the parameters. The values of the above-mentioned characteristics are shown in Table 3. All of them prove a proper fit each model to the statistical data. The estimates of the parameters of the static models changes over time. However, prior to the year 2008 the changes had been subject to some regularity with the exclusion of the 2009 model. Its being different can be spotted especially in the parameters of the trend that reflects the main spatial tendency of the examined phenomenon. The years 2004–2008 in Poland were the period of economic prosperity. The unemployment rate was decreasing every subsequent year, investment and salaries were increasing. At that time Poland was perceived as a quickly developing country and was attracting foreign investors. The 2008 crisis slowed down all world economies significantly. Obviously, this affected the Polish economy. Because of that two models for the years 2004–2008 and 2004–2009 will be considered in the dynamic approach.

Table 3. The selected measures of quality of the models determined by the equation (6)

Characteristics	Static models					
	2004	2005	2006	2007	2008	2009
R^2	0,640	0,630	0,600	0,570	0,550	0,536
I	0,032	0,028	0,036	0,013	-0,003	0,003
$(I\text{-E}(I))/\text{S}(I)$	-0,003	-0,003	-0,003	-0,003	-0,003	-0,003
p-value	0,157	0,185	0,128	0,320	0,509	0,439

When analyzing the models presented in Table 2, it can be stated that up to the year 2008 the intercept was decreasing. That means that the unemployment rate was, globally, becoming lower and lower in the period 2004–2008. Moreover, the slope of the surface of the trend, determined by the *b* and *c* values, becomes less steep, so the disproportions in the unemployment rate between individual poviats decrease. These positive effects, as seen from an economic point of view, started to disappear in the year 2009.

Although the signs of the coefficients do not change with the spatial trend, their absolute values increase in relation to the previous year. The regression parameters α, β for all the examined years had negative values. This reflects the beneficial impact of the growth of investments made and the number of economic subjects on the decrease of the unemployment rate in poviats. However, it can be seen that in course of time the influence of the X_1 process is constantly decreasing and the slightly decreasing impact of the X_2 process changed its character in the year 2009.

In all the six models there can be noticed strong spatial autodependence, which gives the average value of the autoregression parameter ρ at the level of 0.63. Moreover, this dependency appeared to be quite stable over time, which resulted in the constant spatial autocorrelation being accepted in the dynamic models. A slight decrease in the autoregression parameter over time indicates the weakening of spatial dependencies of the unemployment rate along with the improvement in the level of social and economic development. This should be read as a positive phenomenon since the existing spatial dependencies create a negative mechanism countering changes in the spatial system of the unemployment rate.

2.2. Space-Time Models

Due to the change in the developmental trend of the major phenomena in Poland both space-time models used for the purposes of the present paper were estimated twice – with and without the inclusion of the observations from the year 2009 ($T = 6$ or $T = 5$).

First the spatial lag model with time-varying structural parameters, the trend parameters and the stable over time spatial autocorrelation was considered. What turned out to be statistically significant during the estimation was the parameters for the trend model of the first degree; hence the final hypothetical form of the model was as follows:

$$Y_t = \rho \cdot \mathbf{W}Y_t + a_t + b_t x + c_t y + \alpha_t X_{1t} + \beta_t X_{2t} + \varepsilon_t , \tag{7}$$

where:

$t = 1,...,T$,

$\mathbf{W} = (w_{ij})$, $i, j = 1,...,n$ is the weight matrix,

x and y are the geographic coordinates of the gravity centres of the poviats,

ε_t stands for the space-time noise.

Tables 4 and 5 include the results of the estimation of the parameters of the models designated by the equation (7) with and without the inclusion of the observations from the year 2009. The analysis of the results presented allows a statement to be formulated that the autoregression parameter and the parameters for the determinants model of the unemployment rate are statistically valid. Also, in the case of the spatial trend all the parameters proved to be statistically significant. It needs to be emphasized that in order to identify a major spatial trend it is enough to have a significant parameter at least for one of the spatial coordinates.

