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Long memory or parameter inconstancy in GARCH models? An empirical illustration

1. Structural breaks or long memory?

It is well known that many financial data (log returns of stock indices, exchange rates, etc.) exhibit some peculiar properties which are often characterized as „long range dependence effect" (LRD). This includes the fact that the sample autocorrelation functions (ACF) of absolute and squared returns are found to be, they decay relatively fast for the first few lags, and at larger lags are low but decay very slowly (cf. Mikosch and Starica (2004)). The long range dependence effect can be related to the so called *IGARCH effect* whose name comes from the observation that the fitting of GARCH(1,1) models on log returns very often results in obtaining estimates such that $\alpha + \beta$ is close to one. This is true in particular for longer samples, in shorter subsamples the sum of both coefficients is found to be substantially smaller than one (see Bollerslev et al. (1992) and the references therein).

However, it has been argued that both LRD and IGARCH effects can be easily generated by nonstationarity due to shifts in unconditional variance rather than by the genuine long memory in the data. This view has been recently advocated by Mikosch and Starica (2004). Their work was anticipated by Lamoureux and Lastrapes (1990) who were among the first ones who noted the relationship between breaks in unconditional volatility and estimated volatility persistence. However, the latter did not provide any theoretical explanation of this phenomenon nor they offered systematic testing procedure able to detect such

such breaks. Some theoretical perspective can be found in Granger and Hyung (1999) and Diebold and Inoue (2001).

This issue is particularly relevant for financial data because it has been shown that the widely used GARCH(1,1) model given by

$$\varepsilon_t = h_t^{1/2}\xi_t$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$$

where $\{\xi_t\}$ is i.i.d. with zero mean and unit variance, is not flexible enough to capture these stylized facts (see Malmsten and Teräsvirta (2004), among others). One of the reasons is that GARCH(1,1) process is strongly mixing with geometrical rate which implies that it „forgets" its past quickly. The same applies to the processes of its absolute values and squares which should also exhibit short memory property, i.e. their sample ACF should vanish quite quickly. Note that for weakly stationary GARCH(1,1) models the change of the unconditional variance $\omega/(1-\alpha-\beta)$ corresponds to change in the parameters ω, α, β.

There are several tests of structural stability which can be applied at GARCH models. In this paper, we shall concentrate ourselves to Lundbergh and Teräsvirta (2002) (hereafter LT) and modified Inclan and Tiao (1994) (hereafter IT) tests. LT test of parameter constancy of GARCH model was derived as a part of a unified framework aimed to check the adequacy of an estimated GARCH model. As the alternative a model with smoothly changing parameters is considered:

$$h_t = \left[\omega_1 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}\right]\left[1 - F(t)\right] + \left[\omega_2 + \alpha_2\varepsilon_{t-1}^2 + \beta_2 h_{t-1}\right]F(t)$$

where $F(t) = \dfrac{1}{1+\exp(-\gamma t)}, \gamma > 0$ is the logistic transition function. Under the null $H_0 : \omega_1 = \omega_2, \alpha_1 = \alpha_2, \beta_1 = \beta_2$ the parameter γ of the transition function is not identified. Nonetheless, this can be circumvented by replacing the transition function by a first-order Taylor approximation. This yields the following auxiliary model

$$h_t = \delta_1 + \delta_2\varepsilon_{t-1}^2 + \delta_3 h_{t-1} + \delta_4 t + \delta_5\varepsilon_{t-1}^2 t + \delta_6 h_{t-1} t$$

Now the null becomes $H_0' : \delta_4 = \delta_5 = \delta_6 = 0$ which can be tested by means of the Lagrange multiplier test. In practice, the test is carried out using an artificial regression (for more details, see Lundbergh and Teräsvirta (2002)). Of course, it is possible to test the constancy of a subset of parameters only.