Table 4. The results of the estimation of the models determined by the equation (7) in the years 2004–2009

Parameter		Dynamic model					
		2004	2005	2006	2007	2008	2009
a_t	Estimates	16.87	13.71	12.45	11.60	11.30	11.69
	p-value	≈0.00	0.590	0.170	0.010	0.001	0.015
b_t	Estimates	0.68	0.63	0.52	0.44	0.43	0.50
	p-value	≈0.00	≈0.00	≈0.00	≈0.00	0.039	0.003
c_t	Estimates	-0.86	-0.74	-0.63	-0.44	-0.31	-0.45
	p-value	≈0.00	≈0.00	≈0.00	0.003	0.003	≈0.00
α_t	Estimates	-1.20	-1.27	-1.07	-0.68	-0.48	-0.41
	p-value	≈0.00	≈0.00	≈0.00	≈0.00	≈0.00	≈0.00
β_t	Estimates	-0.73	-0.67	-0.59	-0.56	-0.53	-0.65
	p-value	≈0.00	≈0.00	≈0.00	≈0.00	≈0.00	≈0.00
ρ	Estimates	0.65					
	p-value	≈0.00					

The statistical quality of both dynamic models was evaluated applying the same measures that were used for static models. The aggregate results are shown in Table 7 and they show that the models were well fitted to the empirical data and that the autocorrelation in the residuals did not occur.

Table 5. The results of the estimation of the models determined by the equation (7) in the years 2004–2008

Parameter		Dynamic model				
		2004	2005	2006	2007	2008
a_t	Estimates	16.68	16.14	15.74	15.33	15.28
	p-value	≈0.00	0.600	0.180	0.011	0.001
b_t	Estimates	0.67	0.62	0.51	0.43	0.42
	p-value	≈0.00	≈0.00	≈0.00	0.005	0.041
c_t	Estimates	-0.88	-0.74	-0.63	-0.43	-0.31
	p-value	≈0.00	≈0.00	0.001	0.003	0.004
α_t	Estimates	-1.20	-1.27	-1.07	-0.67	-0.48
	p-value	≈0.00	≈0.00	≈0.00	≈0.00	≈0.00
β_t	Estimates	-0.73	-0.66	-0.59	-0.55	-0.52
	p-value	≈0.00	≈0.00	≈0.00	≈0.00	≈0.00
ρ	Estimates			0.68		
	p-value			≈0.00		

The comparison of the results presented in Table 4 and Table 5 with the results contained in Table 2 shows that the values of the parameters of the two dynamic models are similar to the corresponding parameters of the static models. Consequently, their economic interpretation will be similar.

The analysis of the changes in the time of the parameters of the dynamic models indicates that for the model in which the last observations made (i.e., from the year 2009) were omitted, the changes are linear relative to time. The linear property concerns both the trend and the structural parameters of the model. The consequence of the assumption of the linear changes of the parameters over time was the acceptance of the theoretical model reflecting this property:

$$\begin{aligned} Y_t &= \rho \cdot \mathbf{W} Y_t + a_0 + a_1 t + (b_0 + b_1 t)x + (c_0 + c_1 t)y + \\ &+ (\alpha_0 + \alpha_1 t) X_{1t} + (\beta_0 + \beta_1 t) X_{2t} + \varepsilon_t, \end{aligned} \tag{8}$$

where:

$t = 1, ..., T$,

$\mathbf{W} = (w_{ij})$, $i, j = 1, ..., n$ is the weight matrix,

x and y are the geographic coordinates of the gravity centres of the poviats,

ε_t stands for the space-time noise.

Despite the fact that the analysis of the results presented in Table 4 and Table 5 showed that in 2009 the linear character of the changes of the parameters relative to time was altered, the estimation of the models designated by the equation (8) was made both with and without the inclusion of the observations from that year. The estimation results are shown in Table 6 and the residuals of both models are contained in Table 7.

For both models the autoregression parameter and the parameter for the per capita investment are statistically significant. In the case of the number of economic subjects the linear variation over time is insignificant for the model estimated in the years 2004-2009. However, for the years 2004–2008 it is just on the edge of significance. Attention must be paid to the fact that the analysis covered five or six years. Increasing the number of periods would certainly enhance the significance of the parameters in the estimated models. The estimate of the autoregression parameter indicates strong spatial dependencies of the unemployment rate and the estimates obtained for the parameters of the determinants show negative dependencies between the unemployment rate, investment, and the number of economic subjects per capita.