On the other side, the test suggested by Inclan and Tiao (1994) is a CUSUM-type test. Define the cumulative sum of squares $C_k = \sum_{t=1}^{k} a_t^2, k = 1,...,T$ and the corresponding centered and normalized variable $D_k = C_k / C_T - k / T$. Inclan and Tiao proposed the statistic $IT = \sup_k \left|\sqrt{T/2} D_k\right|$ whose asymptotic distribution can be shown to be $IT \xrightarrow{L} \sup_r \left|W^*(r)\right|$, where $W^*(r) \equiv W(r) - rW(1)$ is a Brownian bridge and *W(r)* is a standard Wiener process, under the assumption that a_t are normally and *iid* random variables with zero mean and constant variance. Whereas the assumption of normality is not crucial and can be easily relaxed, the independence requirement turns out to be a more serious issue. If the test is applied to conditionally heteroskedastic processes like GARCH, it suffers from severe size distorsions which precludes its application to financial time series. Therefore, there is a need for its modification which would cover this case of dependence in the data. Fortunately, Sansó et al. (2004) proposed the modified IT statistics $IT_m = \sup_k \left|T^{-1/2} \hat{\omega}_4^{-1/2} D_k\right|$ where $\hat{\omega}_4$ is some consistent estimator of long-run fourth moment of a_t. If multiple structural breaks are supposed to occur in the analysed series, the use of the iterated cumulative sum of squares (ICSS) algorithm is recommended (see Inclan and Tiao (1994) for a detailed description).

2. Data

In this paper, we concentrate ourselves on daily log-returns of Prague Stock Exchange index PX 50 and of Warsaw General Index WIG 20. The series covers the period between January 6,1995 and January 3, 2005 for PX 50 and between January 3, 1995 and December 30, 2004 for WIG 20, so that we have 2490 and 2608 observations, respectively. The both series are plotted in Figure 1 and Table 1 presents the descriptive statistics[1].

[1] The dataset was obtained from the website of the Prague Stock Exchange and from Datastream. The analysis of the series has been carried out using GAUSS 6.0 (the purchase of the software was funded by the grant of FRVŠ nr. 1306Ab/2004). To a large extent, we built on the code written by Dick van Dijk and Andreu Sansó.

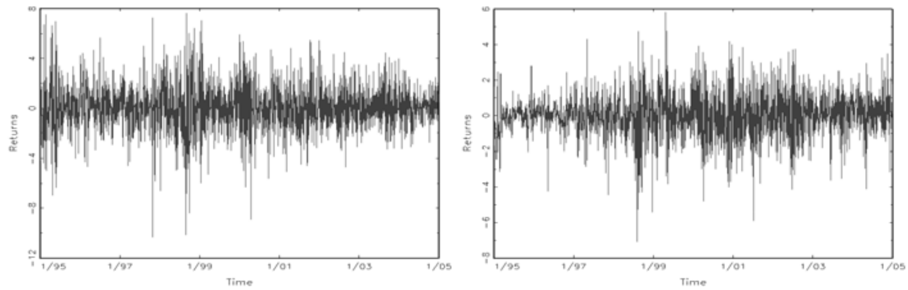

Figure 1. Plots of log-returns of WIG 20 and PX 50

Table 1. Summary statistics for daily returns

	Mean	Minimum	Maximum	St. dev.	Skewness	Kurtosis
WIG 20	0.0385	-10.3231	7.8669	1.8314	-0.0496	5.4517
PX 50	0.0246	-7.0772	5.8200	1.2055	-0.1867	5.0360

We can also look at the sample autocorrelations functions of both series if we are interested to check whether the „long-memory" effect is present (see Figure 2). We can note that for PX 50 the sample ACF decays indeed very slowly remaining positive at large lags, however, for WIG 20 this effect is much weeker, if any.

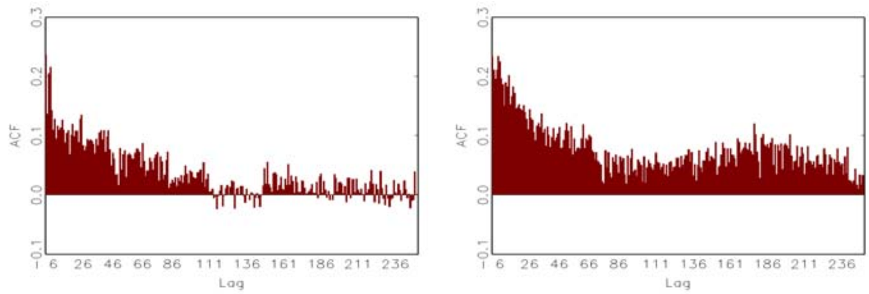

Figure 2. Sample autocorrelation functions of absolute log-returns for WG20 (left panel) and PX50 (right panel)

Furthermore, an AR(1)-GARCH(1,1) model was estimated. The results are not reported here in order to save space but they are available upon request. However, it is worth mentioning that the estimate of the sum $\alpha + \beta$ is equal to 0.9739 in case of WIG 20 and 0.9892 for PX 50 which suggests a high amount of persistency in the GARCH process, especially for the Prague stock exchange index. In order to check whether this could be caused by nonstationarity in unconditional variance, we performed both LT and modified IT tests. The results are reported in Table 2 and 3, respectively. The LT test rejects the null of parameter constancy in both cases which indicates that the models are misspecified[2]. However, LT test is not informative about the location of the possible break (or multiple breaks) in the analysed series. Modified IT test suggests five breaks for WIG20 returns and four breaks for PX 50 index.

Table 2. P-values of Lundbergh-Teräsvirta test of parameter constancy (χ^2 form)

test	WIG 20	PX 50
intercept	0.020	0.004
ARCH parameter	0.8441	0.758
all parameters	0.1168	0.016

Table 3. Location of breaks in unconditional variance

WIG 20	5.6.1995 (110)	23.10.1997 (733)	29.1.1999 (1064)	5.2.2002 (1851)	18.3.2004 (2403)
PX 50	29.3.1995 (52)	29.3.1995 (504)	1.7.1998 (857)	5.8.2002 (1886)	

Changing volatility can be confirmed by looking at the evaluation-weighted estimate for volatility which was obtained using one-sided estimator with normal kernel (see Figure 3). For instance, for case of Prague stock index, the picture reflects slowly growing volatility between the years 1995 and 1998 and the consequent sudden rise. On the contrary, the last period beginning at the end of 2002 seems to be more tranquile. For WIG 20, the most volatile period seems to be that one located between second and third break (i.e. between October 23, 1997, and January 29, 1999) and after that we observe a gradual decline in the level of volatility. Nonetheless, contrary to the PX 50 case, for Warsaw stock index the effect of time-varying variance does not seem to be strong enough to cause a spurious long-memory effect.

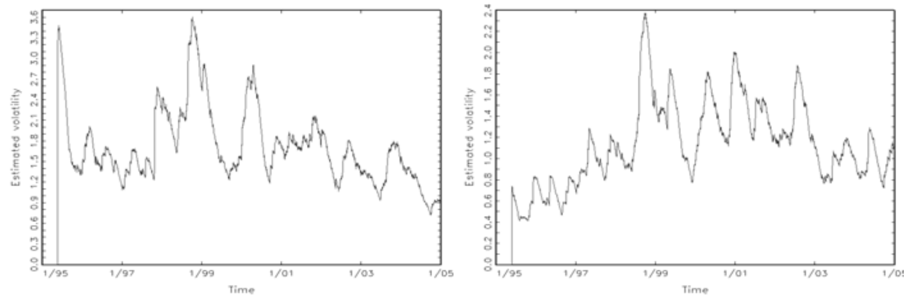

Figure 3. Estimated time-varying standard deviation using one-sided smoothing for WG20 (left panel) and PX50 (right panel)

[2] Moreover, in case of WG20, there seems to be remaining ARCH effect not captured by our GARCH(1,1) model.

3. Conclusion

The adequacy of GARCH models widely used in applied econometric research has been called into question in the last years. Since they often fail to capture stylized facts observed in real data, one possible solution is to „augment“ the baseline GARCH model with more sophisticated features like more complex nonlinearity or fractional integration which should allow for a richer dynamics and which are supposed to be relevant. However, there exists another explanation why GARCH models possibly do not do their job well and this stresses the role of structural breaks in the data. In fact, both approaches have distinct implications for forecasting: a genuine long memory (generated typically by fractional integration) implies that even observations in the distant past have influence on the present and therefore are still relevant for forecasting whereas the presence of structural breaks suggests exactly the opposite. For this reason, it is very likely that this problem will keep the academic community busy in the future.

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