Table 6. The results of the estimation of the models determined by the equation (8)

Parameter	Model for the years 2004-2009		Model for the years 2004-2008	
	Estimates	p-value	Estimates	p-value
a_0	17.44	≈0.00	18.81	≈0.00
a_1	-1.28	≈0.00	-1.81	≈0.00
b_0	-0.91	≈0.00	-1.01	≈0.00
b_1	0.10	0.005	0.14	0.004
c_0	0.65	≈0.00	0.74	≈0.00
c_1	-0.04	0.210	-0.07	0.140
α_0	-1.57	≈0.00	-1.58	≈0.00
α_1	0.20	≈0.00	0.21	≈0.00
β_0	-0.70	≈0.00	-0.76	≈0.00
β_1	0.02	0.300	0.05	0.110
ρ	0.66	≈0.00	0.65	≈0.00

Table 7. The selected measures of quality of the models determined by the equation (7) and (8)

Property	Models described by the equation (7)		Models described by the equation (8)	
	2004-2009	2004-2008	2004-2009	2004-2008
R^2	0.69	0.70	0.69	0.69
I	0.019	0.023	0.012	0.022
$(I\text{-E}(I))/\text{S}(I)$	1.436	1.546	0.915	1.483
p-value	0.075	0.061	0.180	0.069

When analyzing the results presented in Table 6 and Table 7 it can be inferred that the observations from the year 2009 did not affect considerably the obtained models either in the statistical or the interpretational aspect. The acceptance of the observation for the years following 2009, where the economic slow down would be continued, would undoubtedly affect the evaluation of the parameters and the properties of the model. Both models are characterized by a high degree of fitting to the empirical data and the differences in the estimates of the parameters remain slight. If the change of the character of the unemployment rate observed in the year 2009 were to be of a lasting nature, then the model created on the basis of all the observations would certainly not be correct from a forecasting point of view, either.

Conclusions

The article discussed a spatial analysis of the registered unemployment rate in Polish poviats at the end years 2004–2009. Within the first approach, a static one, spatial models of the unemployment rate in subsequent years were estimated. Next, the analysis was enhanced by a time dimension and within the dynamic approach estimations were made of appropriate space-time models. A spatial linear trend as well as two determinants of the unemployment rate, investment made and the number of economic subjects per capita were assumed for all the models.

The analysis conducted allows the identification of a linear spatial trend and negative dependencies occurring between the unemployment rate, investment made and the number of economic subjects per capita.

For the trend parameters and assumed processes of determinants a linear character of the time-varying changes was determined. All of the estimated models were characterized by good statistical properties and economic interpretability of the parameters.

The question of the dissimilarity of the observations from the year 2009 remains unsettled. If the economic cycle in Poland and the related unemployment, level of investment and general condition of enterprises changed permanently, then the model with a linear dependency of time-varying parameters could result in an erroneous simplification of changes of the parameters.

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Przestrzenno-czasowe modelowanie stopy bezrobocia w Polsce

Z a r y s t r e ś c i. Celem artykułu jest modelowanie stopy bezrobocia w ujęciu przestrzennym i przestrzenno-czasowym. Dane wykorzystane w badaniach empirycznych dotyczyły powiatów Polski w latach 2004-2009. Wykorzystano modele o charakterze regresyjno-autoregresyjnym z macierzą sąsiedztwa opartą na zasadzie wspólnej granicy powiatów. Analiza zmian w czasie parametrów modeli przestrzennych dała podstawy do zastosowania modeli czasowo-przestrzennych z parametrami liniowo zależnymi od czasu. Podejście takie pozwoliło na otrzymanie modeli o dobrych własnościach statystycznych oraz charakteryzujących się poprawną ekonomiczną interpretowalnością parametrów.

S ł o w a k l u c z o w e: ekonometria przestrzenna, model przestrzenny, model przestrzenno-czasowy.